

# Effects of Heat Source and Sink on Radiative Hybrid Nanofluid Flow Past a Stretching Surface

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**Abstract:** This current communication aims on significance of nanofluid flow past an exponential velocity stretching sheet with heat generation and absorption effects. Mathematical model for fluid flow is developed by employing boundary layer approximation theory and solved with the help of similarity solution. System of equations are solved by applying shooting method and importance of various fluid flow parameters are visualised by drawing profiles of momentum and energy equations. Skin friction and heat transfer rate is calculated and analysed. In this study it is observed that magnetic field acts as opposing force on boundary layer thickness and reduces and velocity profile.

**Keywords:** Numerical Simulation, Nanofluid Flow, MHD, Shooting Method.

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## I. INTRODUCTION

Heat transfer is one of the crucial and significant aspect in many industrial applications such as mechanical, aerospace, food processing systems, cooling of nuclear reactor etc. Based on the system specification and industrial requirements various mechanisms are used for heat exchange applications. Convection is also one of the widely used technique for many mechanical processes in which heat is carried out by motion of fluid either liquid or gas. Most of the cases the heat transfer takes place between solid surface and fluid adjacent to that surface which are known as boundary layer flows. Sakiadis was first researcher to investigate such flow over surface problems by making use of Prandtl boundary layer approximation. Sakiadis [1] introduced the mechanism of flow over continuously moving surfaces and formation of boundary layer withing the vicinity of fluid structure interface. Later Sakiadis [2] extended this work with derivation of mentum using boundary layer approximation by taking flat surface. The same author Sakiadis [3] chosen another cylindrical geometry successfully to discuss laminar and turbulent flow regime. This has created a new research area and many researchers investigated under distinct flow configurations. Kuznetsov and Neild [4] introduced an analytical solution for fluid flow past a vertical directed plate. Bachok et al.[5] studied about fluid flow over a moving surface by considering moving surface and static fluid condition and also moving fluid over a static surface. In the last decade there is a sudden switch over to modifying the conventional heat carrying fluids usually water or oils. The researchers identified the major drawback in conventional fluids and tried to modify the thermal conductivity of the fluid by suspending nano sized metal particles into base fluid.

These engineered fluids are known as nanofluids Many experimental investigations ( See [6], [7], [8], [9], [10] ) proved that the inclusion of nanoparticles will increases the thermal conductivity of base fluid hence better heat transfer rate can be obtained. Nurul Amina Zainal et al.[11] discussed time dependent nanofluid flow past a moving surface. Vishnu Ganesh et al.[12] studied impact of non linear inertia in a porous medium with suction and blow effects.Sulochana et al.[13] studied roles of electric and magnetic fields in controlling flow characteristics in boundary layer flows. Riaz[14] et al.investigated Darcy Forchheimer effect on nanofluid flow over a stretching and shrinking surfaces. Ramesh et al.[15] studied nanofluid flow with an exponential velocity moving surface and their results shows that heat transfer rate decelerate with dissipated heat. Gangadhar et al.[16] performed sensitivity analysis to understand the role of fluid influencing parameters in understanding influential aspects of boundary layer thickness formation. In some studies mixed base fluids are also used instead of hybrid nanoparticles in order to enhance thermal conductivity and better viscosity for industrial and lubrication applications.Sulochana et al.[17] reported the hybrid base fluid flow over an elongating surface. Pavithra et al.[18] used the similarity approach to handle laminar flow over a surface. Entropy generation analysis is performed by Venkateswarulu et al.[19] with an exponential stretching sheet problem. Their results show that enhancing magnetic field strength will cause drastic increment in friction at the surface. In another study by Sibanda et al.[20] on fluid flow over an exponential stretching sheet depicts the role of Brownian motion and thermophoresis in improving fluid heat exchange capabilities. In view of the above literature review ,it is observed that many authors focused on heat transfer with Newtonian and

non-Newtonian models and some authors have produced results with the help of nanofluids. But in the above cited literature all authors neglected the internal heat generated and understanding flow pattern. So this work is mainly focused on the impact of internal heat generation and absorption and using nanofluid to enhance the heat transfer rate and np such work is initiated to the best of authors knowledge.

**II. MATHEMATICAL MODEL**

Consider a steady laminar incompressible hybrid nanofluid flow induced by a stretching sheet which is moving with an exponential velocity  $u_w(x) = ce^{\frac{x}{L}}$  and the stretching sheet is taken in x direction and transverse magnetic field is applied in the direction opposite to sheet. Full specifications of the fluid flow set up is given in Figure 1. It is assumed that nanoparticles and carrying fluid are in thermally equilibrium condition and shape of the nano particle is spherical and size is negligible in this study. Based on these assumptions governing partial differentials can be written as follows.

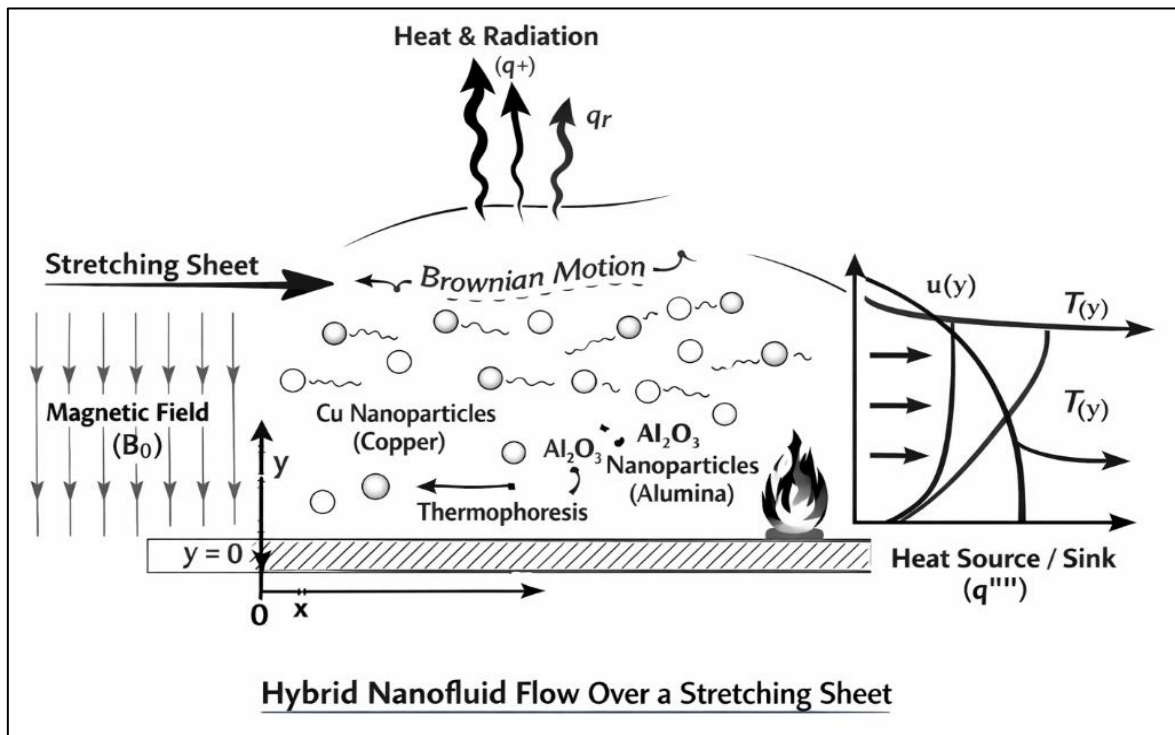


Fig 1 Physical Sketch of Hybrid Nanofluid Boundary Layer Flow.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\rho_{hnf} \left( u \frac{\partial u}{\partial y} + v \frac{\partial u}{\partial x} \right) = \mu_{hnf} \left( \frac{\partial^2 u}{\partial y^2} \right) - \sigma_{hnf} B_0^2(x) u \tag{2}$$

$$(\rho c_p)_{hnf} \left( u \frac{\partial T}{\partial y} + v \frac{\partial T}{\partial x} \right) = k_{hnf} \left( \frac{\partial^2 T}{\partial y^2} \right) - \frac{\partial q_r}{\partial y} + Q_0(T - T_\infty) \tag{3}$$

Boundary conditions at the surface can be written as:

$$\text{At } y = 0, u = \omega_1 u_w, v = v_w \text{ and } T = T_w \text{ and } u \rightarrow 0, T \rightarrow T_\infty \text{ as } y \rightarrow y_\infty \tag{4}$$

Where  $v_w$  is mass flux velocity,  $T_w$  is surface temperature,  $q_r$  indicates radiative heat flux and  $\omega_1$  is stretching parameter when  $\omega_1 > 0$  sheet is in stretching mode,  $\omega_1 < 0$  represents shrinking sheet.

Consider the transformed variable  $\eta = y \sqrt{\frac{U_0}{2\nu_f L}} e^{x/2L}$  (See [21]) then equations (2)-(4) becomes  $\psi =$

$$\sqrt{2U_0\nu_f L} f(\eta), u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}, T = T_\infty + (T_w - T_\infty)\theta(\eta) \text{ .where } \psi \text{ is stream function and } (u, v) \text{ represents velocity components in x and y directions.}$$

Now the governing equations (2)-(5) are expressed in single variable  $\eta$  as shown below.

$$\left( \frac{\mu_{hnf}}{\mu_f} \right) f'''(\eta) + f(\eta)f''(\eta) - 2f'(\eta)f'(\eta) - Mf'(\eta) = 0 \tag{6}$$

$$\left( \frac{k_{hnf}}{k_f} + \frac{4Nr}{3} \right) \theta''(\eta) + A_3 Pr f(\eta)\theta(\eta) - f(\eta)'(\eta)\theta(\eta) + Q\theta(\eta) = 0 \tag{7}$$

The Boundary Conditions:

$$f(0) = \lambda, f'(0) = S, \theta(0) = 1, f'(\infty) = 0 \text{ and } \theta(\infty) = 0 \tag{8}$$

**III. RESULTS AND DISCUSSION**

Let us discuss some of the important and unique results of this study in this section. Basically, boundary layer

problems are solved by converting into initial value problems and then a guess is made to know the unknown boundary conditions using shooting technique. Governing equations (6) to (7) along with boundary conditions mentioned in equation (8) are solved numerically by Runge Kutta method and results are visualised by velocity and energy profiles.

The variation in the velocity distribution under the influence of the magnetic parameter is illustrated in Figure-2 and Figure-3 for stretching and shrinking surface cases, respectively. It is clearly observed that the velocity profile decreases with increasing magnetic parameter in the stretching sheet case, whereas the reverse behaviour is noticed in the shrinking sheet configuration. Physically, the application of a transverse magnetic field to an electrically conducting fluid generates a Lorentz force due to the interaction between the induced electric current and the magnetic field. In the stretching case, this Lorentz force acts opposite to the direction of fluid motion and therefore suppresses the velocity field by increasing the resistive drag within the momentum boundary layer. As a result, the momentum boundary layer thickness reduces and the fluid motion slows down near the surface. However, in the shrinking case, the magnetic force supports the inward motion of the fluid towards the surface, stabilizing the boundary layer structure and enhancing the velocity distribution. This assisting nature of the Lorentz force leads to an increase in momentum transport in the shrinking regime.

The influence of magnetic field strength on the temperature distribution is presented in Figure-3 and Figure-4 for both stretching and shrinking surfaces. It is observed that the temperature profile increases with increasing magnetic parameter for both nanofluid and hybrid nanofluid models. The physical reason behind this behaviour is that the applied magnetic field produces Joule heating due to electrical resistance within the fluid. This additional heat generation converts mechanical energy into internal thermal energy and increases the fluid temperature inside the thermal boundary layer. Furthermore, the reduction in fluid velocity caused by the magnetic field weakens convective heat transport and allows more heat to accumulate near the surface, which further enhances the temperature distribution.

The effect of thermal radiation on temperature distribution is depicted in Figure-4 and Figure-5. From these figures, it is evident that the temperature profile increases significantly with increasing radiation parameter values. Physically, thermal radiation contributes an additional mode of heat transfer in the form of radiative energy flux, which supplements conductive heat transport within the boundary layer. As the radiation parameter increases, more thermal energy is absorbed by the fluid particles, leading to an increase in thermal boundary layer thickness and elevation of fluid temperature. This effect becomes particularly significant in high-temperature industrial processes such as polymer extrusion, glass manufacturing, and nuclear cooling systems, where radiative heat transfer dominates conduction.

The influence of heat generation and absorption on temperature distribution is illustrated in Figure-6 and Figure-

7 for stretching and shrinking cases. It is observed that the temperature profile increases in the presence of internal heat generation because additional thermal energy is supplied directly into the fluid domain, which enhances molecular activity and raises the thermal energy level of the system. Consequently, the thermal boundary layer becomes thicker and the temperature gradient near the surface reduces. In contrast, heat absorption removes thermal energy from the fluid, which weakens molecular motion and reduces temperature distribution across the boundary layer. Therefore, the heat source/sink parameter plays a crucial role in controlling thermal regulation within the flow region.

The variation of the skin friction coefficient and Nusselt number is analysed by fixing selected governing parameters while varying the nanoparticle volume fraction in order to understand the role of hybrid nanofluid on surface drag force and heat transfer rate. Figure-8 and Figure-9 present the influence of nanoparticle volume fraction on the skin friction coefficient and Nusselt number under magnetic field effects. It is observed that the skin friction coefficient decreases with increasing nanoparticle volume fraction, whereas the Nusselt number increases. Physically, the addition of nanoparticles enhances the effective thermal conductivity of the base fluid, which improves the rate of heat transfer from the surface to the fluid. At the same time, the presence of nanoparticles modifies viscosity and momentum transport characteristics, which reduces the velocity gradient at the surface and consequently lowers the skin friction coefficient. This indicates that hybrid nanofluids are more efficient in enhancing heat transfer performance while maintaining moderate surface drag.

A similar behaviour is observed for radiation and heat generation/absorption parameters combined with nanoparticle volume fraction, as illustrated in Figure-10, Figure-11, and Figure-12. The inclusion of nanoparticles strengthens radiative heat absorption capability and increases thermal diffusivity within the fluid, which further enhances temperature distribution and heat transfer rate. Moreover, internal heat generation combined with nanoparticle dispersion intensifies energy transport mechanisms inside the boundary layer, leading to improved thermal performance of hybrid nanofluids compared to conventional nanofluids.

Finally, the computed numerical values of the skin friction coefficient and Nusselt number for various governing parameters are shown in Figure 12 to Figure 14 and it is observed that the Nusselt number increases with increasing thermal radiation parameter and nanoparticle volume fraction due to enhancement in radiative and conductive heat transport mechanisms. However, the Nusselt number decreases with increasing magnetic parameter and heat generation parameter because the magnetic field suppresses fluid motion through Lorentz force effects and internal heat generation reduces the temperature gradient at the surface. Since the Nusselt number is directly proportional to the surface temperature gradient, any reduction in this gradient leads to a decline in heat transfer rate from the surface. These observations confirm the significant role of electromagnetic forces, nanoparticle dispersion, and internal heat sources in controlling thermal

transport behaviour within hybrid nanofluid boundary layer flows.

#### IV. CONCLUSIONS

The present statistical investigation examined the role of nanoparticle volume fraction on magnetohydrodynamic boundary-layer flow of a copper–alumina hybrid nanofluid. Few important and unique results are listed below:

- The analysis indicates that the heat transfer rate improves significantly with increasing thermal radiation parameter and nanoparticle volume fraction, as reflected by higher Nusselt number values.

- It is also observed that thermal radiation enhances the temperature distribution for both stretching and shrinking surface conditions. Furthermore, the presence of internal heat generation or absorption reduces the overall heat transfer performance in both nanofluid and hybrid nanofluid cases.
- The skin-friction coefficient decreases with an increase in the magnetic parameter due to the resistive influence of the applied magnetic field on the fluid motion.
- Nanoparticle volume fraction concentration and thermal radiation in controls heat transfer characteristics in magnetohydrodynamic hybrid nanofluid flows.

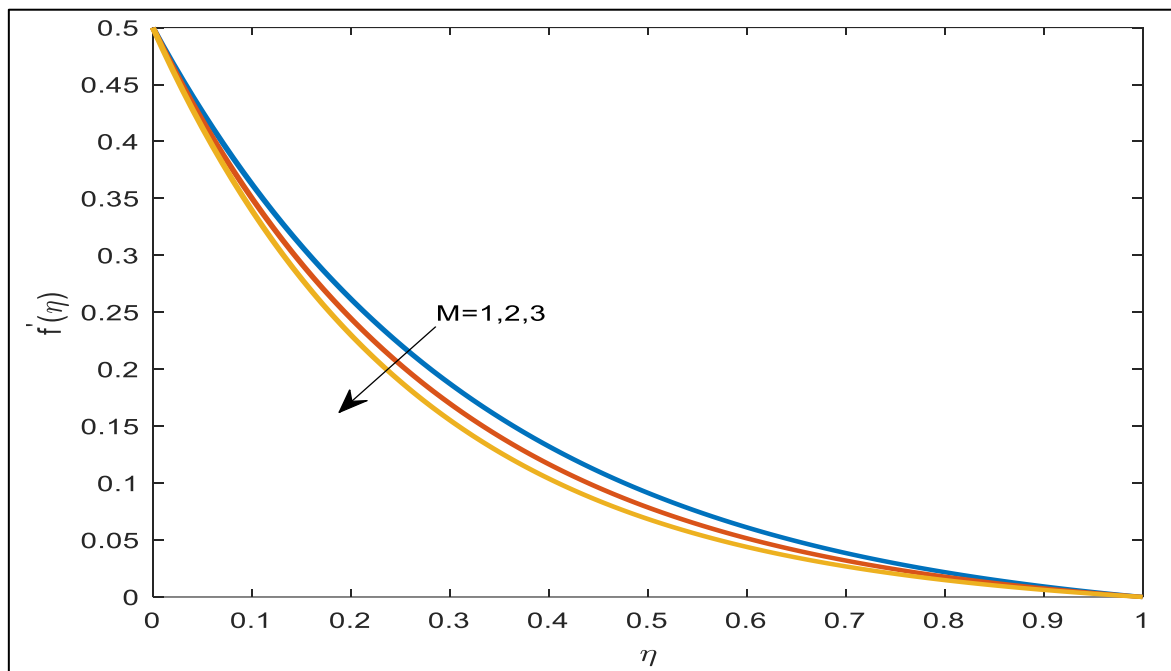


Fig 2 Velocity Profile for Magnetic Field for Stretching Case.

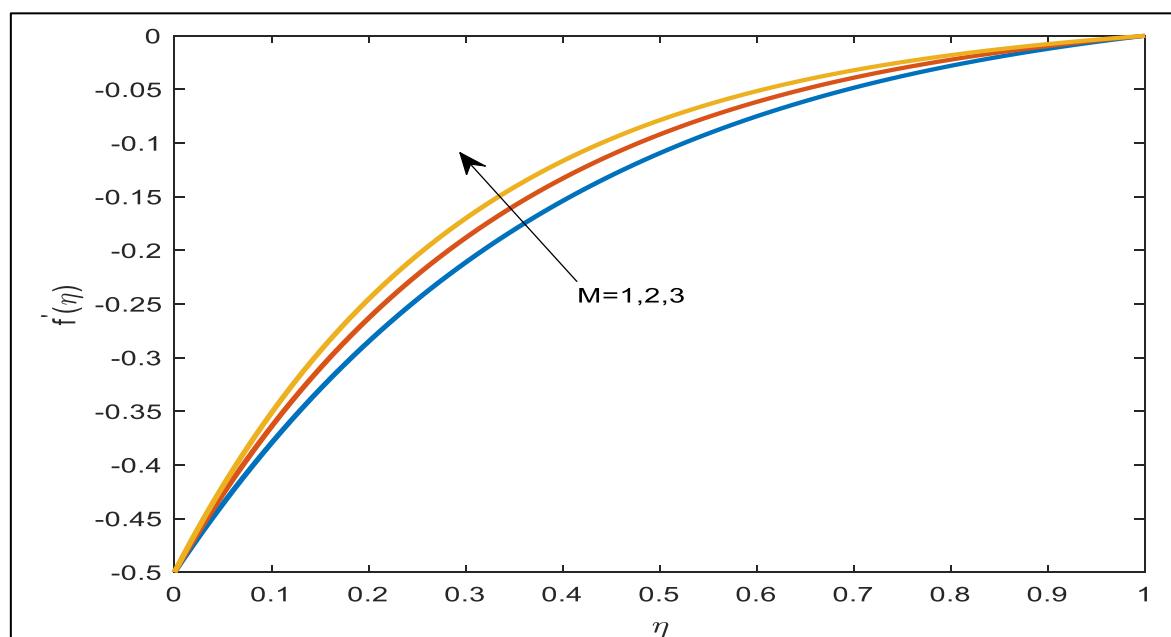


Fig 3 Velocity Profile for Magnetic Field in Shrinking Case.

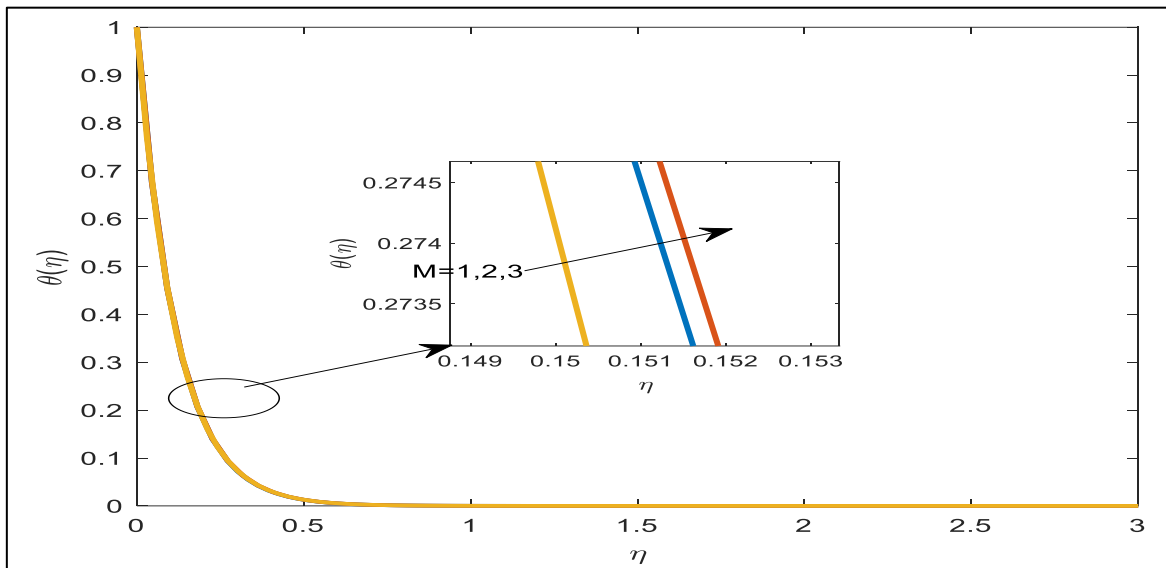


Fig 4 Temperature Profile for Magnetic Field in Stretching Case.

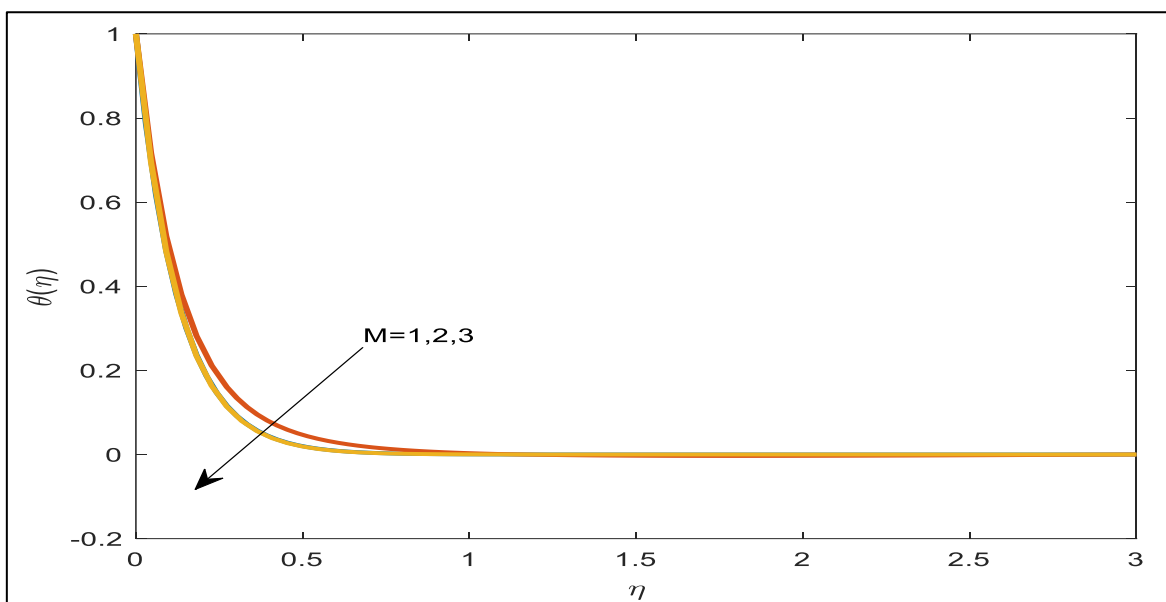


Fig 5 Temperature Profile for Magnetic Field in Shrinking Case.

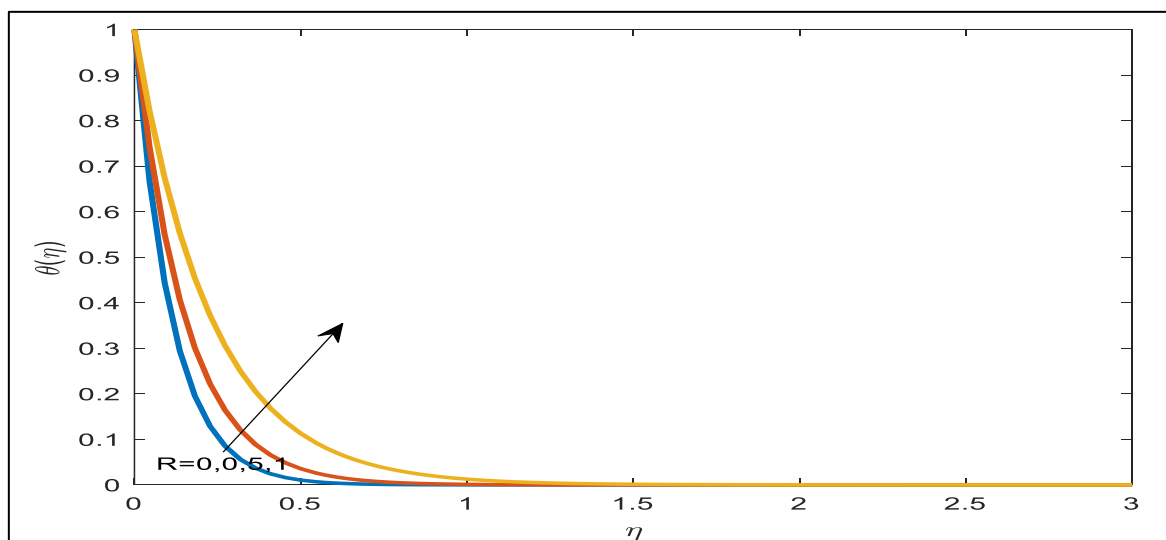


Fig 6 Temperature for Radiation in Stretching Case.

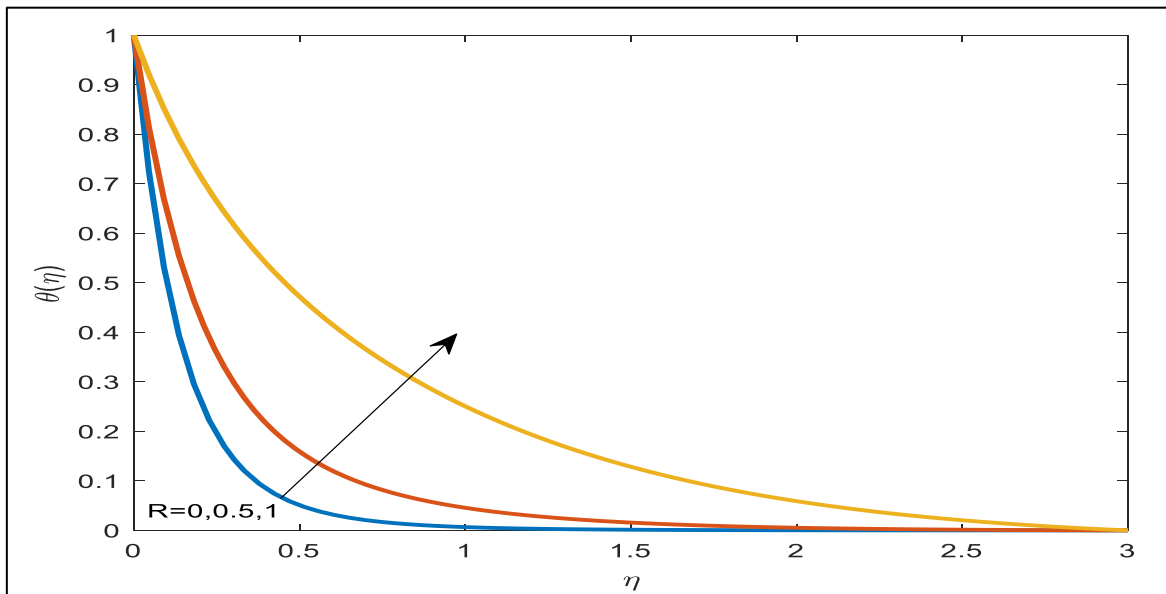


Fig 7 Temperature for Radiation in Shrinking Case.

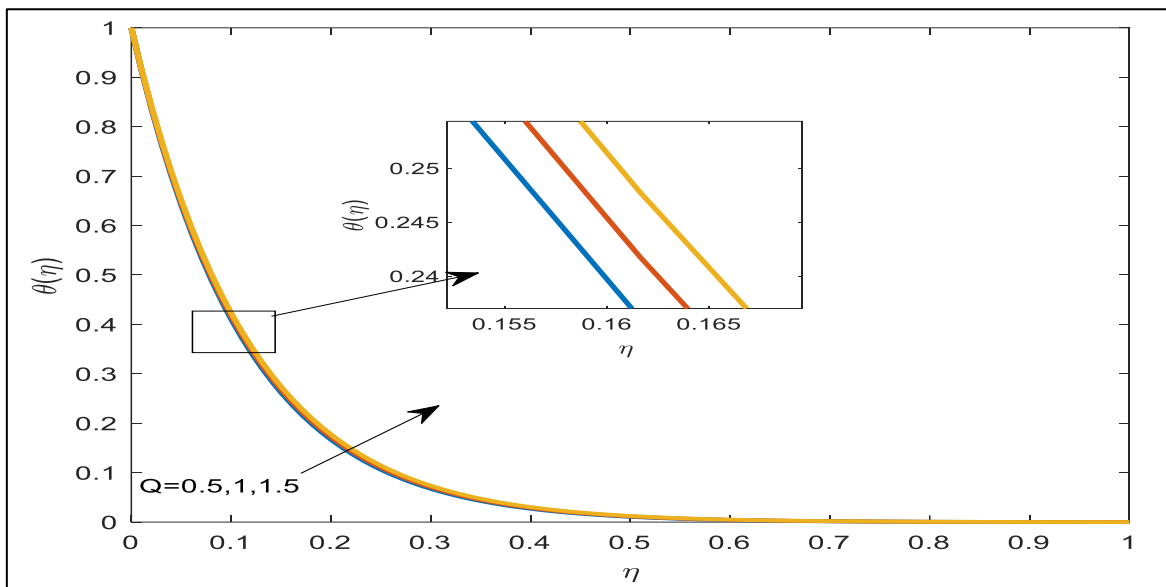


Fig 8 Temperature for Heat Generation in Stretching Case.

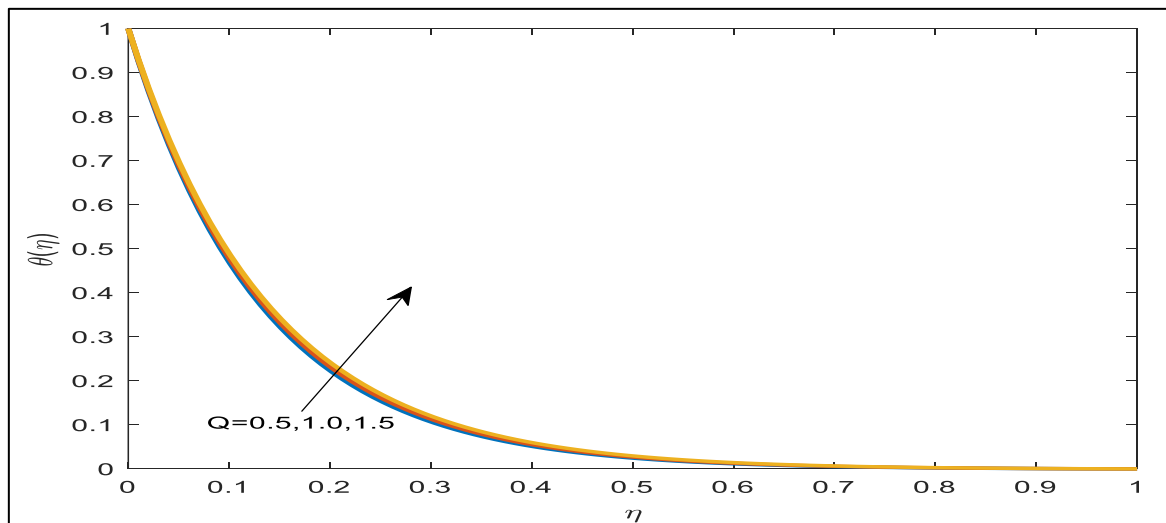


Fig 9 Temperature for Heat Generation in Shrinking Case.

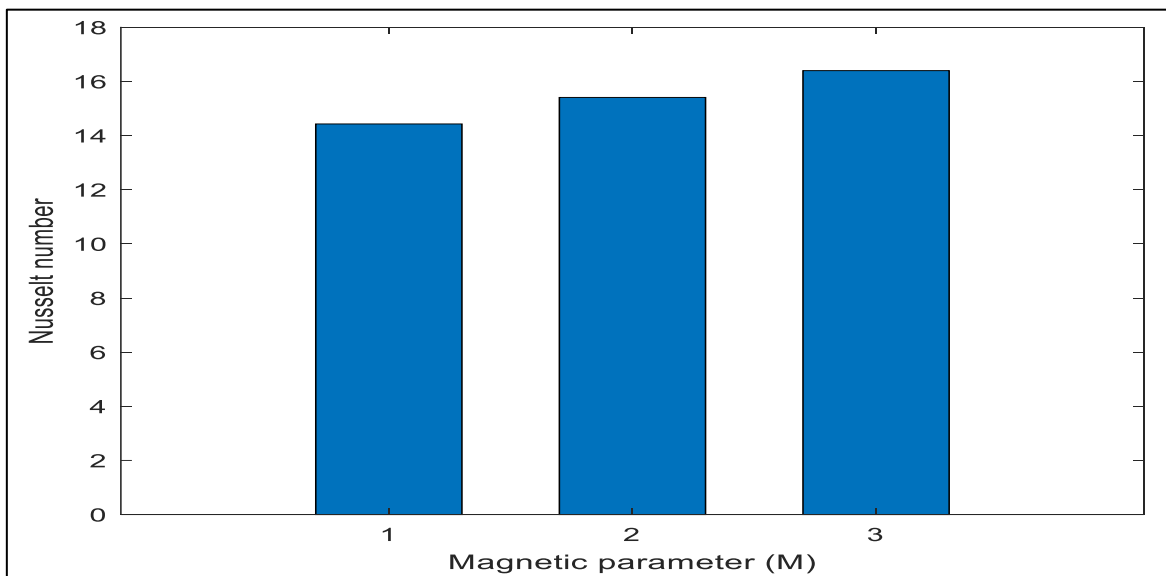


Fig 10 Friction at the Surface for Different Volume Fractions

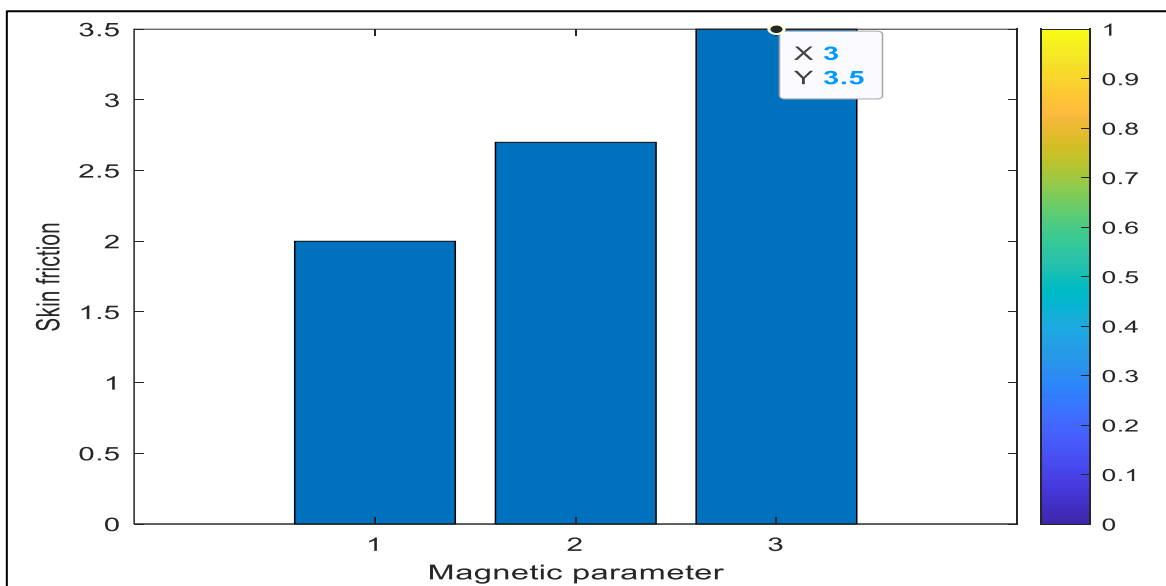


Fig 11 Heat Exchange at the Surface for Different Volume Fractions

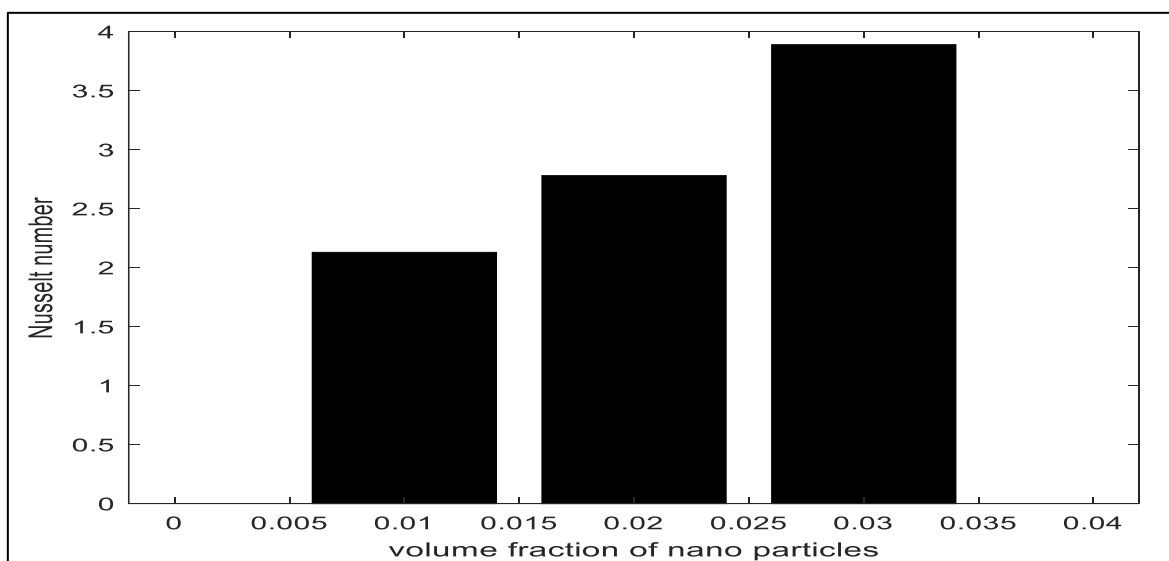


Fig 12 Heat Transfer Rate for Volume Fraction Change

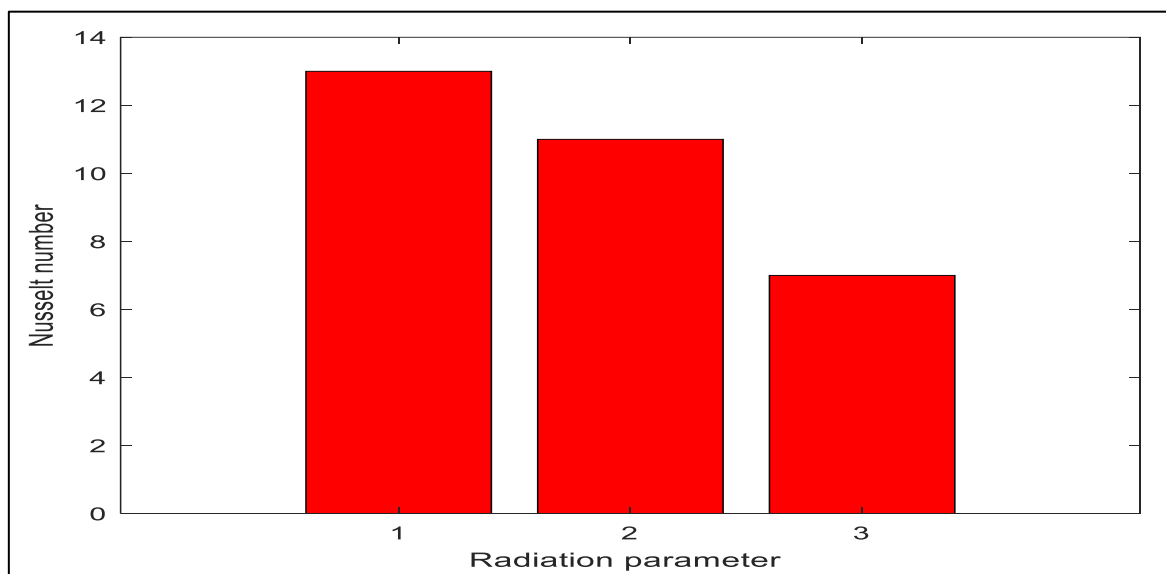


Fig 13 Nusselt Number at the Surface for Radiation Parameter.

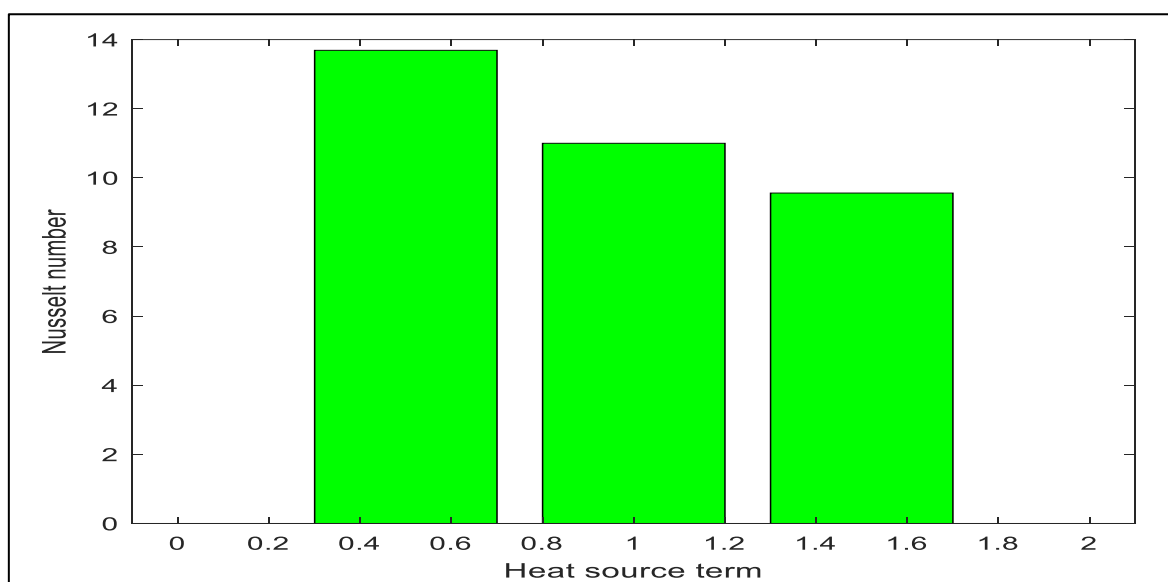


Fig 14 Heat Generation and Absorption at the Surface.

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