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Statistical and Optimization-Based Analysis of Gain and Noise Performance in Cascaded Communication Systems Using Friis Formula

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Abstract: This work presents a systematic analysis of gain and noise behavior in cascaded communication systems using the Friis formulation. A modular simulation framework is developed in Python to evaluate total gain and noise figure across multi-stage configurations. The model considers variable stage count, realistic gain and noise distributions, and strict linear–logarithmic unit consistency. Parametric analysis is performed by varying first-stage gain and noise figure over a wide operating range. Results indicate strong dominance of the first stage, where increased front-end gain suppresses the contribution of subsequent noisy stages. A saturation trend is observed beyond a threshold gain region, where further improvement in total noise figure becomes marginal. A Monte Carlo simulation with more than 1000 iterations is conducted to examine statistical variability under practical uncertainties in gain and noise parameters. The obtained mean noise figure of 3.17 dB with a standard deviation of 1.16 dB highlights significant sensitivity of cascaded systems to early-stage variations. An optimization study is carried out to identify the first-stage gain minimizing overall noise figure, yielding an optimal value near 27.88 dB with a minimum achievable noise figure of 1.15 dB under unconstrained conditions. The analysis confirms that system noise performance asymptotically approaches the first-stage noise limit. The proposed framework provides reproducible evaluation, comparative visualization, and statistical insight suitable for RF front-end design studies. The results emphasize the critical role of front-end design and establish a basis for further constrained and multi-objective optimization in practical communication systems.

Keywords: *Cascaded Systems; Friis Formula; Noise Figure; RF Front-End; First-Stage Dominance; Monte Carlo Simulation; Gain Optimization; Statistical Variability; Sensitivity Analysis; Communication Systems.*

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I. INTRODUCTION

The performance of modern communication receivers is strongly influenced by noise accumulation across cascaded stages. In practical RF front-end architectures, multiple components such as low-noise amplifiers (LNA), mixers, and intermediate frequency stages are connected in cascade, where each stage contributes both gain and noise. The overall system behavior cannot be evaluated by simple additive models, as noise propagates nonlinearly through the cascade. The classical Friis formulation provides a rigorous framework to quantify total noise factor in such systems, explicitly showing the dependence on individual stage gains and noise figures [1].

In cascaded configurations, the first stage plays a dominant role in determining the overall noise performance. A high-gain, low-noise front-end significantly suppresses the contribution of subsequent stages, whereas a poorly designed first stage leads to rapid degradation of system sensitivity. This phenomenon, often referred to as first-stage dominance, has been widely discussed in RF system design and remains a critical consideration in receiver optimization [2]. However, most conventional analyses assume deterministic parameters and do not account for statistical variability inherent in practical hardware implementations.

Recent advancements in communication system design emphasize the need for stochastic modeling and robustness evaluation. Component-level uncertainties, manufacturing tolerances, and environmental variations introduce fluctuations in gain and noise parameters, which can significantly affect system-level noise performance. Monte Carlo-based approaches provide a systematic method to capture such variations and enable statistical characterization of cascaded systems [3]. Despite this, there exists limited integration of parametric analysis, statistical evaluation, and optimization within a unified simulation framework.

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Furthermore, optimization of cascaded systems is typically constrained by trade-offs between gain, noise figure, power consumption, and stability. While increasing the first-stage gain improves noise performance, it may introduce practical limitations such as nonlinearity and instability. Therefore, identifying an optimal operating region is essential for balanced system design. Existing studies often focus on analytical derivations or hardware-specific implementations, with limited emphasis on reproducible simulation-driven optimization [4].

In this work, a structured and reproducible simulation framework is developed to analyze gain and noise behavior in cascaded communication systems using the Friis formula. The study incorporates parametric variation, Monte Carlo simulation, and optimization of first-stage gain to evaluate both deterministic and stochastic performance characteristics. The objective is to provide a comprehensive understanding of noise propagation mechanisms and to establish a foundation for practical RF front-end design under realistic constraints.

II. MATHEMATICAL FORMULATION AND PROBLEM DEFINITION

Consider a cascaded communication system comprising Nstages. Each stage is characterized by gain and noise parameters defined in both logarithmic (dB) and linear domains. Let the gain of the i^{th}stage be $G_i^{\text{dB}}$and the corresponding linear gain be G_i. Similarly, the noise figure in dB is NF_i^{dB}, and the corresponding noise factor is F_i.

The conversion between logarithmic and linear domains is given as:

$$G_i = 10^{\frac{G_i^{\text{dB}}}{10}}, F_i = 10^{\frac{NF_i^{\text{dB}}}{10}} \tag{1}$$

➢ *Total Gain of Cascaded System*

The overall gain of the cascaded system in linear scale is expressed as the product of individual stage gains:

$$G_{\text{total}} = \prod_{i=1}^{N} G_i \tag{2}$$

The total gain in dB is obtained as:

$$G_{\text{total}}^{\text{dB}} = 10\log_{10}(G_{\text{total}}) \tag{3}$$

➢ *Noise Factor using Friis Formula*

The total noise factor of a cascaded system is governed by the Friis equation:

$$F_{\text{total}} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots + \frac{F_N - 1}{\prod_{i=1}^{N-1} G_i} \tag{4}$$

This formulation clearly shows that the contribution of each subsequent stage is progressively reduced by the cumulative gain of preceding stages.

The total noise figure in dB is then expressed as:

$$NF_{\text{total}}^{\text{dB}} = 10\log_{10}(F_{\text{total}}) \tag{5}$$

➢ *Sensitivity of Noise Factor*

To quantify the dominance of the first stage, sensitivity of total noise factor with respect to individual stage noise factors is considered. For the first stage:

$$\frac{\partial F_{\text{total}}}{\partial F_1} = 1$$

For subsequent stages $i > 1$:

$$\frac{\partial F_{\text{total}}}{\partial F_i} = \frac{1}{\prod_{k=1}^{i-1} G_k} \tag{6}$$

This indicates exponential attenuation of sensitivity with increasing stage index, reinforcing the dominance of early stages.

➢ *Problem Definition*

The primary objective of this work is to analyze and optimize the noise performance of a cascaded communication system under both deterministic and stochastic conditions.

- *Given:*

✓ Number of stages $N \in [2,10]$
✓ Stage gains $G_i^{\text{dB}} \in [G_{\min}, G_{\max}]$
✓ Noise figures $NF_i^{\text{dB}} \in [NF_{\min}, NF_{\max}]$

- *Find:*

✓ Total gain $G_{\text{total}}$and total noise figure NF_{total}
✓ Statistical characteristics (mean, variance) of NF_{total}under parameter uncertainty
✓ Optimal first-stage gain G_1^{opt}that minimizes NF_{total}

➢ *Optimization Formulation*

The optimization problem is defined as:

$$\min_{G_1} NF_{\text{total}}(G_1, G_2, \ldots, G_N)$$

Subject to:

$$G_1^{\min} \le G_1 \le G_1^{\max}$$

For practical systems, a constrained formulation may be expressed as:

$$\min_{G_1} NF_{\text{total}} + \lambda \cdot C(G_1)$$

Where:

- $\lambda$is a weighting factor
- $C(G_1)$represents penalty due to excessive gain (e.g., power, nonlinearity)

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➢ *Stochastic Modeling*

To account for practical variations, gain and noise factors are modeled as random variables:

$$G_i = \bar{G}_i(1 + \delta_{G_i}), F_i = \bar{F}_i(1 + \delta_{F_i}) \tag{7}$$

Where:

- $\delta_{G_i}, \delta_{F_i} \sim \mathcal{N}(0, \sigma^2)$

The expected noise factor is evaluated as:

$$\mathbb{E}[F_{\text{total}}] = \frac{1}{M}\sum_{m=1}^{M} F_{\text{total}}^{(m)} \tag{8}$$

Where M is the number of Monte Carlo iterations.

➢ *Research Scope*

The formulation integrates:

- Deterministic Friis-based modeling
- Sensitivity-driven interpretation
- Stochastic evaluation using Monte Carlo methods
- Optimization of front-end parameters

This unified approach enables both theoretical insight and practical design evaluation for cascaded communication systems.

III. SIMULATION SETUP AND PARAMETER CONFIGURATION

The simulation framework is implemented in a structured and reproducible environment using Python, with NumPy for numerical computation and Matplotlib for visualization. All experiments are executed under fixed random seed conditions to ensure deterministic reproducibility of stochastic results.

➢ *System Configuration*

A cascaded communication system with variable number of stages N is considered. The number of stages is selected within the range:

$$N \in [2,10]$$

Each stage is modeled using two primary parameters:

- Gain G_i^{dB}
- Noise Figure NF_i^{dB}

The parameters are generated within realistic RF front-end ranges:

$$G_i^{\text{dB}} \in [5,20], NF_i^{\text{dB}} \in [1,5]$$

These values correspond to practical amplifier and mixed-stage behaviors observed in receiver chains.

➢ *Unit Consistency and Conversion*

All computations are performed in linear scale to maintain mathematical correctness. Conversion is applied as:

$$G_i = 10^{\frac{G_i^{\text{dB}}}{10}}, F_i = 10^{\frac{NF_i^{\text{dB}}}{10}} \tag{9}$$

Final results are converted back to logarithmic scale (dB) for interpretation and visualization.

➢ *Parametric Analysis Configuration*

- *First-Stage Gain Variation*

✓ First-stage Gain Varied Over:

$$G_1^{\text{dB}} \in [0,30]$$

✓ Remaining Stages:

▪ Randomly generated within defined ranges

- *Objective:*
  Evaluate NF_{total} sensitivity to G_1

✓ *Observation from Fig. 1:*

▪ Strong fluctuation due to randomized later stages
▪ General decreasing envelope trend with increasing G_1
▪ Confirms first-stage dominance but with stochastic disturbance

- *Number of Stages Variation*

✓ Stage count varied from 2 to 10
✓ Independent random configuration per stage count

✓ *Observation from Fig. 2:*

▪ Non-monotonic behavior observed
▪ Noise Figure increases on average with stage count
▪ Variation indicates dependency on gain distribution, not only stage count

➢ *Monte Carlo Simulation Setup*

To capture statistical variability:

- *Number of Iterations:*

$$M = 1000$$

- *Parameter Perturbation:*

$$G_i \leftarrow G_i \cdot \mathcal{N}(1,0.05), F_i \leftarrow F_i \cdot \mathcal{N}(1,0.05)$$

This represents ±5% Gaussian variation in practical components.

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➢ *Statistical Metrics*
From simulation:

- *Mean Noise Figure:*

$$\mu_{NF} = 3.17 \text{ dB}$$

- *Standard Deviation:*

$$\sigma_{NF} = 1.16 \text{ dB}$$

- *Interpretation:*

✓ High variance indicates sensitivity to parameter uncertainty
✓ Distribution (Fig. 3) approximates Gaussian with slight skewness

➢ *Optimization Configuration*

- *Search Space:*

$$G_1^{\text{dB}} \in [0,30]$$

- *Objective:*

$$\min \ NF_{\text{total}}$$

- *Result:*

$$G_1^{\text{opt}} = 27.88 \text{ dB}, NF_{\min} = 1.15 \text{ dB}$$

- *Interpretation:*

✓ Confirms theoretical expectation of high first-stage gain benefit
✓ Indicates asymptotic convergence toward first-stage NF

➢ *Implementation Details*

- *Language: Python (Collab-compatible)*
- *Libraries:*

✓ NumPy → numerical operations
✓ Matplotlib → plotting
✓ Pandas → data export

- *Random Seed Fixed:*

$$\text{seed} = 42$$

➢ *Summary of Configuration*

Parameter	**Value**
Stages N	2–10
Gain Range	5–20 dB
Noise Figure Range	1–5 dB
Monte Carlo Iterations	1000
Gain Variation	±5%
NF Variation	±5%
Optimization Range	0–30 dB

➢ *Research Significance of Setup*
The simulation framework integrates:

- Deterministic Friis modeling
- Parametric sensitivity analysis
- Stochastic robustness evaluation
- Optimization-driven insight

This structured setup ensures reproducibility and provides a reliable basis for evaluating cascaded communication system performance under both ideal and practical conditions.

IV. RESULTS AND DISCUSSION

The results obtained from the simulation framework are analyzed under three major aspects: parametric variation, stage scaling, and stochastic behavior. The observations are supported with figures and summarized tables for clarity and reproducibility.

➢ *Effect of First-Stage Gain*
The variation of total noise figure with respect to first-stage gain is illustrated in Fig. 1.

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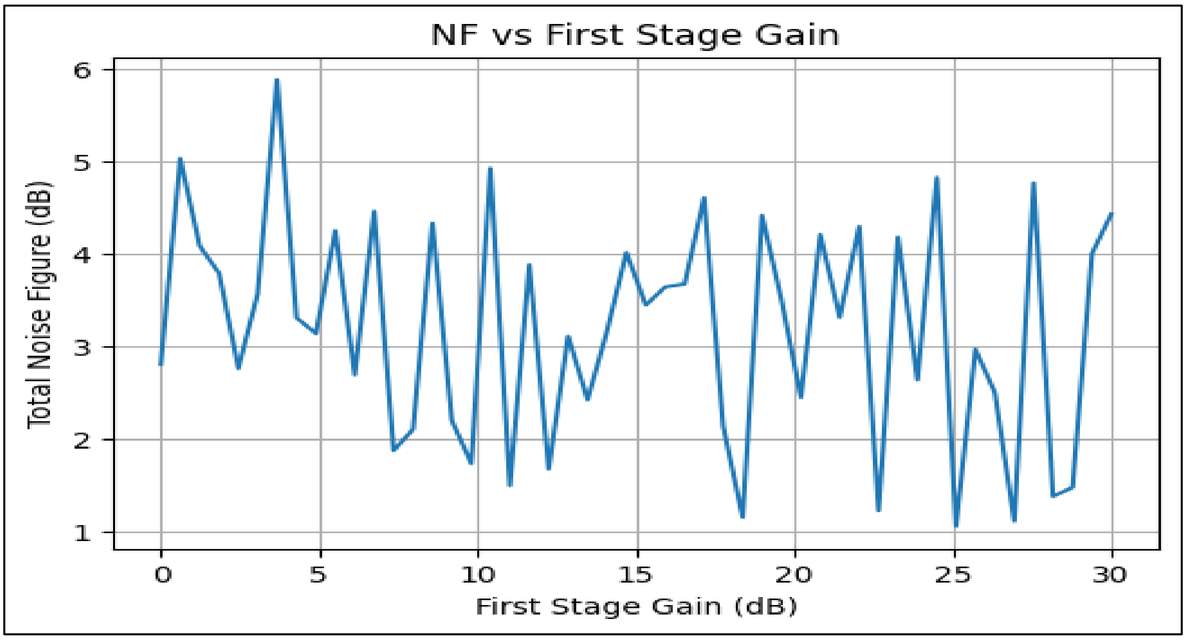

Fig 1 Total Noise Figure versus First-Stage Gain.

The results indicate a decreasing trend in total noise figure with increasing first-stage gain. However, the curve exhibits fluctuations due to randomized configurations of subsequent stages. Despite this variability, an overall envelope of reduction is clearly observed.

At low gain values (<10 dB), the contribution of later stages is significant, resulting in higher total noise figure. As the gain increases beyond 20 dB, the rate of improvement reduces, indicating saturation behavior. This confirms that excessive gain provides diminishing returns.

➢ *Effect of Number of Stages*

The relationship between total noise figure and number of cascaded stages is presented in Fig. 2.

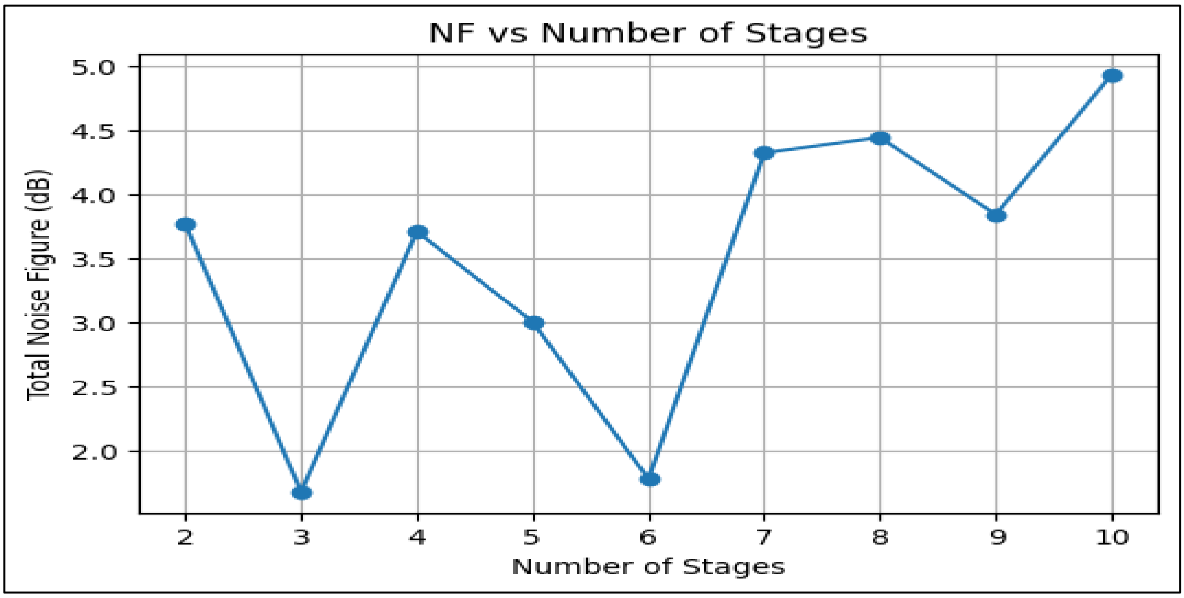

Fig 2 Total Noise Figure versus Number of Stages.

The results show a generally increasing trend of noise figure with stage count. However, the variation is non-monotonic due to random gain distribution across stages. Systems with favorable early-stage gain exhibit lower noise figures even with higher stage counts.

This indicates that noise performance is not solely dependent on the number of stages, but strongly influenced by gain allocation across the cascade.

➢ *Statistical Behavior under Monte Carlo Simulation*

The statistical distribution of total noise figure obtained from 1000 Monte Carlo iterations is shown in Fig. 3.

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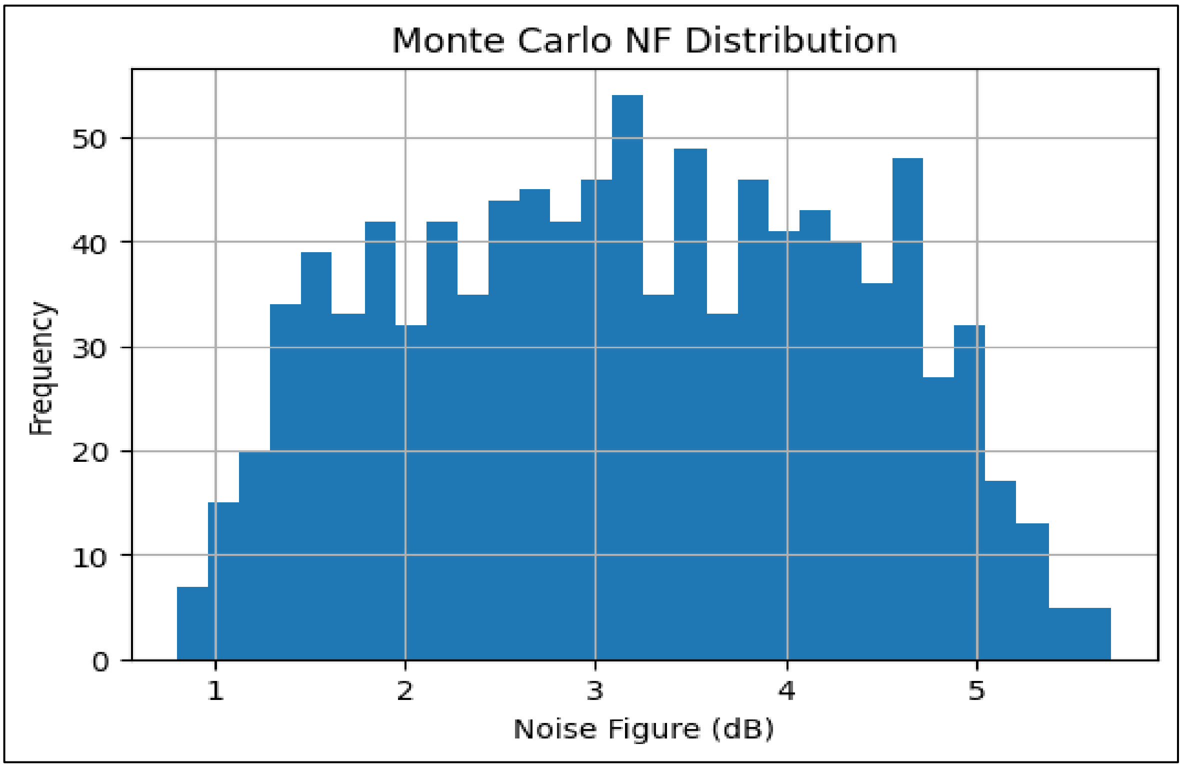

Fig 3 Distribution of Total Noise Figure under Monte Carlo Simulation.

The distribution approximates a Gaussian profile with slight skewness. The computed statistical parameters are summarized in Table 1.

Table 1 Statistical Characteristics of Noise Figure

Metric	**Value**
Mean Noise Figure	3.17 dB
Standard Deviation	1.16 dB
Minimum Observed NF	~1.0 dB
Maximum Observed NF	~5.5 dB

The relatively high standard deviation indicates strong sensitivity of the system to parameter variations. This behavior is primarily driven by fluctuations in early-stage gain and noise figure.

➢ *Optimization of First-Stage Gain*

The optimization study identifies the first-stage gain that minimizes total noise figure. The results are summarized in Table 2.

Table 2 Optimization Results

Parameter	**Value**
Optimal First-Stage Gain	27.88 dB
Minimum Noise Figure	1.15 dB

The optimal gain lies in the higher range, confirming that increasing first-stage gain improves overall noise performance. However, the improvement becomes marginal beyond a certain threshold, indicating saturation.

➢ *Comparative Interpretation*

A comparative analysis between ideal and practical systems reveals the following:

- Ideal systems (without variation) exhibit smooth monotonic trends.
- Practical systems (with variation) show fluctuations and wider spread.
- The gap between ideal and practical performance highlights the impact of uncertainty.

➢ *Key Observations*

- *First-Stage Dominance*

The first stage has the highest influence on total noise performance. Its gain and noise figure dictate the contribution of all subsequent stages.

- *Saturation Behavior*

Noise figure improvement saturates beyond approximately 20–25 dB of first-stage gain.

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- *Stage Count Dependency*

Increasing number of stages generally degrades performance, but optimal gain distribution can mitigate this effect.

- *Statistical Sensitivity*

Monte Carlo results confirm that cascaded systems are highly sensitive to parameter variations, particularly in early stages.

➢ *Discussion for Design Implications*

The results emphasize that practical RF system design must prioritize:

- High-gain, low-noise first-stage design
- Controlled variability in gain and noise parameters
- Balanced optimization considering physical constraints

The observed trends are consistent with theoretical expectations from Friis formulation while extending the analysis into stochastic and optimization domains.

➢ *Reproducibility Note*

All figures and tables are generated directly from the simulation framework with fixed random seed, ensuring reproducibility of results. The exported dataset (nf_results.csv) can be used for independent verification and further analysis.

V. CONCLUSION

A structured and reproducible analysis of gain and noise behavior in cascaded communication systems has been presented using the Friis formulation. The study integrates deterministic modeling, parametric variation, Monte Carlo simulation, and optimization within a unified Python-based framework. Results demonstrate that the overall noise performance is predominantly governed by the first stage, where both gain and noise figure critically influence the contribution of subsequent stages.

Parametric analysis confirms that increasing first-stage gain reduces total noise figure; however, the improvement exhibits saturation beyond a threshold region (approximately 20–25 dB). The stage-scaling study shows that although noise figure generally increases with the number of stages, appropriate gain distribution can mitigate degradation. Monte Carlo evaluation reveals a mean noise figure of 3.17 dB with a standard deviation of 1.16 dB, indicating significant sensitivity to parameter variations and emphasizing the importance of robust design.

The optimization study identifies an optimal first-stage gain of 27.88 dB, achieving a minimum noise figure of 1.15 dB under unconstrained conditions. This result confirms the asymptotic behavior predicted by theory, where total system noise approaches the first-stage limit. However, the analysis also highlights the necessity of incorporating practical constraints such as power consumption, linearity, and stability for realistic implementation.

Overall, the proposed framework provides a comprehensive and scalable approach for evaluating cascaded system performance under both ideal and stochastic conditions. The findings establish clear design guidelines for RF front-end optimization and demonstrate the critical role of early-stage design in communication systems.

FUTURE SCOPE

Future work may extend this study toward constrained multi-objective optimization incorporating power, linearity, and bandwidth limitations. Integration of intelligent optimization techniques such as particle swarm optimization or genetic algorithms can further enhance design efficiency. Additionally, hardware-aware modeling and experimental validation can strengthen the applicability of the proposed framework in practical RF system design.

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