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Computational Naval Architecture: Optimizing Hull Displacement Calculations Using a 2D Tensor-Product Adaptation of Simpson's 3/8 Rule

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Abstract: Calculating the exact volumetric displacement of a marine vessel is a fundamental challenge in naval architecture, as modern ship hulls consist of continuous "fair curves" that lack singular analytical definitions. Traditional volume calculations rely on sequential, one-dimensional numerical integration. This paper presents an optimized methodology that applies a two-dimensional tensor-product adaptation of Simpson's 3/8 Rule to a discrete matrix of hull offsets. By processing both longitudinal and vertical spatial axes simultaneously using procedurally generated cubic weights, this algorithm calculates total displacement with $O(1)$ auxiliary memory complexity. The findings demonstrate how higher-order numerical methods can bridge pure mathematics and marine engineering, offering high-precision volume approximation for complex hull forms.

Keywords: *Naval Architecture, Volumetric Displacement, Numerical Integration, Simpson's 3/8 Rule, Tensor-Product Algorithm, Hull Offsets, Computational Efficiency, Marine Engineering.*

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I. INTRODUCTION

The displacement volume of a ship directly dictates its buoyancy, cargo capacity, and stability parameters. Because ship hulls are not simple geometric solids, engineers cannot rely on standard Euclidean volume formulas. Instead, physical or digital measurements—known as offsets—are taken at discrete intervals along the hull's surface.

To convert these discrete data points into a continuous volume, naval architects utilize numerical integration. While Simpson's 1/3 Rule (based on quadratic interpolation) is the industry standard due to its simplicity, it is occasionally insufficient for modeling the sweeping, high-curvature profiles of modern hydrodynamic vessels. This paper explores the application of Simpson's 3/8 Rule—a cubic interpolation method—adapted into a two-dimensional algorithm to process hull grids simultaneously, improving mathematical precision over highly curved hull sections.

II. MATHEMATICAL FOUNDATION: SIMPSON'S 3/8 RULE

Simpson's numerical methods approximate the area under a curve by fitting polynomials through discrete data points. The 3/8 Rule fits a cubic polynomial (degree 3) through four consecutive points.

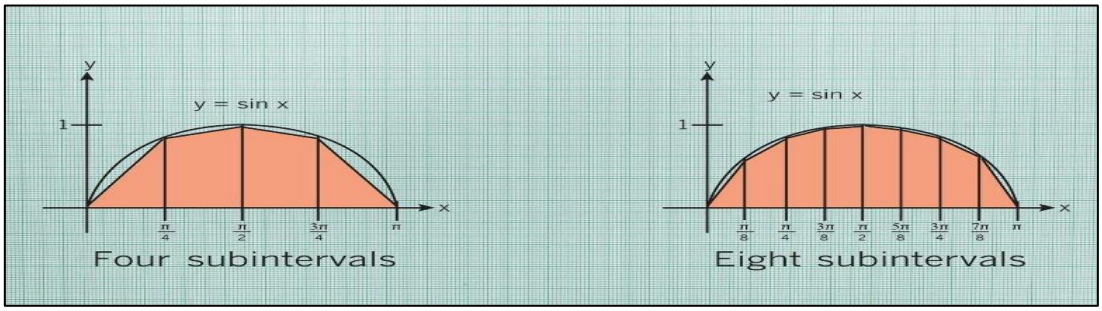

Fig 1 Discretizing a Continuous Curve into Subintervals for Numerical Integration.
Source: Tribune Media Services

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For an interval $[a, b]$ divided into n equal subintervals of width h (where n must be a multiple of 3), the one-dimensional rule is defined as:

$$\int_a^b f(x)dx \approx \frac{3h}{8}\sum_{i=0}^{n} w_i f(x_i)$$

The weight vector w_i follows a strict repeating sequence of $(1,3,3,2,3,3,2,\ldots,1)$. The local truncation error for this method is proportional to $O(h^5)$, making it highly accurate for smooth, continuous curves.

III. DISCRETIZATION OF THE HULL SURFACE

To apply this integration mathematically, the continuous, three-dimensional hull must be discretized into an orthogonal grid. Because ship hulls are symmetrical along the centre line, calculations are performed on a "half-hull" and the final result is doubled.

➢ *The Integration Domain is Established Using Two Axes:*

- Longitudinal Axis (Stations): The ship's Length Between Perpendiculars (L) is divided into n_x equal intervals, establishing vertical cross-sections. The step size is $h_x = \frac{L}{n_x}$.
- Vertical Axis (Waterlines): The ship's draft (T) is divided into n_z equal intervals, establishing horizontal planes. The step size is $h_z = \frac{T}{n_z}$.

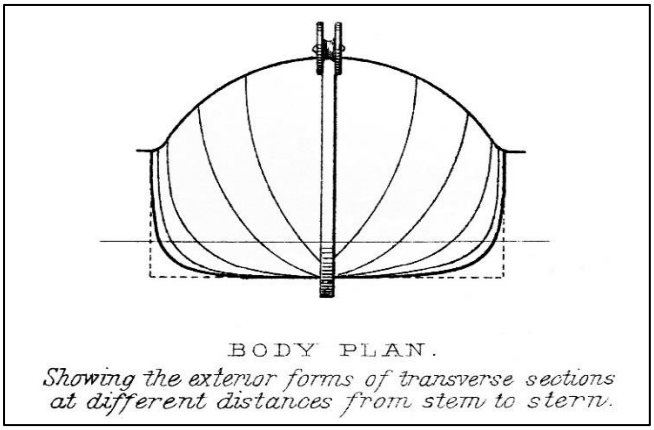

Fig 2 A Ship's Body Plan Showing Transverse Cross-Sections (Stations) Used for Offset Measurement. Source: mikroman6 / Getty Images

The continuous function being integrated is $y(x, z)$, representing the measured half-breadth (width) of the hull at a specific length x and depth z.

Table 1 Half-Breadth Hull Offsets at Discrete Stations and Waterlines4. The 2D Tensor-Product Algorithm for Displacement

	Station 0 (Bow)	Station 1	Station 2	Station 3	Station 4	Station 5	Station 6 (Stern)
WL 3 (Draft)	0.00	2.10	4.50	5.00	4.50	2.20	0.00
WL 2	0.00	1.50	3.80	4.20	3.70	1.40	0.00
WL 1	0.00	0.80	2.10	2.50	2.00	0.60	0.00
WL 0 (Keel)	0.00	0.00	0.50	0.80	0.50	0.00	0.00

The total volumetric displacement (∇) is analytically defined by the double integral of the offset surface:

$$\nabla = 2\int_0^L \int_0^T y(x, z)\, dz\, dx$$

Rather than calculating the cross-sectional area of each station sequentially, the tensor-product method evaluates the entire offset matrix in a single computational pass. By nesting the 3/8 rule across both the x and z axes, the scalar multiplier for any specific measurement node is the product of its independent 1D weights ($W_{i,k} = w_i \times w_k$).

The discrete volume approximation becomes:

$$\nabla \approx 2\left(\frac{9h_x h_z}{64}\right)\sum_{i=0}^{n_x}\sum_{k=0}^{n_z}(w_i \times w_k)\, y_{i,k}$$

➢ *Computational Implementation*

To optimize memory, the algorithm avoids storing a full two-dimensional weight matrix. Instead, it utilizes conditional logic based on modulo arithmetic to generate the w_i and w_k weights dynamically during the nested spatial iteration. This procedural generation ensures the algorithm operates with $O(1)$ auxiliary space complexity, allowing it to efficiently process highly dense point-cloud data from modern 3D hull scanners.

IV. DISCUSSION: ACCURACY AND GEOMETRIC ANOMALIES

The primary advantage of the cubic 3/8 rule over the quadratic 1/3 rule lies in its ability to model inflection points and sweeping sheer lines typical in midship sections.

However, mathematical integration assumes a continuously fair curve. Discontinuities in modern ship design—such as flat transom sterns, sharp skegs, or protruding bulbous bows—violate the assumptions of cubic polynomials. In these boundary regions, the 3/8 rule may introduce Runge's phenomenon (artificial oscillations between data points). To counteract this, practical software implementations must allow for localized grid refinement (reducing h_x and h_z) at the extreme bow and stern, isolating geometric anomalies from the primary numerical integration.

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V. CONCLUSION

Applying Simpson's 3/8 rule as a two-dimensional tensor product offers a mathematically robust and computationally lightweight algorithm for calculating ship hull displacement. By processing longitudinal and vertical spatial axes simultaneously, naval architecture software can approximate complex volumes with high precision. Future research in this domain should focus on dynamic step-size adjustments, allowing the algorithm to automatically transition between Simpson's rules based on the localized curvature of the hull matrix.

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