Volume 11, Issue 8, August– 2026 International Journal of Innovative Science and Research Technology
ISSN No:-2456-2165 https://doi.org/10.38124/ijisrt/26aug240

Nonlinear Dynamics of Dispersive Delay Coupled Two Phase Oscillator System

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Publication Date: 2026/08/20

Abstract: Dynamics of dispersive delay coupled system of two stable limit cycle oscillators is presented. The effect of dispersive delay coupling on synchronization dynamics of two phase oscillators having different frequencies is investigated. First, we determine fixed points, and then analyze their Lyapunov stability and phase space trajectories over practically realizable range of system parameters. It is shown that large number of phase locked solutions is obtained in contrast to constant delay coupled system. Both, the limit cycle and the chaotic dynamics are seen to occur in different regions of the parametric spaces. The motivation behind this study is whether this dispersive delay coupled oscillator system can be used as chaos generator for communication applications. This study contributes to a good understanding of the impact of dispersive time delay on interacting oscillators.

Keywords: *Synchronization; Coupled Phase Oscillators; Dispersive Delay.*

How to Cite: Shashank S. Jha (2026) Nonlinear Dynamics of Dispersive Delay Coupled Two Phase Oscillator System. *International Journal of Innovative Science and Research Technology*, 11(8), 1212-1224. https://doi.org/10.38124/ijisrt/26aug240

I. INTRODUCTION

The dynamics of coupled oscillators is an efficient and very popular paradigm for the study of interacting oscillatory phenomena in physical and biological sciences [1-8]. To model the collective behaviour of real systems, the dynamics of interacting units in a system creates the foundation of curiosity. Synchronization is the most appealing phenomena in coupled systems, where the individual units of the system oscillate at a common frequency while the phase difference between two individuals becomes locked. Synchronization may be studied through simpler phase models as this phenomenon can be observed under weak coupling schemes in coupled systems where amplitude stability is assumed [7-14].

The study of nonlinear dynamics of delay coupled oscillators is now become a subject of great interest in various fields of Physics, Engineering and Biology [15-22]. Time delay is a ubiquitous phenomenon in physical and biological systems. The first study on delay coupled phase oscillators was reported by Schuster and Wagner [19] and they found that the system becomes synchronized and phases become locked with increase in coupling parameter. Further system of many phase oscillators with same delay coupling scheme have been considered by Kim et al. [23, 24]. Presence of delay produces an additional complexity besides the complexity due to coupling or system structures. In general delay systems are infinite dimensional and therefore can display complex dynamics in form of stable oscillations and chaos. In comparison to non delay coupled systems, existence of multitude of synchronized solutions and amplitude death are two major phenomena in delay coupled systems [25-36]. While the importance of time delays in coupled systems is now clear, studies in this area have so far been limited only to discrete or constant delays. In other words, it has been assumed that information reaches from one unit to another after a fixed time which is unchanging as the system evolves, and, moreover, the units act only on the instantaneous value of the received information and forget any previous values. Practically in many real coupled systems, the delay may be continuous time varying with the system evolution, popularly known as distributed delay which leads to more realistic models in science [37-39], while in some cases delay perturbations in systems depend on the frequency variations which is very much familiar in radar communication systems and generally named as dispersive delay [40-42].

In this paper, we consider a coupled system of two non-identical stable limit cycle oscillators. The outputs of the two oscillators are coupled by dispersive delay, in which the perturbations in delay are controlled with the frequency variations. The condition of amplitude stability is assumed in the coupled system, the coupling is therefore due to phase exchange between the two oscillators. We have analyzed the conditions on different system parameters under synchronization. The stability of these synchronized states has also been analyzed [43-45]. On the basis of stability analysis the parametric space plots are presented among different system parameters to separate out stable limit cycle solutions with chaotic solutions. The paper is organized as

ISSN No:-2456-2165 https://doi.org/10.38124/ijisrt/26aug240

follows. First we introduce the dispersive delay coupled system. Further the dependence of synchronization frequencies and phase locked solution on different system parameters have been worked out. Next stability analysis of synchronized solution is discussed. Finally we conclude with results and discussions.

II. SYSTEM OF LINEAR DISPERSIVE DELAY COUPLED PHASE OSCILLAORS

Figure 1 shows the block diagram of the proposed configuration of dispersive delay coupled system of two stable self-sustained limit cycle oscillators. The output signal of the two oscillators in complex form are $v_{1,2}(t) = v_{1,2}^0 e^{j(\omega_{1,2}t + \theta_{1,2}^0)}$, where $v_{1,2}^0$, $\omega_{1,2}$ denotes the constant amplitude, normal frequencies and $\theta_{1,2}^0$ are constant initial phases of two oscillators respectively. The two oscillators are coupled by a dispersive delay $\tau_c(\omega)$, in which the frequency dependent delay function is assumed in the form $\tau_c(\omega'_{1,2}) = \tau_0 + \dfrac{\omega'_{1,2} - \omega_0}{2\pi\mu}$, where τ_0 is the group delay corresponding to centre frequency ω_0. If B is the bandwidth and T represents the duration of the chirp then the chirp rate μ is defined by $\mu = B/T$.

Let us assume the weak coupling of the two oscillators by dispersive delay that is the amplitudes of the two oscillators remain stable. The form of output of the two oscillators after coupling is $v_{1,2}(t) = v_{1,2}^0 e^{j\varphi_{1,2}(t)}$. The total phases of the two oscillators after coupling is represented by $\varphi_{1,2}(t) = \omega'_{1,2}t + \theta_{1,2}(t)$, where $\theta_{1,2}(t)$ represents the time dependent phases and $\omega'_{1,2}$ define the frequencies of two oscillators after coupling. The system of time dependent phase equations is given by following [19, 22 & 36]:

$$\dot{\varphi}_1(t) = \omega_1 + K\cos[\varphi_2\{t - \tau_c(\omega'_2)\} - \varphi_1(t)] \quad \text{(1a)}$$

$$\dot{\varphi}_2(t) = \omega_2 + K\cos[\varphi_1\{t - \tau_c(\omega'_1)\} - \varphi_2(t)] \quad \text{(1b)}$$

Where K is defined as coupling constant, quantifying the strength of coupling between the two oscillators. One can see from the above system that the phase from one oscillator is transmitted to the other by frequency dependent delay terms $\varphi_{2,1}\{t - \tau_c(\omega'_{2,1})\}$. The synchronization dynamics is discussed for above system where the phase difference is bounded, that is $|\varphi_2(t) - \varphi_1(t)| < \text{const}$. We start the investigation of system (1) by introducing two new variables that is phase difference $\psi_d(t) = \varphi_2(t) - \varphi_1(t)$ and mean phase $\psi_m(t) = \dfrac{\varphi_1(t) + \varphi_2(t)}{2}$. The modified equations of system (1) in terms of $\psi_d(t)$ and $\psi_m(t)$ can be obtained as:

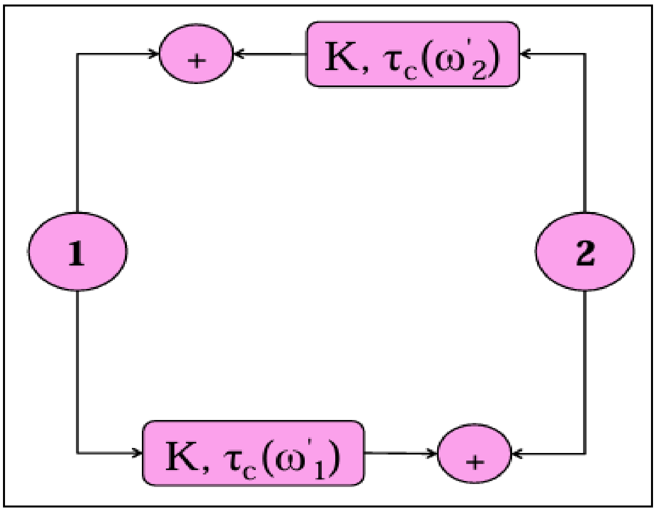

Fig 1 Schematic Representation of Two Interacting Oscillators (Indicated by Big Circles) with Dispersive Delay Coupling.

$$\dot{\psi}_d(t) = \omega_d - 2K\sin\left[\psi_d(t) - \frac{1}{2}\{\omega'_2\tau_c(\omega'_2) - \omega'_1\tau_c(\omega'_1)\}\right]\sin\left[\frac{1}{2}\{\omega'_2\tau_c(\omega'_2) + \omega'_1\tau_c(\omega'_1)\}\right] \quad \text{(2a)}$$

$$\dot{\psi}_m(t) = \omega_m + K\cos\left[\psi_d(t) - \frac{1}{2}\{\omega'_2\tau_c(\omega'_2) - \omega'_1\tau_c(\omega'_1)\}\right]\cos\left[\frac{1}{2}\{\omega'_2\tau_c(\omega'_2) + \omega'_1\tau_c(\omega'_1)\}\right] \quad \text{(2b)}$$

Where $\omega_d = \omega_2 - \omega_1$ is the natural frequency detuning and $\omega_m = \dfrac{\omega_1 + \omega_2}{2}$ defines the mean frequency of the two oscillators. After substituting the values of $\tau_c(\omega'_{1,2})$ and some mathematical simplifications, system (2) can be rewritten as:

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$$\dot{\psi}_d(t) = \omega_d - 2K\sin\left[\psi_d(t) - \frac{\omega'_d\tau_0}{2} - \frac{\omega'_d(2\omega'_m - \omega_0)}{4\pi\mu}\right]\sin\left[\omega'_m\tau_0 + \frac{\{\omega_1'^2 + \omega_2'^2 - (\omega'_1 + \omega'_2)\omega_0\}}{4\pi\mu}\right] \quad (3a)$$

$$\dot{\psi}_m(t) = \omega_m + K\cos\left[\psi_d(t) - \frac{\omega'_d\tau_0}{2} - \frac{\omega'_d(2\omega'_m - \omega_0)}{4\pi\mu}\right]\cos\left[\omega'_m\tau_0 + \frac{\{\omega_1'^2 + \omega_2'^2 - (\omega'_1 + \omega'_2)\omega_0\}}{4\pi\mu}\right] \quad (3b)$$

Where $\omega'_d = \omega'_2 - \omega'_1$ and $\omega'_m = \frac{\omega'_1 + \omega'_2}{2}$ denote frequency detuning and mean frequency of the two oscillators after coupling.

➢ *Synchronization*

Synchronization is the most fundamental phenomenon associated with oscillations. It is directly and widely spread consequence of the interaction of different system with each other. In case of phase synchronization the phase difference between the two oscillators becomes time invariant which implies that the time rate of phase difference is equal to zero, that is $\frac{d\psi_d}{dt} = 0$. In weakly coupled systems the different units pull their natural frequencies towards a common frequency that is synchronization frequency of that system. If $\psi^* = \psi_d(t)$ denotes phase locked solutions or fixed point solutions then substituting $\omega'_1 = \omega'_2 \approx \Omega$ in eqn. (3a), we obtain:

$$\omega_d - 2K\sin(\psi^*)\sin\left[\Omega\tau_0 + \frac{\{\Omega^2 + \Omega^2 - (\Omega + \Omega)\omega_0\}}{4\pi\mu}\right] = 0 \quad (4)$$

After some steps of mathematical manipulations eqn (4) can be simplified as:

$$\psi^* = \sin^{-1}\left(\frac{\omega_d}{2K\sin(\Theta)}\right) \text{ if } \sin(\Theta) > 0 \quad (5a)$$

$$= \pi - \sin^{-1}\left(\frac{\omega_d}{2K\sin(\Theta)}\right) \text{ Otherwise} \quad (5b)$$

Where $\Theta = \Omega\tau_0 + \frac{\Omega(\Omega - \omega_0)}{2\pi\mu}$.

Substituting ψ^* from eqn (5) in eqn (4), the expression for synchronization frequency in terms of system parameters K, ω_d, ω_m, τ_0 and μ can be obtained as:

$$\Omega = \omega_m \pm K\cos(\Theta)\sqrt{1 - \frac{\omega_d^2}{4K^2\sin^2(\Theta)}} \quad (6)$$

Any frequency Ω calculated from eqn (6) defines a single phase locked solution in form of eqns (5), where the first and second values in eqn (6) correspond to the sign '+' and '-' in eqn (5), respectively.

➢ *Stability of Synchronized Solutions*

Stability of a coupled system is an extremely crucial issue. The stability of these synchronized solutions can be discussed with the use of linear stability analysis [43]. Let us assume the perturbation around the fixed points (Ω, ψ^*) is proportional to $\exp(\lambda t)$. The Jacobian matrix corresponds to system (1) after linearization, is obtained as:

$$J = \begin{pmatrix} -K\sin(\Theta + \psi^*) & Ke^{-\lambda\tau_c(\Omega)}\sin(\Theta + \psi^*) \\ Ke^{-\lambda\tau_c(\Omega)}\sin(\Theta - \psi^*) & -K\sin(\Theta - \psi^*) \end{pmatrix} \quad (7a)$$

To characteristic roots λ of linearized system which are just the Lyapunov exponents that measure the stability of synchronized solutions are calculated by equating the following determinant to zero.

$$|J - \lambda I| = \begin{pmatrix} -K\sin(\Theta + \psi^*) - \lambda & Ke^{-\lambda\tau_c(\Omega)}\sin(\Theta + \psi^*) \\ Ke^{-\lambda\tau_c(\Omega)}\sin(\Theta - \psi^*) & -K\sin(\Theta - \psi^*) - \lambda \end{pmatrix} = 0 \quad (8a)$$

$$\lambda^2 + \lambda K\{\sin(\Theta_c + \psi^*) + \sin(\Theta - \psi^*)\} + K^2(1 - e^{-2\lambda\tau_c(\Omega)})\sin(\Theta + \psi^*)\sin(\Theta - \psi^*) = 0 \quad (8b)$$

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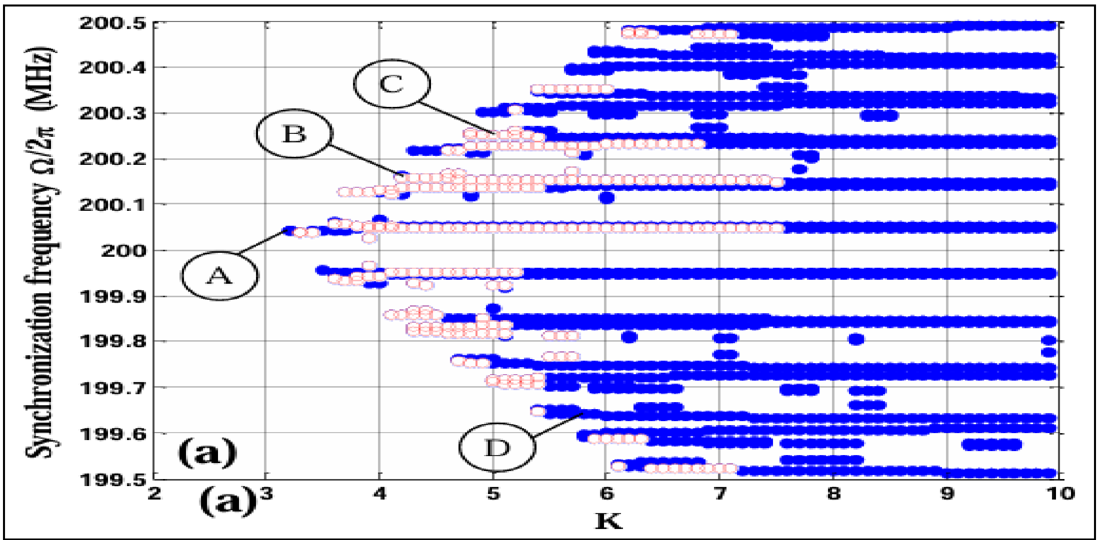
Eqn (8b) is in the form of quasi-polynomials, so in general it is not possible to solve this eqn analytically, except for some special cases and a numerical solution is necessary to obtain characteristic roots of this eqn. It is also obvious that infinite complex roots are possible for a particular synchronized solution. It is obvious that $\lambda = 0$ is always a solution for the above equation. If the real parts of all roots are negative corresponding to a fixed point, it is stable. If a fixed point has at least one eigen value with positive real part, it becomes unstable. The imaginary parts of characteristic roots are concerned with the oscillatory behaviour of trajectories with time after perturbation, around the fixed points.

III. RESULTS & DISCUSSION

We proceed our analysis for two different combinations of system parameters. In first case the difference between the two frequencies, that is frequency detuning ω_d is assuming large in comparison to the second case. The parameter values in first case are considered as: $f_1 = \frac{\omega_1}{2\pi} = 199.5$ MHz, $f_2 = \frac{\omega_2}{2\pi} = 200.5$ MHz, $B = 10$ MHz, $T = 10\mu\text{sec}$, $\mu = \frac{B}{T} = 1.0$, $\tau_0 = 5\mu\text{sec}$.

The variation in synchronization frequency $\Omega/2\pi$ and related phase differences ψ^*/π are depicted with the increasing strength of coupling K and shown in figs. 2(a & b). Two types of regions are appearing in these figures which are separated on the basis of the sign of real parts of characteristic eigen values of fixed points. The blue regions are stable which correspond to the fixed points having all negative real parts of eigen values while red regions represents unstable solutions which correspond to fixed points having at least one eigen value with positive real part. We can see that the phase locked solutions are stable for larger values of coupling strengths while at lower K maximum solutions are unstable. It is clear from $\Omega - K$ plot that for a particular value of K, multi values of Ω are available which is a key feature of delay coupled phase oscillators system, but in our system the number of total values of Ω for a particular K is larger in comparison to constant delay coupled system for the same combination of parameters. This is the main feature of the proposed configuration in which linear dispersive delay coupling is assumed in place of constant delay. Fig 2(a) shows that more and more synchronization frequencies are possible with the increasing values of coupling strength. In this combination of system parameters there are no Ω available for $K < 3.2$. The synchronization frequency varies linearly up to a certain value of coupling strength K and some new frequencies appear, resulting in emergence of new phase locked states with respect to mean Ω along with the middle branch at higher K - values. As we further increase value of K new phase locked solutions are found and this phenomenon of emergence of new synchronization frequencies is up to a certain value of coupling strength $K = 6.1$. On the basis of linear perturbation analysis of fixed points (Ω, ψ^*), stable limit cycle and chaotic regions are shown in fig. 2(c) in $K - \tau_c(\Omega)$ parametric plane. The blue regions correspond to stable periodic solutions while red regions show chaotic solutions. It is clear from $K - \tau_c(\Omega)$ plane that maximum region at lower side of coupling strength is chaotic besides few points. As the value of K is increasing, it is seen that the stable solutions are found in the broad range while very few region show unstable chaotic solutions. Another parametric space is plotted between coupling strength K and frequency detuning ω_d on the basis of stability analysis and stable periodic and chaotic solutions are shown by blue and red regions in fig. 2(d) respectively. From $K - \omega_d$ plane, it is clear that larger area of stable regions are appeared for lower frequency detuning ω_d between two oscillators while for higher values of ω_d, the stable regions are very few and also very small in area.



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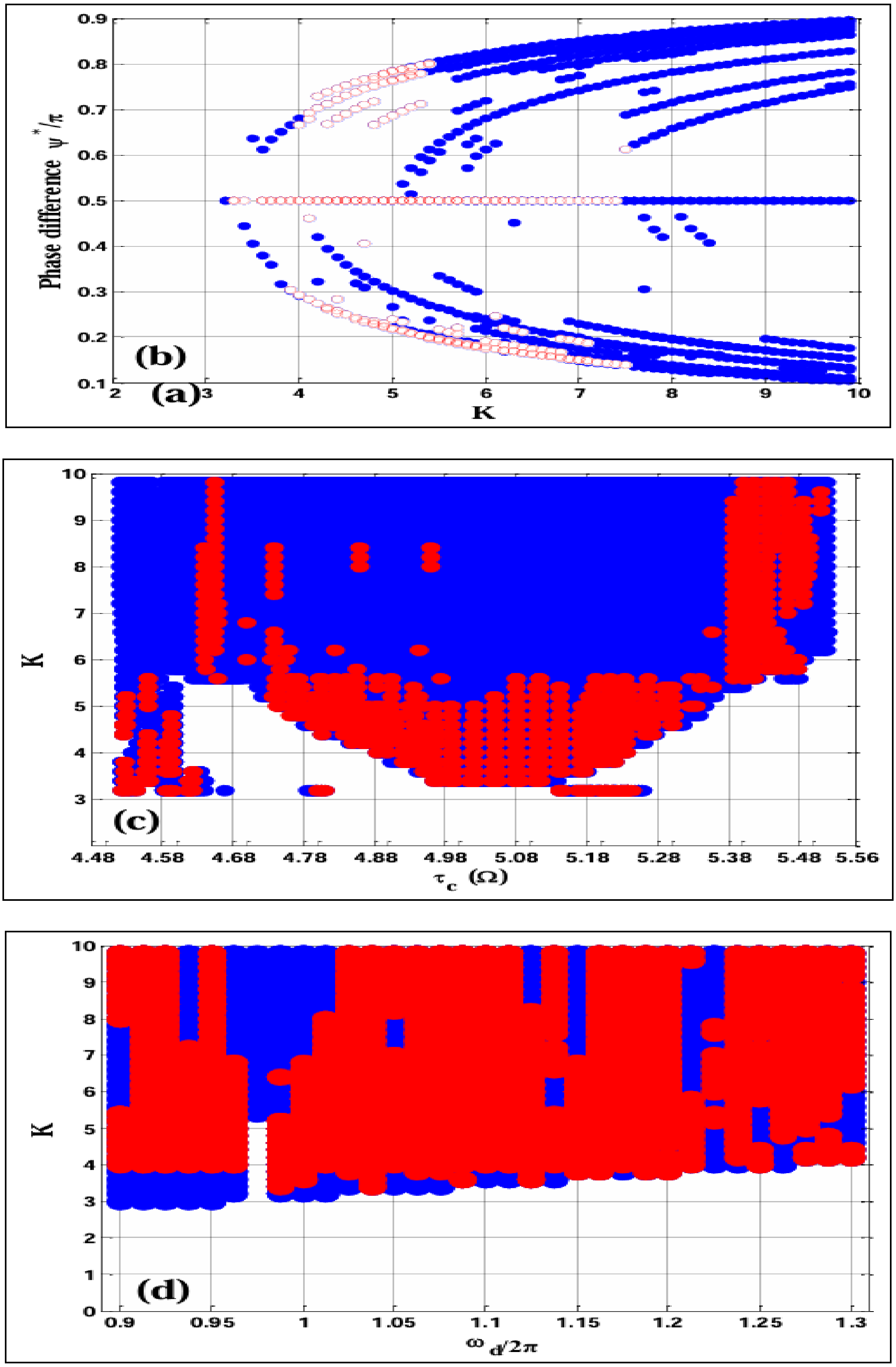
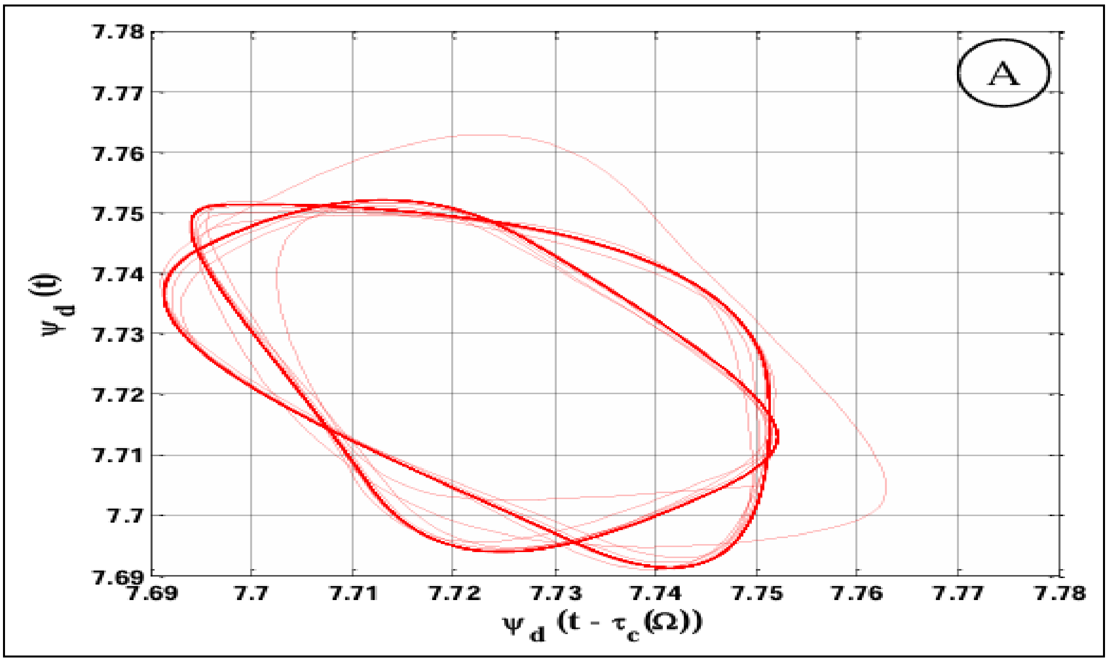


Fig 2 Phase-Locked States of System (1). (a) Synchronization Frequencies Ω and (b) the Corresponding Values of the Phase Differences ψ^* are Depicted Versus Coupling Parameter K. Blue and Red Regions in (a) Indicate Stable and Unstable Phase-Locked States, Respectively. Stable and Unstable Regions in (c) $K-\tau_c(\Omega)$ Plane & (d) $K-\omega_d$ Plane. The Selected Parameters in this Case are $\omega_d = 1.0$ MHz, $\omega_m = 200.0$ MHz, $\mu = 1$ and $\tau_0 = 5.0\ \mu\sec$.

ISSN No:-2456-2165 https://doi.org/10.38124/ijisrt/26aug240

The phase locked states studied above serve as the origin of synchronized dynamics of proposed system. The attracting states of this system for this parameter combination are shown in fig. 3(A, B, C & D) in the $(\psi_d(t-\tau_c(\Omega)), \psi_d(t))$-plane for different values of K, which are mentioned in fig. 2(a). For small value of coupling strength, $K = 3.2$ & $\Omega = 200.04$ MHz, a period-2 stable limit cycle is appeared in fig. 3(A). That means the phase difference $\psi_d(t)$ oscillates periodically and remains bounded. It is clear from fig. 2(a) that for a range of coupling parameter K, more than one limit cycles can coexists (blue regions). Depending on the initial conditions, a trajectory of system can be attracted by any limit cycle, which results in multistability of different stable periodically modulated synchronized states of the system. We can also see the emergence of period-2 limit cyclic behaviour of the system for $K = 5.8$ & $\Omega = 199.65$ MHz in fig. 3(D) which is comparatively denser that shows the relatively fast oscillatory behaviour of the system at higher K-value. The limit cycles may transform into a chaotic attractor. Solutions for chaotic attractors can be observed for $K = 4.2$, $\Omega = 200.16$ MHz and $K = 5.0$, $\Omega = 200.25$ MHz, shown in fig. 3 (B & C). These two are the examples of chaotic attractors and the phase difference ψ_d of a trajectory of system attracted by these chaotic attractors still remains bounded, which corresponds to state of chaotic phase synchronization.



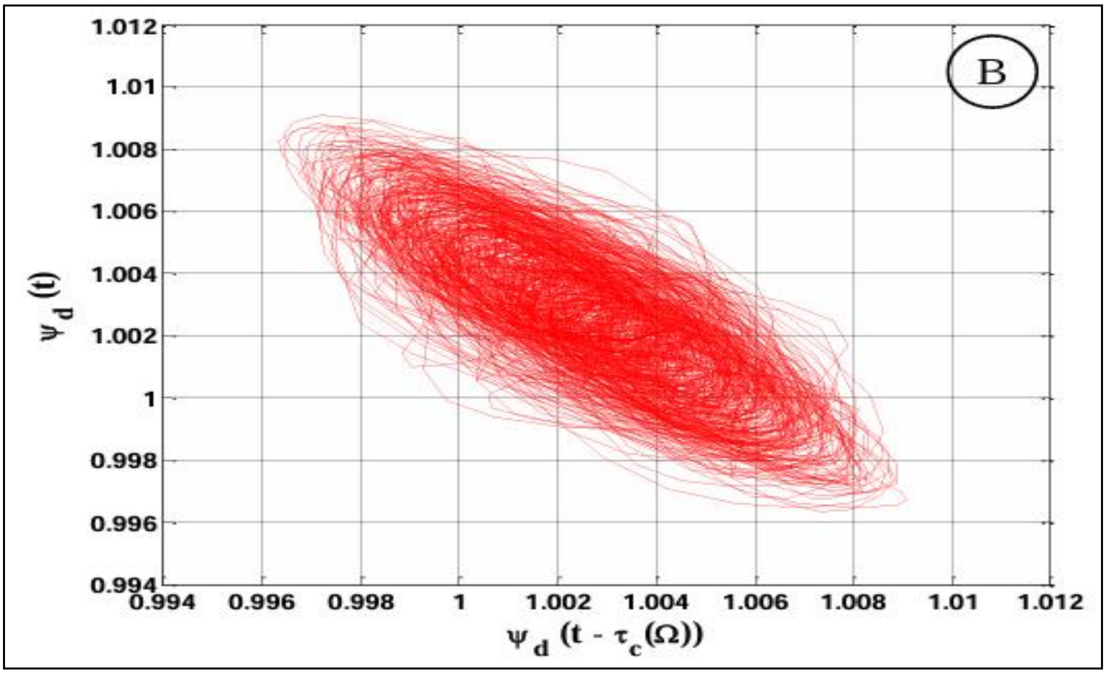


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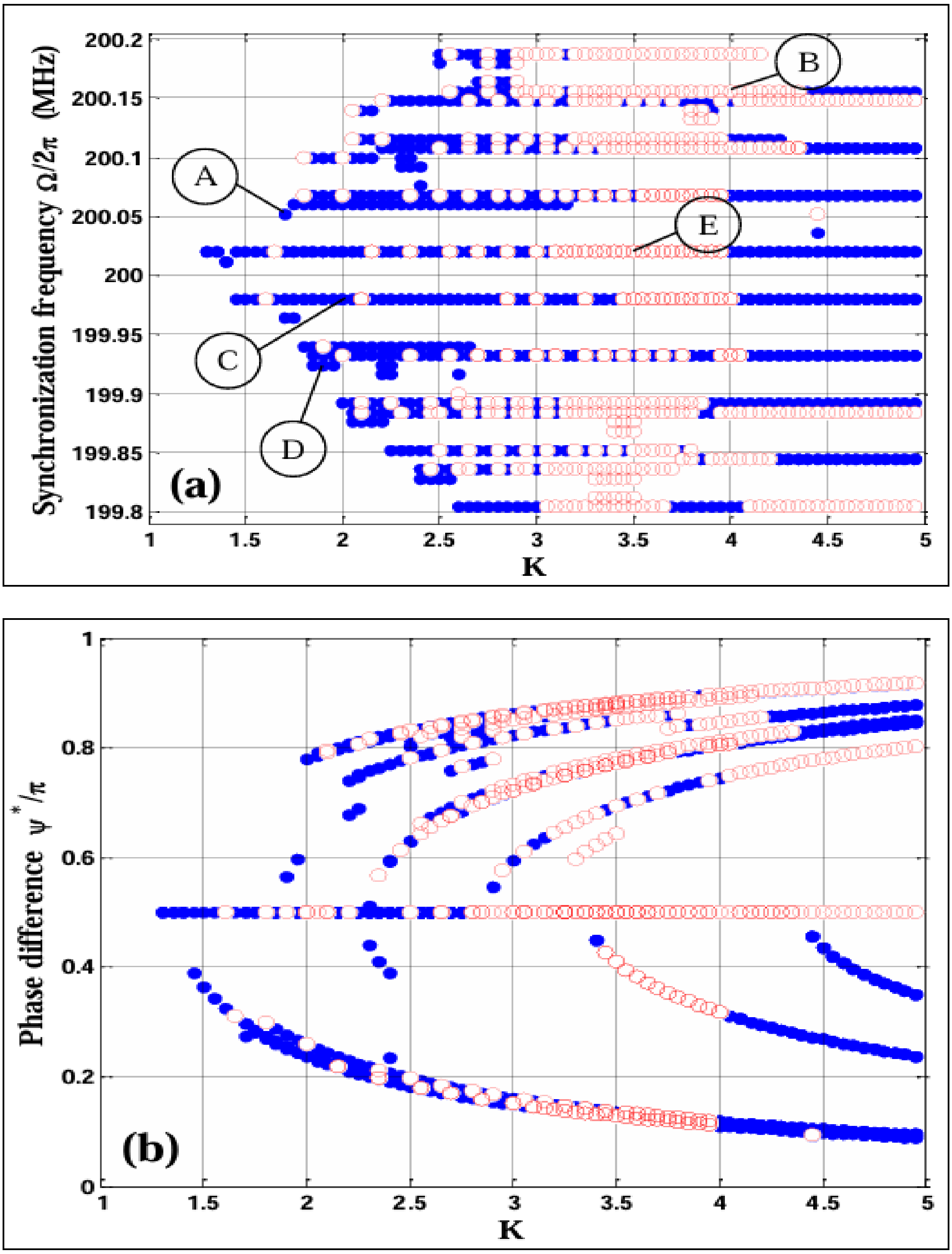
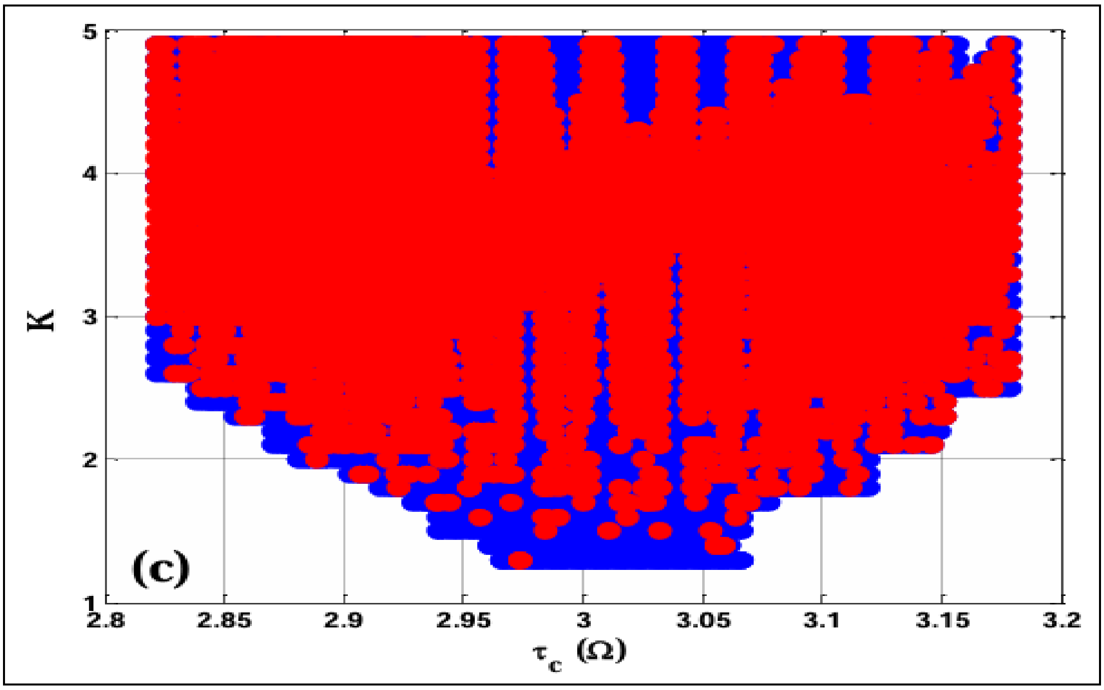
Fig 3 Attracting States of the System for (A) $K = 3.2$ & $\Omega = 200.04$ MHz, (B) $K = 4.2$, $\Omega = 200.16$ MHz, (C) $K = 5.0$, $\Omega = 200.25$ MHz, (D) $K = 5.8$ & $\Omega = 199.65$ MHz. The Other Parameters are $\omega_d = 1.0$ MHz, $\omega_m = 200.0$ MHz, $\mu = 1$ and $\tau_0 = 5.0\ \mu\sec$.

In second case the frequency detuning ω_d is assuming relatively small in comparison of first case. The selected system parameters values are: $f_1 = \frac{\omega_1}{2\pi} = 199.8$ MHz, $f_2 = \frac{\omega_2}{2\pi} = 200.2$ MHz, $\tau_0 = 3\mu\sec$, $B = 15$ MHz, $T = 10\mu\sec$, $\mu = \frac{B}{T} = 1.5$. Fig. 4(a & b) show the variation of phase locked solutions that is synchronization frequency $\Omega/2\pi$ and corresponding phase difference ψ^*/π with increasing coupling strength K. The stable and unstable regions are separated on the basis of linear perturbation analysis of fixed points and indicated by blue and red color respectively. Again it is clear from fig. 4(a) that very large number of synchronization frequencies Ω is available in comparison to constant delay couple phase oscillator system. No phase locked solution is available for $K < 1.3$. First a single branch of phase locked solution varies linearly up to a certain K and new solutions arise in the both side of that single branch after that K-value. On

ISSN No:-2456-2165 https://doi.org/10.38124/ijisrt/26aug240

further increment in values of K results in emergence of new phase locked solutions up to $K = 2.6$. In fig. 4(c), the stable periodic solution and unstable chaotic solutions are mentioned by blue and red colors respectively in parametric space plot between coupling strength K and dispersive delay $\tau_c(\Omega)$. From $K - \tau_c(\Omega)$ plane, we can see that at lower side of K, sufficient region of stable limit cycle solutions appeared while at higher K-values there are few stable periodic solution regions in comparison to chaotic regions. The maximum stable regions are appeared in right side of $K - \tau_c(\Omega)$ plane. The parametric space plot between coupling strength K and frequency detuning ω_d is shown in fig. 4(d) where stable periodic and chaotic solutions are shown by blue and red regions respectively. From $K - \omega_d$ plane, it is clear that maximum area of stable regions are noticed for lower frequency detuning ω_d and K-values, while for higher values of ω_d, the appeared stable regions are very few and also very small in area for smaller values of K. A big area of stable solutions is found at higher ω_d and K-values.





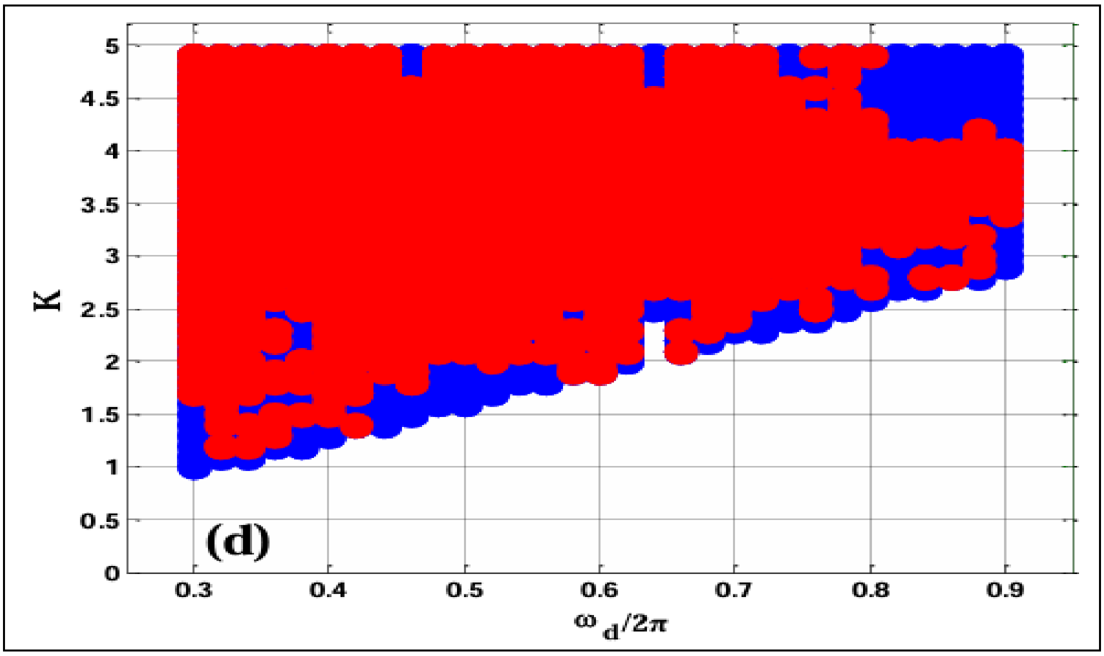


Fig 4 Phase-Locked States of System (1). (a) Synchronization Frequencies Ω and (b) the Corresponding Values of the Phase Differences ψ^* are Depicted Versus Coupling Parameter *K*. Blue and Red Regions in (a) Indicate Stable and Unstable Phase-Locked States, Respectively. Stable and Unstable Regions in (c) $K-\tau_c(\Omega)$ Plane & (d) $K-\omega_d$ Plane. The Selected Parameters in this Case are $\omega_d = 0.4$ MHz, $\omega_m = 200.0$ MHz, $\mu = 1.5$ and $\tau_0 = 3.0\,\mu\text{sec}$.

The attracting states of in the second case are presented in fig. 5(A, B, C, D & E) in the $(\psi_d(t-\tau_c(\Omega)), \psi_d(t))$-plane for mentioned values of K in fig. 3(a). A stable limit cycle is obtained for $K = 1.7$ & $\Omega = 200.05$ MHz, shown in fig. 5(A). A period-2 limit cycle is appeared in fig. 5(D) for $K = 1.82$ & $\Omega = 199.92$ MHz. A period-4 limit is also shown in figure 5(C) and parameter values are $K = 2.0$ & $\Omega = 199.98$ MHz. We can see that in the whole range of coupling strength K there are many continuous stable regions (blue) where more than one limit cycles can coexist and the trajectory of the system at a particular K-value can be attracted by any limit cycle depend on initial conditions. The chaotic attractors in this case are shown in fig. 5(B & E) for the system with parameters $K = 4.0$ & $\Omega = 200.16$ MHz and $K = 3.5$ & $\Omega = 200.02$ MHz.

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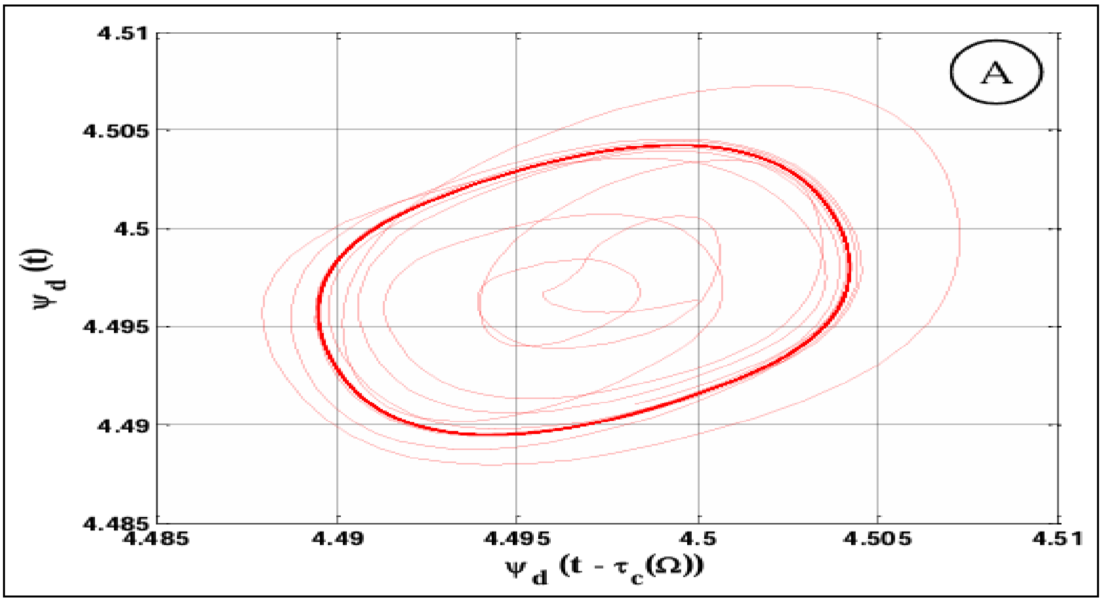

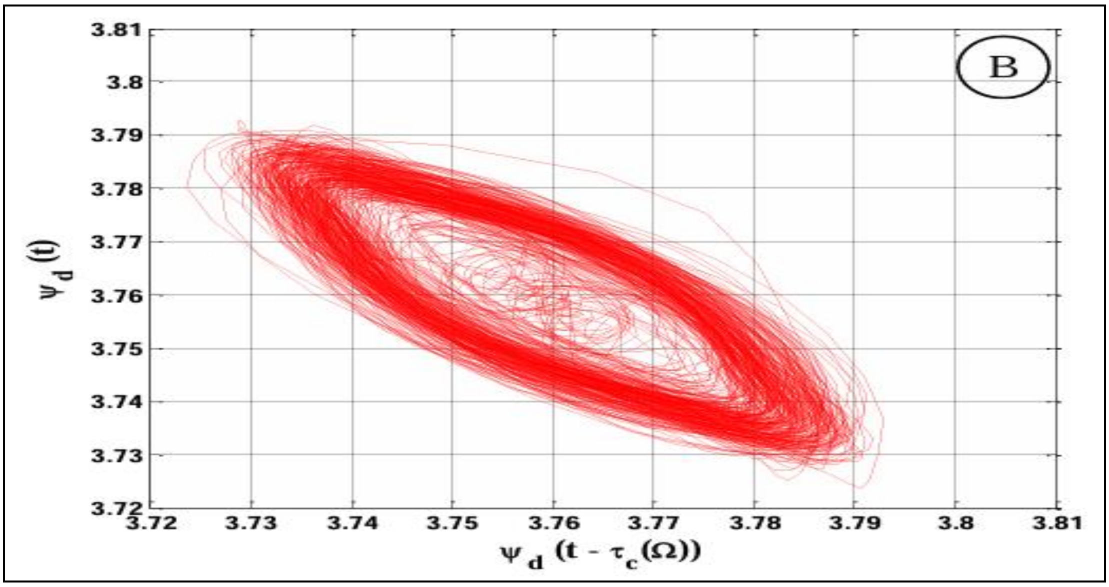

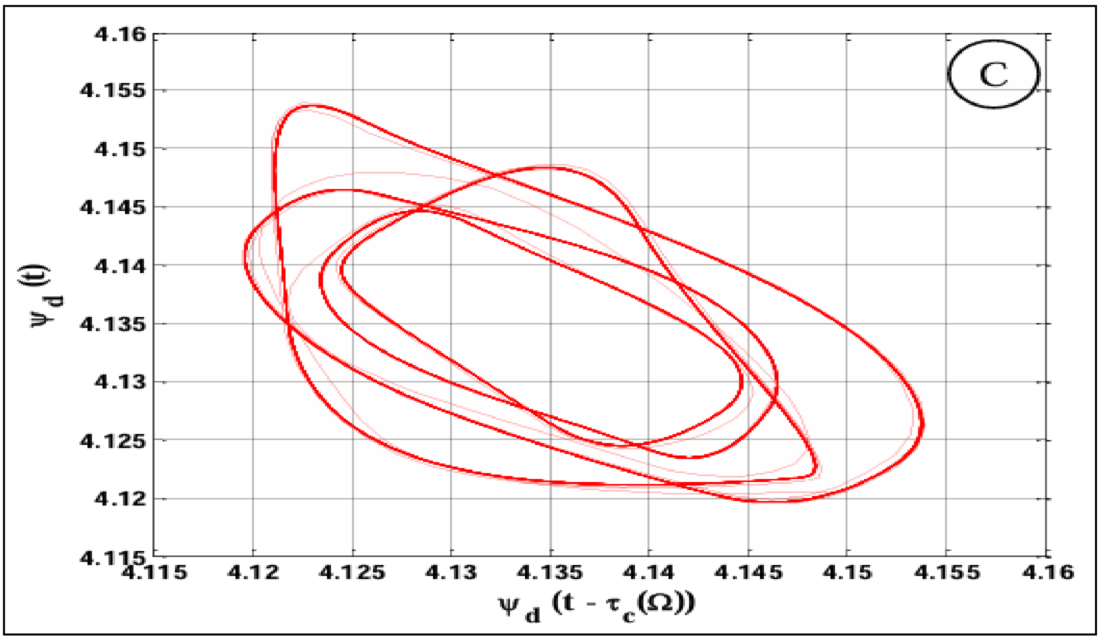

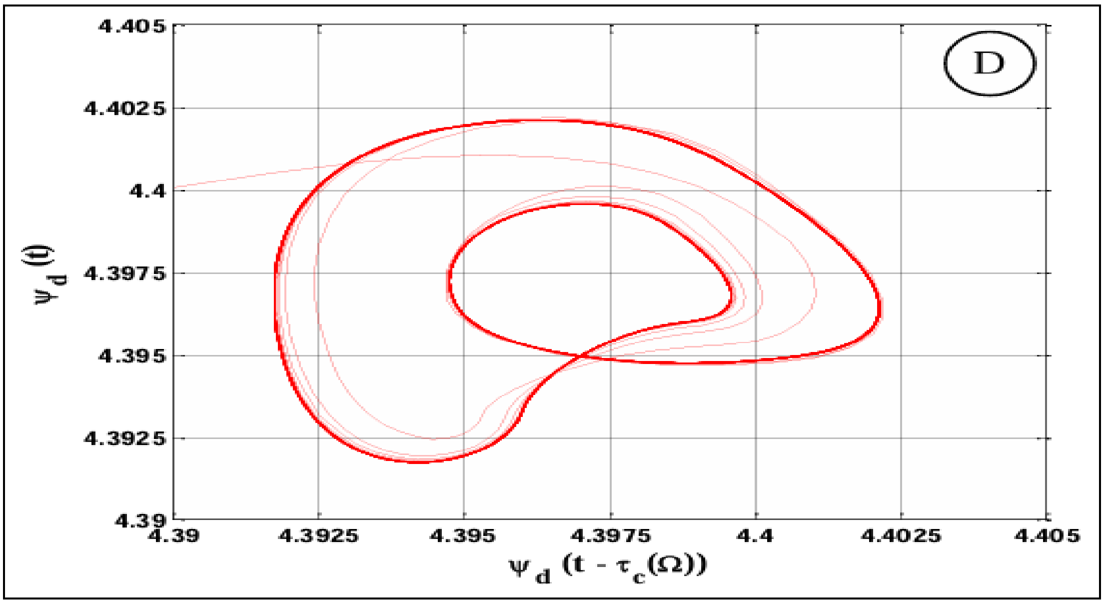

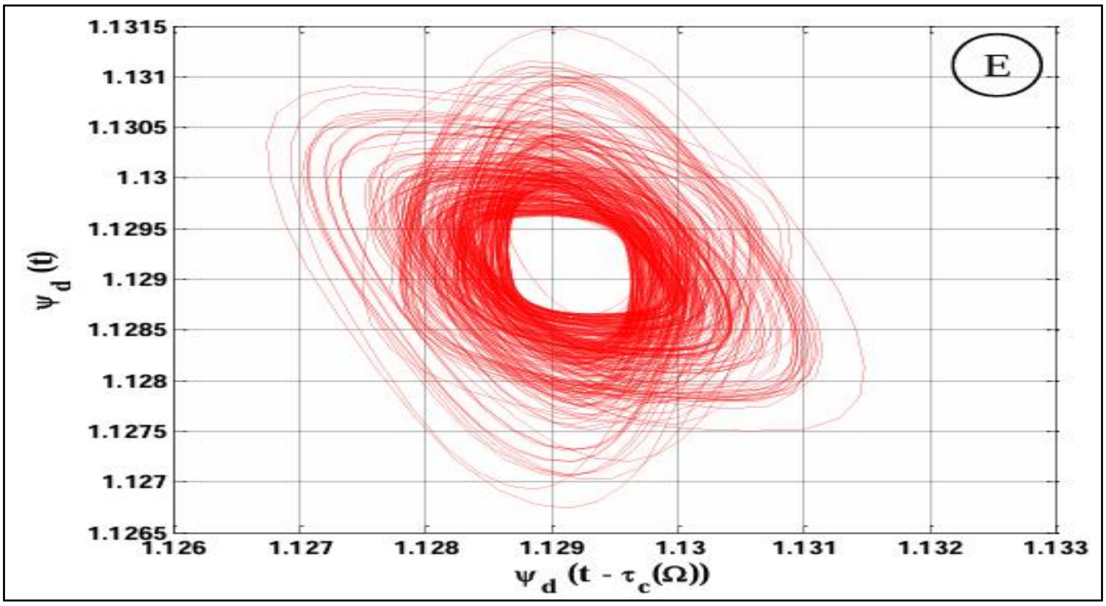

Fig 5 Attracting States of the System for (A) $K = 1.7$ & $\Omega = 200.05$ MHz, (B) $K = 4.0$ $\Omega = 200.16$ MHz, (C) $K = 2.0$ & $\Omega = 199.98$ MHz, (D) $K = 1.82$, $\Omega = 199.92$ MHz & (E) $K = 3.5$ & $\Omega = 200.02$ MHz. The Other Parameters are $\omega_d = 0.4$ MHz, $\omega_m = 200.0$ MHz, $\mu = 1.5$ and $\tau_0 = 3.0\ \mu\text{sec}$.

- *Surface Acoustic Wave (SAW) Dispersive Delay Coupling Implementation In Proposed Configuration*

SAW devices are very popular in communication technology, play a key role as bandpass filters, delay lines with constant or dispersive delays, matched filters, compressors, expanders, convolvers and correlators [46-49]. SAW devices can also be used as sensors. Some physical quantities like temperature directly influence the velocity of the SAW. The sensing of effects like pressure, force, strain or acceleration requires an appropriate mounting and packaging technology [42, 50-51].

The dispersive delay line is designed as a matched filter reducing the sidelobe level of the compressed pulse. The matching condition may be obtained relatively easily by using interdigital transducers of surface waves. The instantaneous frequency of the impulse response varies with time according to the change of the finger spacing and the impulse response amplitude may be made constant by adjusting the finger overlap along the transducer. In a dispersive delay line the delay time τ of a reflected signal depends on the frequency $\omega_0/2\pi$. For a linear dispersion with bandwidth B, dispersion time T and center frequency f_0 we can write for an instantaneous frequency $\omega/2\pi$ within the passband:

$$\tau(\omega) = \tau_0 + \frac{\omega - \omega_0}{2\pi\mu}$$

ISSN No:-2456-2165 https://doi.org/10.38124/ijisrt/26aug240

Where μ is known as chirp rate, defined by $\mu = B/T$. So the transfer function of a SAW dispersive delay line at an instantaneous frequency ω is defined as: $H(\omega) = |H(\omega)|e^{j\omega\tau(\omega)}$. When we are using the SAW dispersive delay as a coupling element the coupling strength is modified in terms of SAW device parameters by assuming amplitude stability, are given by :

$$K_1 = \omega_2' H_c(\omega_2', \tau_c(\omega_2'))$$

$$K_2 = \omega_1' H_c(\omega_1', \tau_c(\omega_1'))$$

Where $\tau_c(\omega_{1,2}') = \tau_0 + \dfrac{\omega_{1,2}' - \omega_0}{2\pi\mu}$

So we can study the synchronization dynamics of SAW dispersive delay coupled two phase oscillators and it is obvious that the dynamics can be controlled be coupling SAW device parameters.

IV. CONCLUSION

In this paper we investigated the dispersive time delay effects on a system of two phase oscillators. It is seen that the system can be synchronized with proper combination of system parameters. The stable phase locked solutions are analysed by using linear perturbation analysis. The attractive states corresponding to stable limit cycle and chaotic phase locked states are presented. Period-1, period-2 and period-4 limit cycles are obtained during the analysis. The trajectory of system attracted by chaotic attractors remains bounded which corresponds to state of chaotic phase synchronization. The islands of stable periodic and chaotic solutions are mentioned in $K - \tau_c(\Omega)$ plane and $K - \omega_d$ plane. It is concluded that the proposed system has the flexibility to use it for stable oscillatory applications as well as for chaotic applications [52-54]. On the basis of dynamics of the proposed system, a configuration of SAW dispersive delay device coupled limit cycle oscillator system can be realized on the basis of this analysis.

ACKNOWLEDGEMENT

Author SSJ is thankful to Principal, Nalanda College, Biharsharif for providing necessary facilities and support during this study.

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