

Domains of Applicability, Transition Evidence and Validation Requirements for the Ergun and Wen-Yu Drag Correlations in Bubbling Fluidized-Bed Roasters

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Abstract. This paper defines the dense and dilute domains represented by the Ergun and Wen-Yu drag correlations and examines the consequences of combining them through the classical Gidaspow switch in a bubbling fluidized-bed roaster. Its specific contribution is a reactor-resolved interpretive and validation framework that separates global fluidization-regime identification from local drag-closure selection and maps the resulting dense, dilute, and switching regions within the roaster. The review is extended with published particle-resolved data, a standard cold-flow benchmark, an original reactor-zone schematic, and quantitative sensitivity calculations. Published lattice-Boltzmann results indicate that an intermediate solids-fraction interval is not adequately represented by either limiting correlation, confirming that the conventional gas-volume-fraction threshold is a numerical selection rule rather than a physical phase boundary. For an illustrative 500 micrometre quartz-air case, the classical switch produces a 7.5% one-sided change in the interphase momentum-exchange coefficient at the threshold, whereas Ergun- and Wen-Yu-based estimates of minimum fluidization velocity differ by 4.4%. These values are calculation examples and do not constitute experimental validation. The paper concludes with a validation matrix and a consistency warning for studies whose reference CFD model uses Syamlal-O'Brien rather than Gidaspow.

Keywords: Ergun; Wen-Yu; Gidaspow; Interphase Drag; Gas-Solid Fluidization; Bubbling Fluidized-Bed Roaster; Lattice-Boltzmann Method; Minimum Fluidization; Model Validation; Start-Up Heating.

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I. INTRODUCTION

Gas-solid fluidized beds support roasting, drying, calcination, combustion, and catalytic operations because vigorous particle motion promotes high rates of momentum and heat transfer. In metallurgical roasting, stable fluidization is particularly important during start-up, when the particle inventory passes from a fixed bed through

incipient fluidization to a heterogeneous bubbling state. The reliability of a numerical model during this transition depends strongly on the constitutive relation used to represent interphase drag, since drag couples the gas and solid momentum balances and directly influences pressure drop, bed expansion, solids circulation, bubble formation, and residence-time distribution [3, 7, 10].

Table 1 Nomenclature and SI Units.

Symbol	Definition	SI unit
β	Gas-solid interphase momentum-exchange coefficient	$\text{kg m}^{-3} \text{s}^{-1}$
α_g, α_s	Local gas and solids volume fractions	-
ρ_g	Gas density	kg m^{-3}
μ_g	Gas dynamic viscosity	Pa s
d_p	Representative particle diameter	m
u_g, u_s	Gas- and solids-phase velocities	m s^{-1}
C_D	Single-particle drag coefficient used in the Wen-Yu closure	-
Re_p	Particle Reynolds number	-
U, U_{mf}	Superficial gas velocity and minimum fluidization velocity	m s^{-1}

A bubbling fluidized-bed roaster does not contain a spatially uniform suspension. Its lower region includes a dense emulsion phase and distributor-induced jets, while bubbles, a fluctuating bed surface, a splash zone, and a dilute freeboard develop above it. These structures coexist and evolve with time. A single reactor-scale classification based only on superficial gas velocity therefore cannot identify the appropriate local drag closure. The model must distinguish the global fluidization regime, commonly assessed through the ratio U/U_{mf} , from the local gas-solid state described by the phase volume fractions.

The Ergun and Wen-Yu correlations represent different limiting conditions. The Ergun relation was derived for pressure loss through a packed particle assembly and captures viscous and inertial resistance in dense regions [2, 7]. The Wen-Yu relation describes drag in a comparatively dilute suspension and incorporates a voidage correction for neighbouring particles [12]. The classical Gidaspow model combines them through a cellwise switch at a local gas volume fraction of 0.80 [3, 4]. Although this hybrid construction is widely implemented, the switching value is often reported incorrectly as a physical phase boundary. Particle-resolved studies instead show a broader intermediate concentration range in which neither limiting expression provides a complete description [1, 5].

This distinction creates a methodological problem for fluidized-bed roaster simulations. An abrupt closure switch can introduce a finite change in the momentum-exchange coefficient even when local state variables vary smoothly. It can also obscure the difference between a minimum-fluidization consistency calculation and full validation of transient, spatially resolved drag. The problem becomes more significant when hydrodynamic results are used to support thermal or energy-efficiency claims without separate heat-transfer closures, wall-loss terms, radiation treatment, and experimental measurements.

This paper therefore defines the physical bases and local applicability domains of the Ergun and Wen-Yu correlations, examines the classical Gidaspow switching convention, evaluates published evidence for the intermediate concentration range, and maps the expected closure allocation within a bubbling fluidized-bed roaster. It also quantifies an illustrative switching discontinuity, compares Ergun- and Wen-Yu-based estimates of minimum fluidization velocity, and proposes a validation matrix for academic and industrial studies. The contribution is methodological and interpretive. It provides a reactor-resolved framework that separates global regime identification, local drag-closure selection, and experimental validation. It does not propose a new drag law or treat illustrative calculations as plant evidence.

II. PHYSICAL BASIS OF THE ERGUN AND WEN-YU CORRELATIONS

➤ Ergun Correlation: Dense-Domain Closure

The Ergun equation was developed to represent the pressure loss of a fluid flowing through a packed particle

bed. In the Gidaspow model, it is reformulated as a local interphase momentum-exchange coefficient for regions with a high solids holdup [2]. A commonly used form is:

$$\beta_{\text{Ergun}} = 150 \frac{\mu_g \alpha_s^2}{\alpha_g d_p^2} + 1.75 \frac{\rho_g \alpha_s |u_g - u_s|}{d_p}. \quad (1)$$

The first term is the viscous contribution and becomes dominant at low particle Reynolds number. The second term is the inertial contribution. This closure is appropriate when multiparticle interactions, crowding, and the short separation distance between particles control the local flow structure. In a bubbling bed, it therefore applies mainly to the dense emulsion phase rather than to the entire reactor volume.

➤ Wen-Yu Correlation: Dilute-Domain Closure

The Wen-Yu correlation represents the drag on particles dispersed in a continuous gas phase, with a voidage correction accounting for neighbouring particles [3, 12]. In a common implementation of the Gidaspow model, the interphase momentum-exchange coefficient is written as:

$$\beta_{\text{Wen-Yu}} = \frac{3}{4} C_D \frac{\rho_g \alpha_g \alpha_s |u_g - u_s|}{d_p} \alpha_g^{-2.65}. \quad (2)$$

The drag coefficient is evaluated from:

$$C_D = \frac{24}{\alpha_g Re_p} \left[1 + 0.15 (\alpha_g Re_p)^{0.687} \right], \quad \alpha_g Re_p < 1000, \quad (3a)$$

And

$$C_D = 0.44, \quad \alpha_g Re_p \geq 1000. \quad (3b)$$

The particle Reynolds number is

$$Re_p = \frac{\rho_g d_p |u_g - u_s|}{\mu_g}. \quad (4)$$

Wen-Yu is appropriate where the gas is clearly the continuous phase and the particles are relatively dispersed. In a fluidized-bed roaster, this condition is commonly found in bubble cores, in the dilute cores of tuyere jets, and in the freeboard. Applying the Wen-Yu closure to the dense emulsion phase would under-represent particle-crowding and packed-bed effects.

➤ Complementarity and Limitations

Ergun and Wen-Yu are not competing models intended to describe exactly the same local state. They represent two limiting hydrodynamic conditions: a dense particle assembly and a dilute suspension. Dimensional analysis places both equations within a common force-balance framework, but this common basis does not justify a sharp, universal boundary between them [7]. Their accuracy depends on particle Reynolds number, local solids volume fraction, particle size distribution, shape, and mesoscale structures such as clusters and bubbles.

➤ Published Evidence for an Intermediate Drag Regime

The conventional switch at a solids volume fraction of 0.20 is best understood as a pragmatic closure-selection rule. It is not a directly measured boundary separating two unique physical mechanisms. Particle-resolved simulations provide a more differentiated picture of the transition between dilute and dense drag behaviour.

Kandhai et al. performed lattice-Boltzmann calculations for particle Reynolds numbers from 1 to 50 and solids volume fractions from 0.10 to 0.50. They reported

that Wen-Yu remained closer to the computed drag near a solids fraction of about 0.30, that neither Wen-Yu nor Ergun adequately represented approximately 0.30-0.43, and that Ergun approached the computed results near a solids fraction of about 0.50 [5]. Beetstra et al. subsequently examined 35 monodisperse combinations up to a Reynolds number of 1000 and 150 systems when bidisperse arrays were included, demonstrating the strong roles of packing fraction, Reynolds number, size ratio, and composition in the drag response [1].

Table 2 Published Evidence Relevant to the Dense-Dilute Transition.

Source	Method and range	Reported result	Implication for roaster modelling
Niven [7]	Dimensional and physical analysis of packed- and fluidized-bed correlations.	Ergun and Wen-Yu can be interpreted within a related force-balance framework.	A common basis does not establish a unique switching voidage.
Kandhai et al. [5]	Particle-resolved lattice-Boltzmann calculations; Re = 1-50; solids fraction = 0.10-0.50.	Wen-Yu closer near 0.30; neither closure adequate over about 0.30-0.43; Ergun closer near 0.50.	The intermediate range is broader than the binary switch suggests.
Beetstra et al. [1]	Lattice-Boltzmann data; 35 monodisperse cases to Re = 1000; 150 systems including bidisperse arrays.	Drag varies materially with packing, Reynolds number, size ratio, and mixture composition.	A single voidage threshold cannot represent all particle systems.

- Note: The reported intervals are study-specific evidence, not universal phase boundaries.

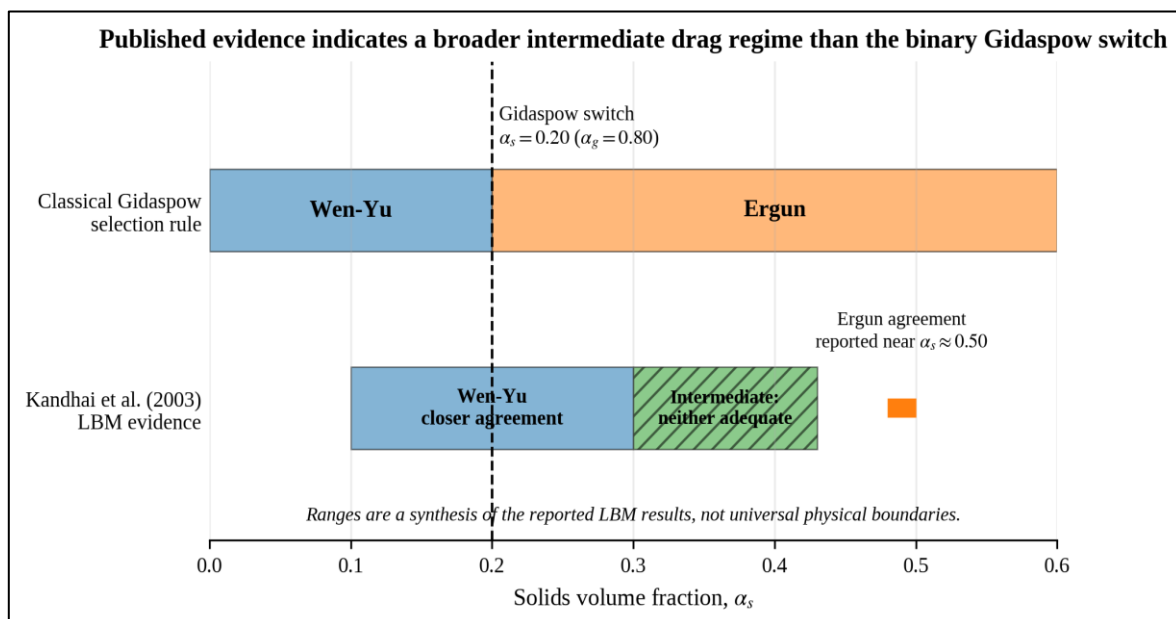


Fig 1 Literature-Derived Comparison of the Binary Gidaspow Switch with the Intermediate Concentration Range Reported by Particle-Resolved Calculations [5]. The Graphic is an Original Synthesis, not a Reproduction of the Source Figure.

Accordingly, the 0.20 solids-fraction threshold should be retained only as the definition of a specific hybrid closure. It should not be described as the experimentally proven point at which dense-bed physics suddenly changes to dilute-suspension physics.

III. LOCAL SWITCHING CRITERION IN THE GIDASPOW MODEL

The local solids volume fraction is related to the gas volume fraction by $\alpha_s = 1 - \alpha_g$. In the classical Gidaspow formulation, the drag closure is selected cell by cell according to:

$$\beta = \begin{cases} \beta_{\text{Ergun}}, & \alpha_g \leq 0.80 \ (\alpha_s \geq 0.20), \\ \beta_{\text{Wen-Yu}}, & \alpha_g > 0.80 \ (\alpha_s < 0.20). \end{cases} \quad (5)$$

Table 3 Classical Gidaspow Local Switching Criterion.

Local region	Volume-fraction condition	Selected closure	Physical interpretation
Dense	$\alpha_g \leq 0.80$; $\alpha_s \geq 0.20$	Ergun	Multiparticle interaction and crowding effects dominate.
Dilute	$\alpha_g > 0.80$; $\alpha_s < 0.20$	Wen-Yu	Particles are dispersed in a continuous gas phase.
Near the threshold	$\alpha_g \approx 0.80$	Repeated switching	A transient cell may alternate between the two closures.

Source: Selection Rule Reported in the Gidaspow Formulation [3, 4].

The value $\alpha_g = 0.80$ is a model convention. It is not a thermodynamic transition and does not define a stationary material interface inside the bed. In a transient simulation, the switching surface moves continuously with bubbles, particle clusters, and jet structures. The scientifically accurate expression is therefore *switching region*, not *competition region* [3, 4].

A criterion based only on the global velocity ratio would state that “Ergun applies when $U < U_{mf}$, whereas Wen-Yu applies when $U > U_{mf}$.” This statement confuses the global fluidization regime with the local drag-model criterion.

In contrast, the ratio U/U_{mf} classifies the global fluidization regime. Within the Gidaspow model, selection between Ergun and Wen-Yu depends on the local gas volume fraction. Consequently, when $U > U_{mf}$, Ergun may remain active in the dense emulsion while Wen-Yu is simultaneously active in bubbles and in the freeboard.

➤ Quantitative Illustration of the Switching Discontinuity

The magnitude of the discontinuity depends on particle size, gas properties, local slip velocity, and the exact implementation of both expressions. To demonstrate the issue without presenting the result as plant data, the two closures were evaluated at identical representative conditions listed in Table 4.

Table 4 Representative Inputs used for the Illustrative Drag and Minimum-Fluidization Calculations.

Quantity	Symbol	Value	Status in this paper
Gas density	ρ_g	1.184 kg m ⁻³	Illustrative air property at 25 °C.
Gas dynamic viscosity	μ_g	1.849×10^{-5} Pa·s	Illustrative air property at 25 °C.
Particle diameter	d_p	500 μ m	Representative quartz-scale particle size used in Figures 2 and 4.
Particle density	ρ_s	2530 kg m ⁻³	Representative quartz particle density used in Figures 2 and 4.
Gas-solid slip speed	$ u_g - u_s $	0.50 m s ⁻¹	Assumption used only for Figure 2.
Voidage at incipient fluidization	ε_{mf}	0.38	Assumption used only for Figure 4.

These inputs define an illustrative calculation set. They are not reported measurements from an operating roaster.

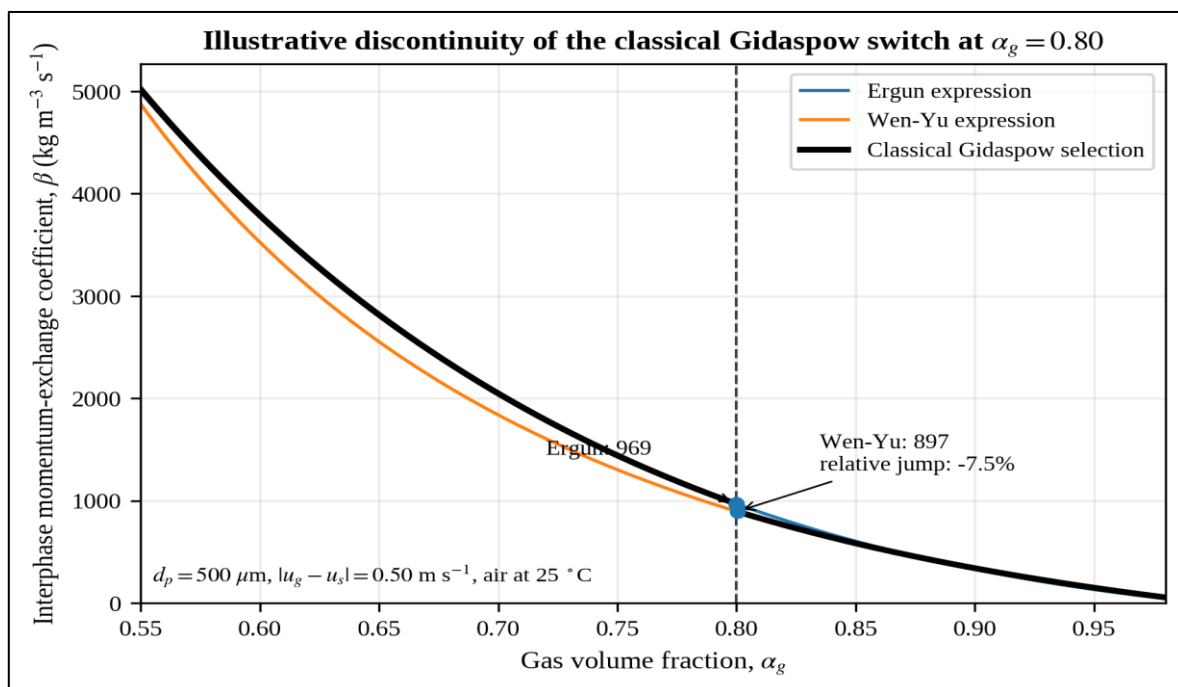


Fig 2 Illustrative Discontinuity of the Classical Gidaspow Selection at a Local Gas Volume Fraction of 0.80. Original Calculation Using the Assumptions in Table 4.

$$J_{\beta} = \frac{\beta_{WY} - \beta_E}{\beta_E} \times 100\% \quad (6)$$

At the switching point, the dense expression gives approximately $969 \text{ kg m}^{-3} \text{ s}^{-1}$ and the dilute expression approximately $897 \text{ kg m}^{-3} \text{ s}^{-1}$. Equation (6) therefore gives a one-sided relative change of about -7.5%. The value is not universal; it quantifies only the selected example. Nevertheless, it demonstrates why cells oscillating around the threshold can experience a non-physical change in drag.

Huilin-Gidaspow-type blending reduces this abruptness but still requires a sensitivity assessment [6].

IV. SPATIAL DISTRIBUTION WITHIN A BUBBLING FLUIDIZED-BED ROASTER

The spatial use of the two correlations must be described from the local gas-solid state rather than from the superficial velocity alone. The main regions expected during start-up and bubbling operation are summarised below [3, 10].

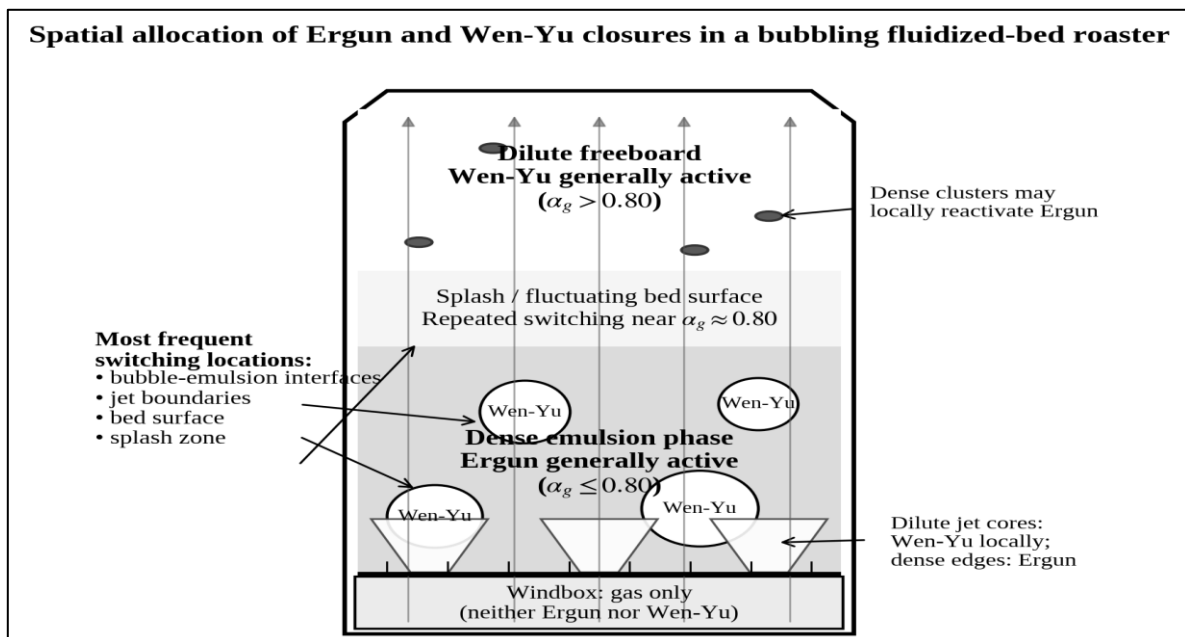


Fig 3 Conceptual Spatial Allocation of Ergun and Wen-Yu Closures in a Bubbling Fluidized-Bed Roaster. The Drawing is an Original Schematic Based on the Local-Voidage Criterion; it is Not a Scale Drawing.

Table 5 Expected Spatial Allocation of the Closures in a Bubbling Fluidized-Bed Roaster.

Roaster region	Local gas-solid state	Probable closure	Physical justification
Fixed bed before fluidization	Strongly packed solids	Ergun	Gas flows through a dense particulate medium.
Dense emulsion phase	Usually $\alpha_g \leq 0.80$	Ergun	This region contains most of the solids inventory and thermal inertia.
Region near the distributor, outside jet cores	Dense, with strong pressure gradients	Ergun	Pressure loss is governed by the high solids concentration.
Bubble cores	Gas-dominated; $\alpha_g > 0.80$	Wen-Yu	Locally dilute suspension with low solids holdup.
Tuyere-jet cores	Frequently depleted in solids	Wen-Yu, locally	The dilute core may use Wen-Yu, while the jet boundary rapidly becomes dense.
Bubble-emulsion interface	α_g fluctuates around 0.80	Repeated switching	Principal region of time-dependent transition between the two closures.
Fluctuating bed surface and splash zone	Alternating dense packets and gas-rich pockets	Ergun/Wen-Yu	Strongly unsteady and spatially heterogeneous region.
Freeboard	Globally dilute	Wen-Yu	Entrained particles are dispersed in a continuous gas phase.
Particle clusters in the freeboard	Locally dense agglomerates	Ergun, locally	A globally dilute region may still contain dense clusters.
Windbox and inside the tuyeres	Gas only	Neither	A single-phase pipe, nozzle, or orifice relation is required.

Source: Original Synthesis from the Local-Voidage Criterion and Published Fluidized-Bed Validation Practice [3, 10].

During the initial fixed-bed stage, when $U < U_{mf}$, Ergun is expected to dominate throughout the particle inventory. Near incipient fluidization, the bed remains mainly dense, although locally open regions may form around the distributor jets. Once bubbling is established, the computational domain becomes a dynamic mosaic: Ergun governs the emulsion phase, whereas Wen-Yu governs bubble interiors and other dilute volumes. In turbulent or circulating regimes, the fraction of cells assigned to Wen-Yu generally increases, but Ergun may remain active in dense clusters and in the lower dense region.

The practical switching region is therefore concentrated at bubble-emulsion interfaces, jet boundaries, the fluctuating bed surface, and the splash zone. Its position is not fixed; it follows bubble motion, particle entrainment, and internal solids circulation.

V. NUMERICAL, THERMAL, AND METHODOLOGICAL CONSEQUENCES

➤ Abrupt Transition at $\alpha_g = 0.80$

In the classical Gidaspow model, switching at $\alpha_g = 0.80$ is discontinuous. A cell in which α_g oscillates between 0.79 and 0.81 can change closure repeatedly. This behaviour may introduce a local discontinuity in β , oscillations in solids velocity, greater sensitivity to time-step and mesh resolution, and more difficult convergence in highly unsteady regions. Smoothed variants, including Huilin-Gidaspow-type blending, reduce the discontinuity but do not remove the need for sensitivity analysis [6].

➤ Published Benchmark and Incipient-Fluidization Consistency Check

A roaster-scale drag model cannot be validated from a single calculated velocity. It should first reproduce well-documented hydrodynamic benchmarks and then be tested against hot-bed measurements. The pseudo-two-dimensional bed studied by Taghipour et al. is frequently used because the geometry, particles, and principal validation observables were reported explicitly [10].

Table 6 Published Cold-Flow Benchmark Frequently Used for Gas-Solid Fluidized-Bed Validation.

Parameter	Published value	Use in validation
Bed geometry	0.28 m wide $\times$ 1.00 m high $\times$ 0.025 m deep	Defines the pseudo-2D computational domain.
Initial static bed height	0.40 m	Reference for bed expansion.
Particles	Spherical glass beads, 250-300 μm ; nominal 275 μm	Defines the particle scale and distribution.
Particle density	2500 kg m^{-3}	Required for buoyancy and drag calculations.
Initial solids volume fraction	0.60	Defines the initial packed state.
Minimum fluidization velocity	Approximately 0.065 m s^{-1}	Reference transition for the cold-flow bed.
Reported observables	Pressure drop, bed expansion, and voidage profiles	Provides multiple validation targets.

Source: Taghipour, Ellis, and Wong [10]. This is a Cold-Flow Hydrodynamic Benchmark, Not a High-Temperature Roaster Validation.

For the representative 500 micrometre quartz-air case in Table 4, a separate minimum-fluidization consistency check was performed. The Archimedes number, the Wen-Yu estimate, and the Ergun force balance at incipient fluidization are written as follows.

$$Ar = \frac{g d_p^3 \rho_g (\rho_s - \rho_g)}{\mu_g^2} \quad (7)$$

$$Re_{mf} = \sqrt{33.7^2 + 0.0408 Ar} - 33.7, \quad U_{mf} = \frac{Re_{mf} \mu_g}{\rho_g d_p} \quad (8)$$

$$150 \frac{\mu_g (1 - \varepsilon_{mf})^2}{\varepsilon_{mf}^3 d_p^2} U_{mf} + 1.75 \frac{\rho_g (1 - \varepsilon_{mf})}{\varepsilon_{mf}^3 d_p} U_{mf}^2 = (\rho_s - \rho_g)(1 - \varepsilon_{mf})g \quad (9)$$

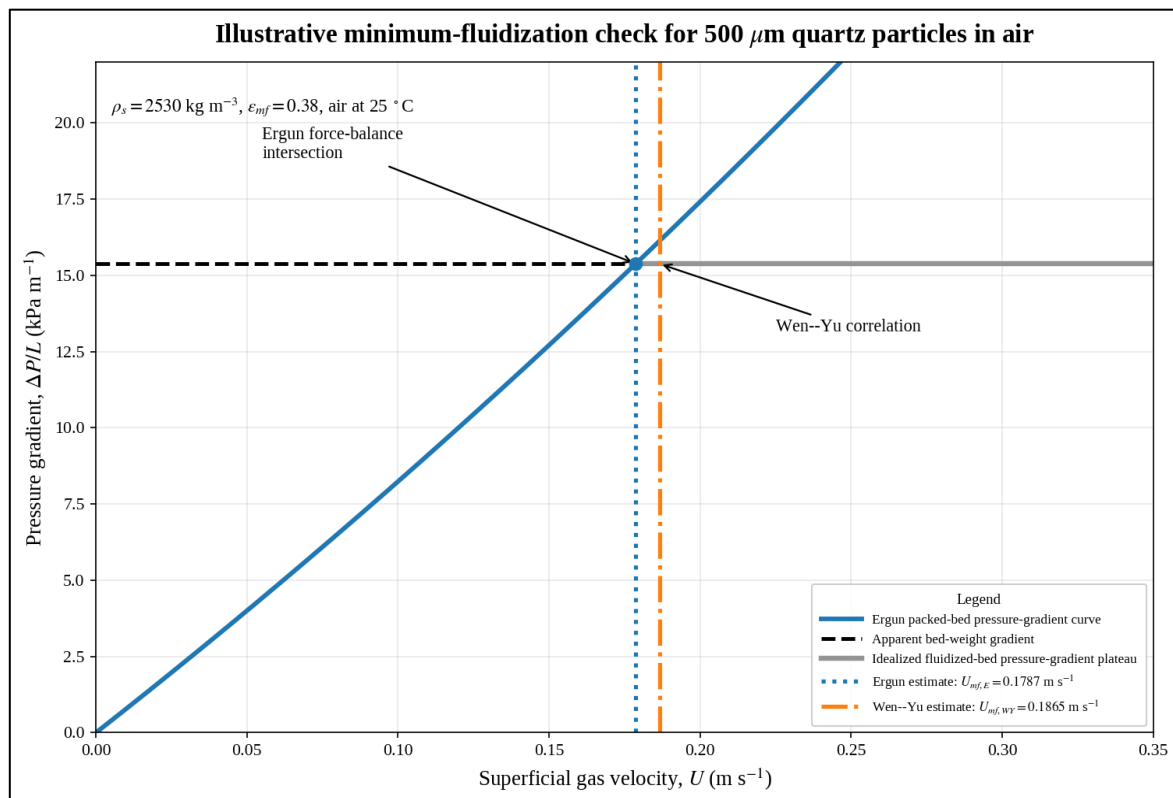


Fig 4 Illustrative Minimum-Fluidization Consistency Check for the Parameter set in Table 4. The Legend Identifies the Ergun Packed-Bed Pressure-Gradient Curve, the Apparent Bed-Weight Gradient, the Idealized Fluidized-Bed Pressure-Gradient Plateau, and the Vertical Minimum-Fluidization Estimates Obtained from the Ergun Force Balance and the Wen-Yu Correlation. Original Calculation; No Experimental fit was Applied.

The calculation gives an Archimedes number of 1.07×10^4 and a Wen-Yu minimum-fluidization Reynolds number of 5.97. The corresponding minimum fluidization velocities are 0.1865 m s^{-1} from Wen-Yu and 0.1787 m s^{-1} from the Ergun pressure-gradient intersection, a difference of 4.4%. This agreement is a useful internal check, but it does not validate the drag closure in bubble cores, the splash zone, the freeboard, or a reacting hot bed.

➤ Recommended Numerical Checks

A defensible CFD study must distinguish physical behaviour from switching artefacts. The diagnostic interval around the conventional threshold may be used in post-processing, but it is only a numerical monitoring band. The minimum validation matrix in Table 7 combines hydrodynamic, thermal, numerical, and conservation checks recommended in published comparative CFD practice [10, 11].

Table 7 Minimum Validation Matrix for a Roaster Drag-Model Study.

Observable or check	Physical or numerical purpose	Minimum reporting requirement
Bed pressure drop	Tests the integrated gas-solid momentum balance.	Transient and mean values compared with measurements or a documented benchmark.
Expanded bed height	Tests global solids suspension and drag strength.	Mean height, fluctuation range, and method of detection.
Local gas volume fraction	Tests bubble, emulsion, cluster, and freeboard structure.	Profiles or maps at specified elevations and times.
Solids entrainment	Tests dilute-region drag and freeboard behaviour.	Outlet solids flux or inventory loss over a defined interval.
Temperature at several elevations	Tests the hydrodynamic consequence for heat transfer.	Measured and calculated histories, not only a final mean temperature.
Closure occupancy	Detects excessive switching near the threshold.	Fraction of cells using each closure and residence time in the monitoring band.
Mesh and time-step sensitivity	Separates numerical artefacts from physical trends.	At least three meshes where practical and a documented transient sensitivity check.
Mass and energy closure	Tests conservation and boundary-condition consistency.	Reported imbalance over the complete calculation period.

Sources: Benchmark and Comparative CFD Validation Principles Synthesised from [10, 11].

➤ *Consistency with the Drag Closure Used in the Underlying Thesis*

The underlying thesis manuscript identifies the Syamlal-O'Brien drag model in its gas-solid parameter table. Under this separate closure, the Ergun/Wen-Yu switch described above is not active in the reference calculation. The switching analysis becomes applicable only if the Gidaspow model or a corresponding blended variant is explicitly selected. The model choice reported in the thesis should therefore be cited separately from the Syamlal-O'Brien formulation documented in the MFIx theory guide [8, 9].

This distinction is methodologically important. It would be internally inconsistent to state that the reference calculation uses Ergun in the dense bed and Wen-Yu in the freeboard while simultaneously declaring Syamlal-O'Brien as the active drag law. In that situation, the Ergun/Wen-Yu discussion must be presented as a theoretical comparison or a sensitivity study, not as a description of the model actually solved.

Finally, selecting a drag correlation does not by itself demonstrate improved heating efficiency. Drag modifies hydrodynamics and therefore the distribution of gas and solids. The thermal consequence must then be established using the selected gas-solid heat-transfer closure, wall heat losses, radiation model, burner heat input, and a complete energy balance. Any conclusion concerning fuel consumption or temperature uniformity requires those quantities and, preferably, experimental validation.

VI. CONTRIBUTION TO FLUIDIZATION KNOWLEDGE, CONCLUSIONS, AND RECOMMENDATIONS

The paper does not introduce a new drag correlation or a new universal solids-fraction threshold. Its contribution is an applied, reactor-resolved framework that connects three levels of analysis that are often conflated: the global onset and regime of fluidization expressed through the ratio U/U_{mf} ; the local selection of a momentum-exchange closure through the gas or solids volume fraction; and the spatial allocation of dense- and dilute-domain closures to the emulsion phase, bubble cores, tuyere jets, splash zone, and freeboard of a bubbling fluidized-bed roaster.

By interpreting the gas-volume-fraction value of 0.80 as a numerical switching convention rather than a material interface, quantifying the illustrative drag discontinuity at the switch, and separating a minimum-fluidization consistency check from validation of local drag, the paper provides a more rigorous basis for drag-model selection, sensitivity analysis, and academic reporting. This is a methodological and interpretive contribution to fluidization modelling. It would constitute a broader physical contribution only after validation with new particle-resolved, pilot-scale, or industrial data under roaster-relevant conditions.

Within this framework, the Ergun and Wen-Yu correlations describe complementary limiting domains of gas-solid flow. In the classical Gidaspow formulation, Ergun is selected in dense regions and Wen-Yu in dilute regions according to the local gas volume fraction. Both can therefore be active simultaneously in different cells of the same bubbling fluidized-bed roaster.

The conventional threshold is not a physical domain of competition. Particle-resolved evidence indicates a broader intermediate concentration range in which neither limiting correlation is fully satisfactory. The illustrative calculations in this paper further show that abrupt switching can create a finite change in drag even when all local state variables vary smoothly.

The academic value of a drag-model comparison therefore depends on data provenance and validation. Published cold-flow benchmarks are useful for hydrodynamics, whereas claims about start-up heating require temperatures, pressure drops, bed expansion, and energy balances measured under conditions relevant to the roaster. The figures and calculations presented here are analytical aids; they are not substitutes for such validation.

➤ *For Rigorous Thesis Reporting, the Following Requirements should be Applied:*

- State explicitly the drag model and software implementation actually selected;
- Distinguish the global calculation of U_{mf} from the local selection of the drag closure;
- describe Ergun/Wen-Yu allocation using α_g or α_s , not the ratio U/U_{mf} alone;
- Quantify hydrodynamic and thermal sensitivity to the drag model;
- Avoid definitive industrial conclusions until pressure drop, bed expansion, and temperature profiles have been validated; and
- Do not claim that Gidaspow is superior to Syamlal-O'Brien without a controlled comparison and relevant validation data.

The new knowledge offered by this paper is the integrated local-voidage, reactor-zone, and validation interpretation of Ergun/Wen-Yu use in bubbling fluidized-bed roasters, not the Ergun and Wen-Yu correlations themselves.

In the Gidaspow drag model, the Ergun correlation is applied to dense gas-solid regions characterised by a local gas volume fraction $\alpha_g \leq 0.80$, whereas the Wen-Yu correlation is applied to dilute regions for which $\alpha_g > 0.80$. In a bubbling fluidized-bed roaster, Ergun mainly governs the dense emulsion phase and regions near the distributor, while Wen-Yu governs bubble cores, dilute gas jets, and the freeboard. The threshold $\alpha_g = 0.80$ is a numerical switching limit rather than a universal physical boundary. The two correlations may therefore be active simultaneously in different reactor regions. This description applies to the

reference simulation only when Gidaspow, or a corresponding blended variant, is actually selected as the drag model.

REFERENCES

- [1]. R. Beetstra, M.A. van der Hoef, and J.A.M. Kuipers, "Drag Force of Intermediate Reynolds Number Flow Past Mono- and Bidisperse Arrays of Spheres," *AIChE Journal*, 53(2) (2007), 489–501, <https://doi.org/10.1002/aic.11065>.
- [2]. S. Ergun, "Fluid Flow through Packed Columns," *Chemical Engineering Progress*, 48 (1952), 89–94.
- [3]. D. Gidaspow, *Multiphase Flow and Fluidization: Continuum and Kinetic Theory Descriptions* (Boston, MA: Academic Press, 1994), <https://doi.org/10.1016/C2009-0-21244-X>.
- [4]. D. Gidaspow and B. Ettehadieh, "Fluidization in Two-Dimensional Beds with a Jet. 2. Hydrodynamic Modeling," *Industrial & Engineering Chemistry Fundamentals*, 22(2) (1983), 193–201, <https://doi.org/10.1021/i100010a008>.
- [5]. D. Kandhai, J.J. Derksen, and H.E.A. Van den Akker, "Interphase Drag Coefficients in Gas-Solid Flows," *AIChE Journal*, 49(4) (2003), 1060–1065, <https://doi.org/10.1002/aic.690490423>.
- [6]. H. Lu, D. Gidaspow, J. Bouillard, and W. Liu, "Hydrodynamic Simulation of Gas-Solid Flow in a Riser Using Kinetic Theory of Granular Flow," *Chemical Engineering Journal*, 95(1–3) (2003), 1–13, [https://doi.org/10.1016/S1385-8947\(03\)00062-7](https://doi.org/10.1016/S1385-8947(03)00062-7).
- [7]. R.K. Niven, "Physical Insight into the Ergun and Wen & Yu Equations for Fluid Flow in Packed and Fluidised Beds," *Chemical Engineering Science*, 57(3) (2002), 527–534, [https://doi.org/10.1016/S0009-2509\(01\)00371-2](https://doi.org/10.1016/S0009-2509(01)00371-2).
- [8]. R.K.M. Rumbu, *Optimisation du chauffage au démarrage des fours de grillage à lit fluidisé bouillonnant par simulation numérique des écoulements: efficacité énergétique et applications industrielles*, MSc thesis (Kinshasa, D.R. Congo: Institut Supérieur de Techniques Appliquées, 2025).
- [9]. M. Syamlal, W. Rogers, and T.J. O'Brien, *MFIX Documentation: Theory Guide*, DOE/METC-94/1004, NTIS/DE9400087 (Morgantown, WV: U.S. Department of Energy, 1993), <https://doi.org/10.2172/10145548>.
- [10]. F. Taghipour, N. Ellis, and C. Wong, "Experimental and Computational Study of Gas-Solid Fluidized Bed Hydrodynamics," *Chemical Engineering Science*, 60(24) (2005), 6857–6867, <https://doi.org/10.1016/j.ces.2005.05.044>.
- [11]. B.G.M. van Wachem, J.C. Schouten, C.M. van den Bleek, R. Krishna, and J.L. Sinclair, "Comparative Analysis of CFD Models of Dense Gas-Solid Systems," *AIChE Journal*, 47(5) (2001), 1035–1051, <https://doi.org/10.1002/aic.690470510>.
- [12]. C.Y. Wen and Y.H. Yu, "Mechanics of Fluidization," *Chemical Engineering Progress Symposium Series*, 62(62) (1966), 100–111.