ISSN No: -2456-2165 https://doi.org/10.38124/ijisrt/26feb393

A Delaporte Innovation Time Series Model for Dependent and Overdispersed Count Data

Enesi Latifat Oyiza[1]; Shobanke Dolapo[2]; Paul Otaru[3]; Benson Onoghojobi[4]; Babatunde O. R.[5]

[1]Department of Mathematical Sciences, Prince Abubakar Audu University Anyigba, Kogi State
[2]Department of Statistics, Federal University Lokoja, Kogi State
[3]Department of Statistics, Federal University Lokoja, Kogi State
[4]Department of Statistics, Federal University Lokoja, Kogi State
[5]Department of Statistics, Federal University Lokoja, Kogi State

Publication Date: 2026/03/02

Abstract: The Delaporte-DCINMA(q) model is a novel integer-valued moving average procedure for overdispersed count time series that is presented in this study. The model preserves discreteness through binomial thinning while overcoming the equidispersion limitation of Poisson-based models by utilizing Delaporte-distributed innovations. We establish the moment structures and important statistical features of the model. Simulation studies show the finite-sample performance and consistency of the estimator. The model's practical usefulness is confirmed by an application to U.S. polio death data, which effectively captures both considerable serial dependence and overdispersion. A versatile and reliable framework for examining correlated count data from a variety of disciplines is offered by the suggested model.

Keyword: *Generalized Method of Moments (GMM), Delaporte Distribution, DCINMA Model, Overdispersion, and Integer-Valued Time Series.*

How to Cite: Enesi Latifat Oyiza; Shobanke Dolapo; Paul Otaru; Benson Onoghojobi; Babatunde O. R. (2026) A Delaporte Innovation Time Series Model for Dependent and Overdispersed Count Data. *International Journal of Innovative Science and Research Technology,* 11(2), 2158-2165. https://doi.org/10.38124/ijisrt/26feb393

I. INTRODUCTION

In several disciplines, such as epidemiology, criminology, insurance, and environmental studies, the study of count data has grown in significance. Since traditional linear time series models might produce non-integer values and presume continuity, they are frequently inappropriate for such data. Because of this restriction, integer-valued models that maintain discreteness through thinning operators have been developed, such as the autoregressive (INAR) and moving average (INMA) families [1], [2]. The Dependent Counting Integer-Valued Moving Average (DCINMA) model is a noteworthy contribution to this field of study. It has demonstrated use in depicting the dynamics of correlated count sequences and introduces dependence via a Bernoulli-driven thinning mechanism [3]. According to Yu and Wang (2021) [3], the DCINMA model's initial iterations were built using Poisson innovations. Although the Poisson option was straightforward and tractable, its applicability to real data, which frequently exhibit higher variability, is limited by its equidispersion feature, which states that the mean equals the variance. Extensions based on more adaptable distributions have been inspired by this. To provide richer variance structures and allow for modest levels of overdispersion, for instance, Hermite innovations have been developed [4]. These changes reflect the increasing understanding that, especially in the presence of excess variability, real-world datasets rarely meet the strict criteria of the Poisson distribution.

By using the Delaporte distribution for the innovation process, this project expands upon the DCINMA framework. A three-parameter structure that represents a broad variety of dispersion patterns, from near-equidispersion to heavy overdispersion, is provided by the Delaporte distribution, which combines characteristics of the Poisson and negative binomial distributions [5, 6]. The DCINMA model is more flexible and more suitable for applications where substantial variability is noticed when Delaporte innovations are incorporated into it. Determining the Delaporte-DCINMA(q) model's primary statistical characteristics and creating estimating techniques based on the Yule–Walker method are the objectives of the study.

➢ *The Delaporte-DCINMA(q) Model*

Two common characteristics of count time series data are temporal dependence and variability that is greater than what the Poisson distribution can account for. While Poisson-based models sometimes enforce an unduly tight equidispersion requirement [3], traditional linear time series

ISSN No: -2456-2165 https://doi.org/10.38124/ijisrt/26feb393

models are inappropriate in this situation because they fail to ensure discreteness [7]. Researchers have created integer-valued moving average techniques that combine more adaptable innovation distributions with thinning operators in order to overcome these problems [1], [2], and [8]. These models provide richer dispersion structures, allow dependence over time, and preserve the discreteness of the data.

This family of processes is expanded by the Delaporte-DCINMA(q) model, which incorporates Delaporte-distributed innovations into a moving average framework with binomial thinning. In formal terms, the procedure is described as.

$$X_t = \varepsilon_t + \sum_{j=1}^{q} \beta_j \circ \varepsilon_{t-j},\ t \in Z \tag{1}$$

Where X_t is the observed integer-valued time series, $\{\varepsilon_t\}$ is a sequence of independent and identically distributed Delaporte random variables, and $\beta_j \circ \varepsilon_{t-j}$ denotes the binomial thinning operator applied to the lagged innovations. This operator ensures that the process remains integer-valued while introducing a simple and interpretable dependence mechanism [2], [8].

The Delaporte-DCINMA(q) model's capacity to distinguish between the functions of dependency and dispersion is one of its main advantages. The memory of the process is controlled by the thinning coefficients β_j, which establish how previous innovations affect present values. In the Delaporte distribution, the degree of overdispersion is uniquely governed by the parameter α, whilst the mean and general distributional form are shaped by the other parameters, λ and p [5], [9]. A tractable yet adaptable model is produced by this separation, and it is especially well-suited for examining overdispersed count data that is observed in practice.

- *Theorem 1*

Let $\{\varepsilon_t\}_{t\in Z}$ be a sequence of i.i.d. random variables following a Delaporte distribution with parameters (λ, α, p). Define the process

$$X_t = \varepsilon_t + \sum_{j=1}^{q} \beta_j \circ \varepsilon_{t-j},\ t \in Z \tag{2}$$

Where the coefficients $\beta_j \in [0,1]$, and the thinning operators are defined independently across time and lags. Then $\{X_t\}$ is a discrete-valued, stationary stochastic process.

- *Proof*

Since the innovation sequence $\{\varepsilon_t\}$ is i.i.d. and each $\varepsilon_t \sim$ Delaporte(λ, α, p), we begin with the probability generating function of the Delaporte distribution:

$$(\phi_\varepsilon(s) = \exp[\lambda(s-1)]\left(\frac{p}{1-(1-p)s}\right)^{\alpha},\ |s| \le 1)$$

The Delaporte distribution is supported on the set of nonnegative integers, so each ε_t is an integer-valued random variable. The binomial thinning operator $\beta_j \circ \varepsilon_{t-j}$ is defined as.

$$\beta_j \circ \varepsilon_{t-j} = \sum_{i=1}^{\varepsilon_{t-j}} B_i^{(j)}$$

Where $\{B_i^{(j)}\}$ are i.i.d. Bernoulli(β_j) random variables independent of ε_{t-j}. This construction guarantees that $\beta_j \circ \varepsilon_{t-j}$ is also an integer-valued random variable.

It follows that X_t is also integer-valued since it is the sum of a finite number of integer-valued components. Moreover, the joint distribution of $(X_t, X_{t-1}, \dots, X_{t-q})$ does not rely on t as the innovation sequence is i.i.d. and the thinning operators are applied independently throughout time. The process $\{X_t\}$ is hence strictly stationary.

- *Theorem 2 (Mean and Variance)*

Let $\{X_t\}$ be the Delaporte-DCINMA(q) process

$$X_t = \varepsilon_t + \sum_{j=1}^{q} \beta_j \circ \varepsilon_{t-j}$$

With $\varepsilon_t \sim$ i.i.d. Delaporte(λ, α, p). Denote

$$\mu_\varepsilon = E[\varepsilon_t] = \lambda + \frac{\alpha(1-p)}{p}, \quad \sigma_\varepsilon^2 = Var(\varepsilon_t) = \lambda + \frac{\alpha(1-p)}{p^2}$$

Then

$$E[X_t] = \mu_\varepsilon\left(1 + \sum_{j=1}^{q} \beta_j\right) \tag{3}$$

And

$$Var(X_t) = \sigma_\varepsilon^2\left(1 + \sum_{j=1}^{q} \beta_j^2\right) + \mu_\varepsilon \sum_{j=1}^{q} \beta_j(1-\beta_j) \tag{4}$$

- *Proof*

Linearity of expectation and the thinning identity $E[\beta_j \circ Y] = \beta_j E[Y]$ give

$$E[X_t] = E[\varepsilon_t] + \sum_{j=1}^{q} E[\beta_j \circ \varepsilon_{t-j}] = \mu_\varepsilon + \sum_{j=1}^{q} \beta_j \mu_\varepsilon = \mu_\varepsilon\left(1 + \sum_{j=1}^{q} \beta_j\right) \tag{5}$$

ISSN No: -2456-2165 https://doi.org/10.38124/ijisrt/26feb393

Use the mutual independence of the thinned terms at a fixed time and the independence of the innovation sequence for the variance (standard INMA assumption).

Then

$$Var(X_t) = Var(\varepsilon_t) + \sum_{j=1}^{q} Var(\beta_j \circ \varepsilon_{t-j})$$

Apply the binomial thinning variance identity

$$Var(\beta_j \circ Y) = \beta_j^2 Var(Y) + \beta_j(1-\beta_j)E[Y]$$

With $Y = \varepsilon$ to obtain

$$Var(X_t) = \sigma_\varepsilon^2 + \sum_{j=1}^{q} (\beta_j^2 \sigma_\varepsilon^2 + \beta_j(1-\beta_j)\mu_\varepsilon)$$

Collecting terms yields

$$Var(X_t) = \sigma_\varepsilon^2 \left(1 + \sum_{j=1}^{q} \beta_j^2\right) + \mu_\varepsilon \sum_{j=1}^{q} \beta_j(1-\beta_j) \quad (6)$$

This completes the proof.

- *Theorem 3 (Autocovariance)*
 Under the same assumptions, define $\beta_0 \equiv 1$. For integer lag $k \geq 1$,

$$Cov(X_t, X_{t-k}) = \begin{cases} \sigma_\varepsilon^2 \sum_{j=0}^{q-k} \beta_j \beta_{j+k}, & k = 1, \dots, q \\ 0, \geq q+1 \end{cases}$$

- *Proof*
 Write $X_t = \sum_{i=0}^{q} \beta_i \circ \varepsilon_{t-i}$ with the convention $\beta_0 \circ \varepsilon_t = \varepsilon_t$. Then

$$Cov(X_t, X_{t-k}) = \sum_{i=0}^{q} \sum_{m=0}^{q} Cov(\beta_i \circ \varepsilon_{t-i}, \beta_m \circ \varepsilon_{t-k-m})$$

Covariance terms are nonzero only when the two thinning expressions feature the same innovation ε_{t-j}, as the innovation sequence is independent over time.

That is, equality $t - i = t - t - i = t - k - m$ or $m = i - k$ must hold. For fixed k, the pairs (i, m) that satisfy this are $m = m = i - k$. Restricting to valid indices $0 \leq i \leq q$ and $0 \leq m \leq q$ gives $i = 0, \dots, q-k$ and $m = i + k$. For each such pair,

$$Cov(\beta_i \circ \varepsilon_{t-i}, \beta_{i+k} \circ \varepsilon_{t-i}) = \beta_i \beta_{i+k} Var(\varepsilon_{t-i})$$

The two thinned sums have means $\beta_i \varepsilon_{t-i}$ and $\beta_{i+k} \varepsilon_{t-i}$, conditional on ε_{t-i}, and the indicators employed in the various thinning are independent. The result of adding $i = 0$ to $q - k$ is

$$Cov(X_t, X_{t-k}) = \sigma_\varepsilon^2 \sum_{i=0}^{q-k} \beta_i \beta_{i+k} \quad (7)$$

When $k \geq q + 1$ there are no overlapping innovations and the covariance is zero.

- *Theorem 4 (Ergodicity).*
 The process $\{X_t\}$ is ergodic.

- *Proof*
 From Theorem 1, the process $\{X_t\}$ is strictly stationary because it is constructed from an i.i.d. innovation sequence and independent thinning operators. To establish ergodicity, it suffices to show that the process is also weakly dependent.

The autocovariance function of $\{X_t\}$, derived in Theorem 3, is

$$Cov(X_t, X_{t-k}) = \begin{cases} \sigma_\varepsilon^2 \sum_{j=0}^{q-k} \beta_j \beta_{j+k}, & k = 1, \dots, q \\ 0, \geq q+1 \end{cases}$$

With $\beta_0 = 1$ and $\sigma_\varepsilon^2 = \lambda + \frac{\alpha(1-p)}{p^2}$.

Because the autocovariance vanishes for all lags greater than q, the process has a finite dependence range. This implies strong mixing with geometric decay of dependence, which in turn ensures ergodicity.

Therefore, the Delaporte-DCINMA(q) process is ergodic.

➢ *Parameter Estimation Process*

The estimation is based on the Generalized Method of Moments (GMM), because the Delaporte-DCINMA(q) model has a tractable moment structure but a complicated likelihood.

The parameters to estimate are:

$$\theta = (\lambda, \alpha, p, \beta_1, \dots, \beta_q), \quad (8)$$

where:
$\lambda > 0$ = Poisson component of the Delaporte innovation,

$\alpha \geq 0$ = overdispersion parameter,

$p \in (0,1)$ = mixing probability,

$\beta_j \in [0,1]$ = thinning coefficients for lags $1, \dots, q$.

Next we compute the sample moments and write the theoretical moments. For the sample moments

- *Mean:*

$$\bar{X} = \frac{1}{T}\sum_{t=1}^{T} X_t \quad (9)$$

- *Variance:*

$$S_X^2 = \frac{1}{T}\sum_{t=1}^{T}(X_t - \bar{X})^2 \quad (10)$$

- *Autocovariances (for $k = 1, \ldots, q$):*

$$\widehat{\gamma_k} = \frac{1}{T}\sum_{t=k+1}^{T}(X_t - \bar{X})(X_{t-k} - \bar{X}) \quad (11)$$

From the Delaporte distribution and the thinning mechanism, we have the theoretical moments as:

- *Innovation Mean and Variance:*

$$\mu_\varepsilon(\lambda, \alpha, p) = \lambda + \frac{\alpha(1-p)}{p}, \quad \sigma_\varepsilon^2(\lambda, \alpha, p) = \lambda + \frac{\alpha(1-p)}{p^2}.$$

- *Mean of the Process:*

$$m_1(\theta) = E[X_t] = \mu_\varepsilon\left(1 + \sum_{j=1}^{q}\beta_j\right). \quad (12)$$

- *Variance of the Process:*

$$m_2(\theta) = Var(X_t) = \sigma_\varepsilon^2\left(1 + \sum_{j=1}^{q}\beta_j^2\right) + \mu_\varepsilon\left(\sum_{j=1}^{q}\beta_j(1-\beta_j)\right). \quad (13)$$

- *Autocovariances (for $\boldsymbol{k} = \mathbf{1}, \ldots, \boldsymbol{q}$):*

$$m_{2+k}(\theta) = Cov(X_t, X_{t-k}) = \sigma_\varepsilon^2\sum_{j=0}^{q-k}\beta_j\beta_{j+k}, \quad \beta_0 \equiv 1. \quad (14)$$

➤ *Construct Moment Conditions*

Define the vector of moment differences:

$$g_T(\theta) = \begin{bmatrix} \bar{X} - m_1(\theta) \\ S_X^2 - m_2(\theta) \\ \widehat{\gamma_1} - m_3(\theta) \\ \vdots \\ \widehat{\gamma^q} - m_{2+q}(\theta) \end{bmatrix}$$

➤ *GMM Objective Function*

We define the GMM objective as:

$$Q_T(\theta) = g_T(\theta)^\top W_T g_T(\theta) \quad (15)$$

Where W_T is a weighting matrix.

- Step 1: Use $W_T = I$ (identity matrix) to get consistent initial estimates $(\hat{\theta})^{(1)}$.
- Step 2: Estimate the covariance of the moment conditions, $\widehat{\Sigma_T}$, and set $W_T = \widehat{\Sigma_T^{-1}}$ for efficient final estimates $(\hat{\theta})^{(2)}$.

➤ *Explicit Recovery of Innovation Parameters*

Once the dependence parameters $(\beta_1, \ldots, \beta_q)$ and mixing probability p are estimated numerically, the Delaporte parameters follow from closed-form formulas:

- *Innovation Mean:*

$$= \frac{\bar{X}}{1 + \sum_{j=1}^{q}\hat{\beta}_j}. \quad (16)$$

- *Innovation Variance:*

$$\hat{\sigma}_\varepsilon^2 = \frac{S_X^2 - \hat{\mu}_\epsilon\sum_{j=1}^{q}\hat{\beta}_j(1-\hat{\beta}_j)}{1 + \sum_{j=1}^{q}\hat{\beta}_j^2} \quad (17)$$

- *Delaporte Parameters:*

$$\hat{\alpha} = \frac{(\hat{\sigma}_\varepsilon^2 - \hat{\mu}_\varepsilon)\hat{p}^2}{(1-\hat{p})^2} \quad (18)$$

$$\hat{\lambda} = \hat{\mu}_\epsilon - \frac{\hat{\alpha}(1-\hat{p})}{\hat{p}} \quad (19)$$

- *Thus, the Final Estimate is:*

$$\hat{\theta} = (\hat{\lambda}, \hat{\alpha}, \hat{p}, \hat{\beta}_1, \ldots, \hat{\beta}_q).$$

➤ *Simulation Study*

This section presents a simulation experiment designed to examine the finite-sample performance of the proposed GMM estimator for the Delaporte–DCINMA (1) model. Three parameter groups were considered to represent different dispersion levels:

$$\text{Model A: } (\lambda, \alpha, p, \beta) = (2,1,0.4,0.3)$$

$$\text{Model B: } (\lambda, \alpha, p, \beta) = (4,1.5,0.6,0.2)$$

$$\text{Model C: } (\lambda, \alpha, p, \beta) = (6,2,0.7,0.4)$$

The Delaporte–DCINMA (1) procedure was used to produce data for each configuration, with sample sizes n=100, 300, 600, and 1000. The experiment was repeated 10,000 times, and the parameters were estimated using the Generalized Method of Moments (GMM) based on the first two theoretical moments.

ISSN No: -2456-2165 https://doi.org/10.38124/ijisrt/26feb393

Table 1 Simulation study Data (A)

Sample Size	$\hat{\lambda}$	$\hat{\alpha}$	$\hat{p}$	$\hat{\beta}$
(a) True values: λ = 2, α = 1, p = 0.4, β = 0.3				
100	2.1542	1.0619	0.3192	0.2895
Bias	0.1542	0.0619	-0.0808	-0.0105
RMSE	0.497	0.293	0.221	0.1523
300	2.0883	1.0418	0.354	0.2963
Bias	0.0883	0.0418	-0.046	-0.0037
RMSE	0.332	0.1932	0.1531	0.0881
600	2.0461	1.0234	0.3764	0.2985
Bias	0.0461	0.0234	-0.0236	-0.0015
RMSE	0.2345	0.1334	0.1034	0.0619
1000	2.0234	1.0125	0.3879	0.3001
Bias	0.0234	0.0125	-0.0121	0.0001
RMSE	0.1699	0.0916	0.0698	0.0475

Table 2 Simulation study Data (B)

Sample Size	$\hat{\lambda}$	$\hat{\alpha}$	$\hat{p}$	$\hat{\beta}$
(b) True values: λ = 4, α = 1.5, p = 0.6, β = 0.2				
100	3.5594	2.7285	0.5065	0.1958
Bias	-0.4406	1.2285	-0.0935	-0.0042
RMSE	0.8834	1.4651	0.1497	0.126
300	3.4159	2.5782	0.5272	0.1984
Bias	-0.5841	1.0782	-0.0728	-0.0016
RMSE	0.7078	1.1553	0.0927	0.0747
600	3.379	2.534	0.5333	0.1988
Bias	-0.621	1.034	-0.0667	-0.0012
RMSE	0.6781	1.0706	0.077	0.0521
1000	3.3689	2.5186	0.5344	0.1998
Bias	-0.6311	1.0186	-0.0656	-0.0002
RMSE	0.6652	1.0409	0.0721	0.04

Table 3 Simulation study Data (C)

Sample Size	$\hat{\lambda}$	$\hat{\alpha}$	$\hat{p}$	$\hat{\beta}$
(c) True values: λ = 6, α = 2, p = 0.7, β = 0.4				
100	4.6656	4.0002	0.6101	0.392
Bias	-1.3344	2.0002	-0.0899	-0.008
RMSE	1.6464	2.2631	0.1178	0.161
300	4.4506	3.7615	0.6257	0.3992
Bias	-1.5494	1.7615	-0.0743	-0.0008
RMSE	1.629	1.8463	0.0845	0.0894
600	4.4026	3.7076	0.6296	0.3993
Bias	-1.5974	1.7076	-0.0704	-0.0007
RMSE	1.6343	1.7492	0.0757	0.0621
1000	4.3865	3.6891	0.631	0.3989
Bias	-1.6135	1.6891	-0.069	-0.0011
RMSE	1.6356	1.7143	0.0723	0.0481

II. REAL DATA EXAMPLE

We used the United States' polio death toll from 1965 to 1996 to demonstrate the usefulness of the suggested Delaporte DCINMA (1) model. The dataset, which includes 32 annual records, shows how polio-related deaths gradually decreased once vaccination campaigns were stepped up. The series shows erratic swings and persistence over years despite this general decline, which encourages the employment of a versatile count model that can manage both overdispersion and serial dependence.

ISSN No: -2456-2165 https://doi.org/10.38124/ijisrt/26feb393

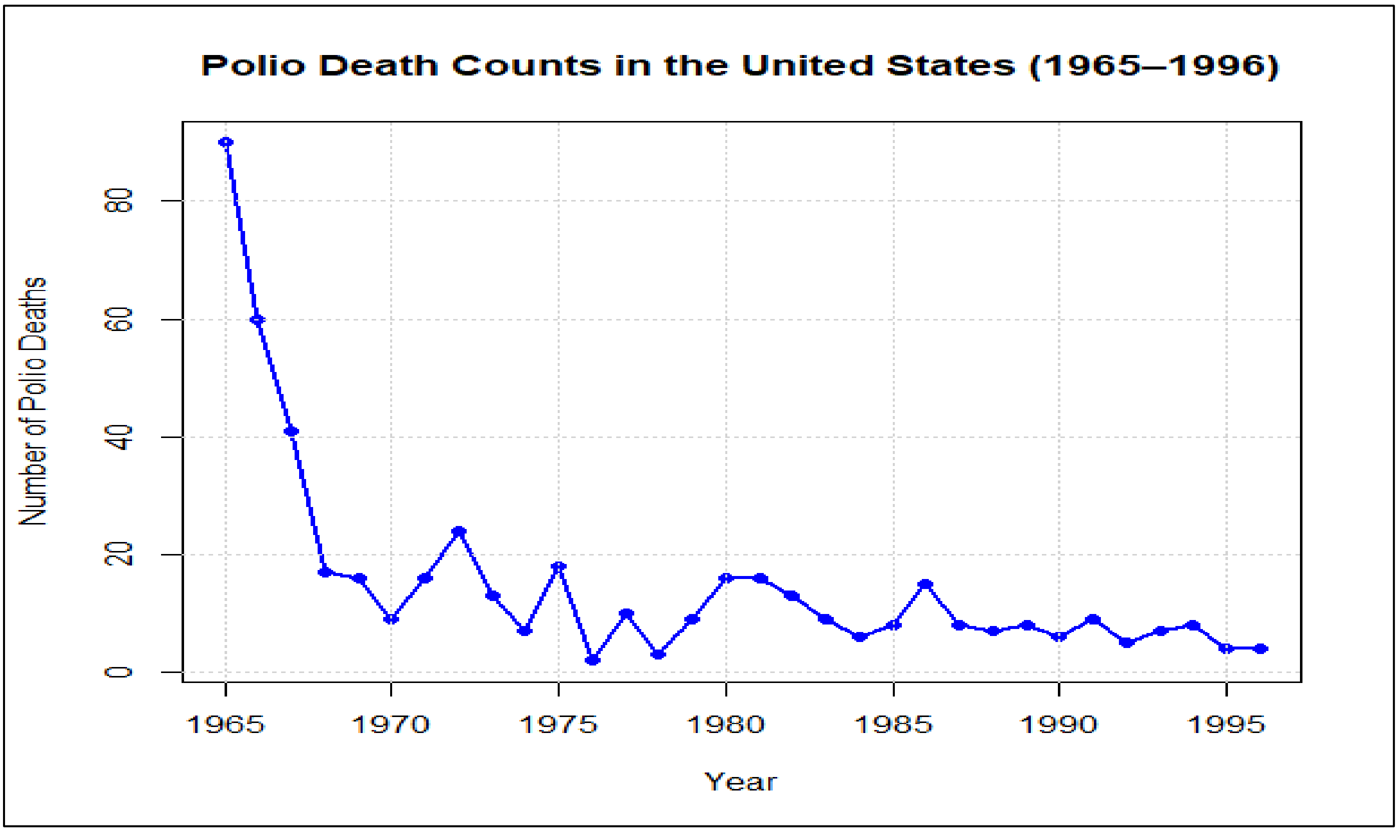

Fig 1 Time Series Plot of Polio Death Count

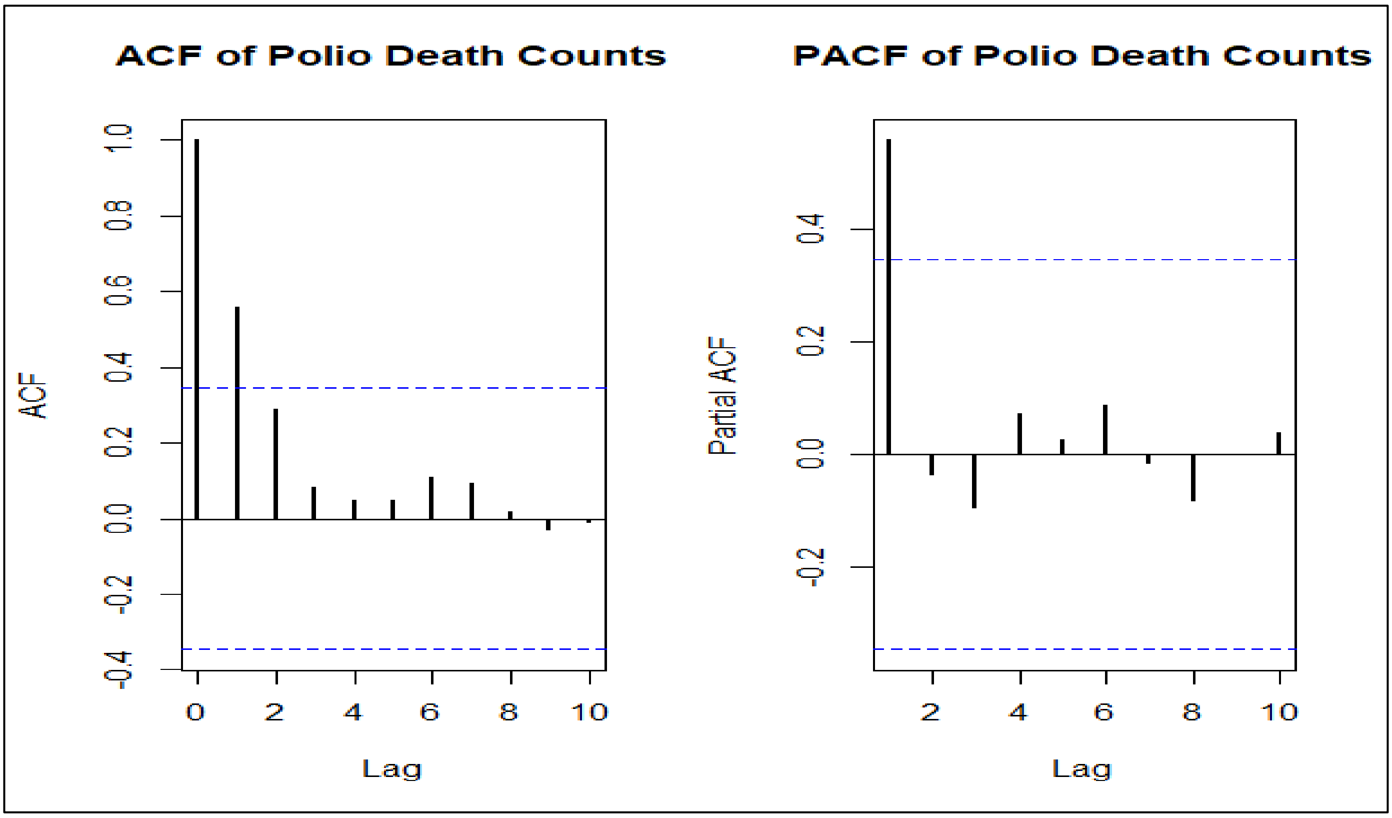

Fig 2 ACF and PACF Plot of Polio Death Count.

Table 4 Parameter Estimation Using the Generalized Method of Moments (GMM)

$\hat{\lambda}$	$\hat{\alpha}$	$\hat{p}$	$\hat{\beta}$
1.794	2.471	0.427	0.712

ISSN No: -2456-2165 https://doi.org/10.38124/ijisrt/26feb393

- *Model Evaluation*

Several diagnostic techniques were methodically used to evaluate the fitted model's appropriateness and forecast performance. Residual analysis will be used to assess the model's reliability for this investigation.

Table 5 Residual Diagnostics (Ljung – Box Test): To Make Sure there was no Residual Autocorrelation, the Box-Ljung Test was Run on the Residuals. The Test Findings are as follows:

Ljung – Box Test		
X-squared: 2.6083	df: 10	p-value: 0.9892

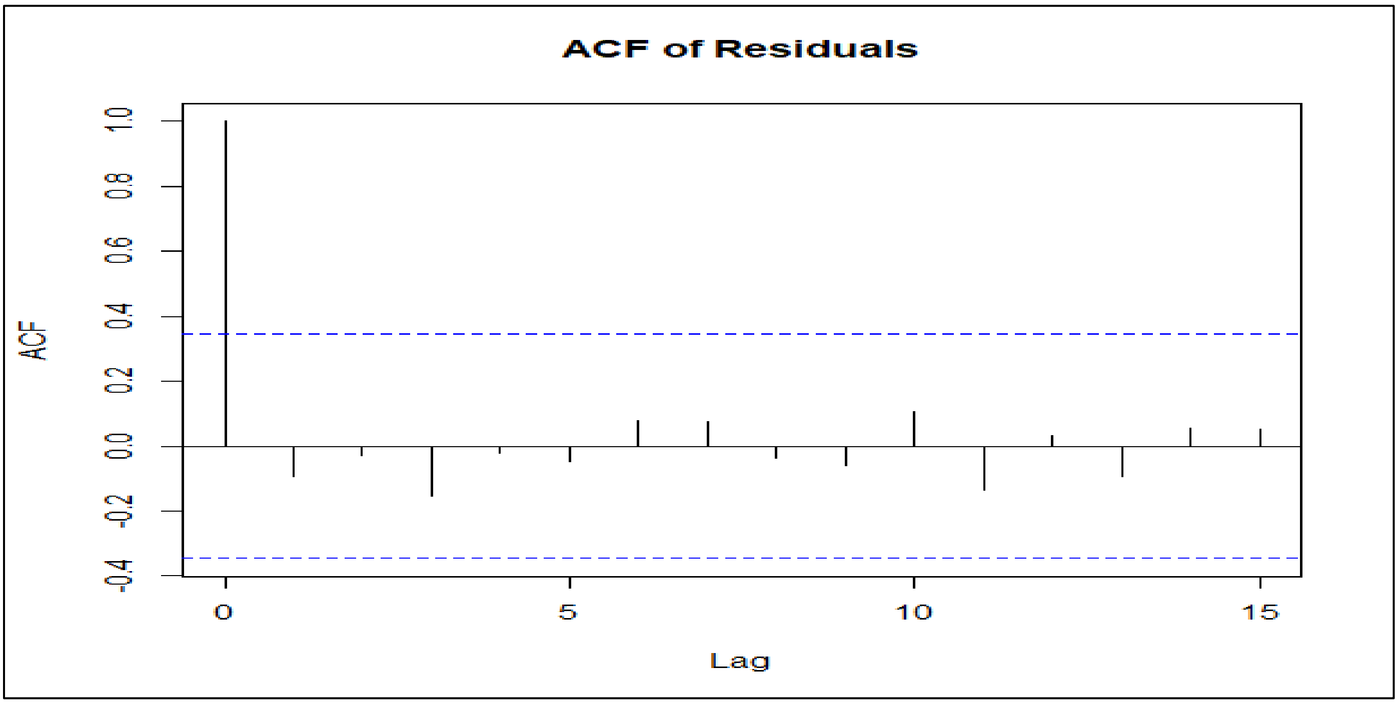

Fig 3 ACF of Residuals

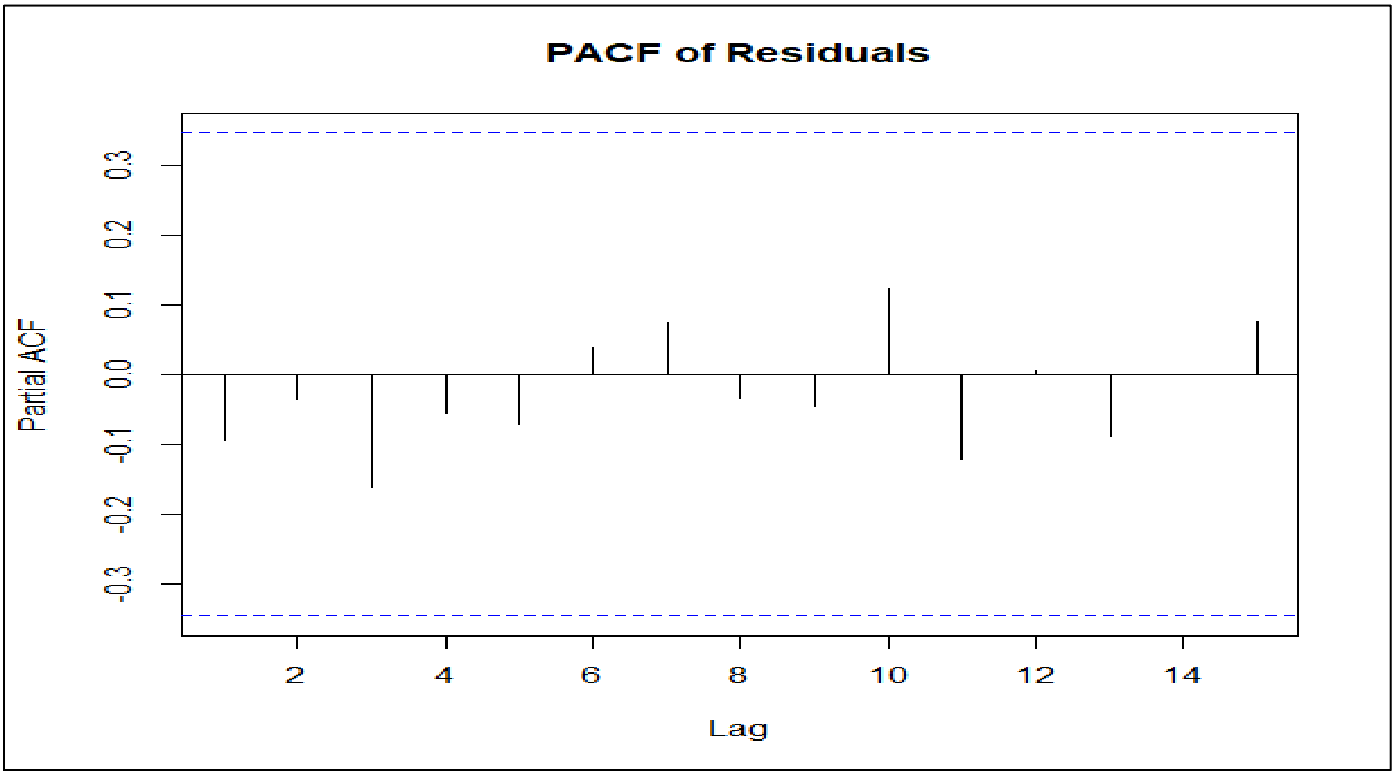

Fig 4 PACF of Residuals

ISSN No: -2456-2165 https://doi.org/10.38124/ijisrt/26feb393

III. DISCUSSION

The estimations' bias and RMSE are shown in Tables 1-3. The estimators performed effectively in every instance, demonstrating a distinct convergence toward the genuine parameter values as the sample size grew. The consistency and effectiveness of the GMM estimator were confirmed by the consistent drop in bias and RMSE with increasing n. A little bias emerged in severely overdispersed data (Model C), which is consistent with known moment-based estimate behavior, even though estimation stayed correct under low and moderate dispersion (Models A and B).

Parameter estimation was carried out using the Generalized Method of Moments (GMM) under parameter constraints $0 < p, \beta < 1$, ensuring valid and interpretable estimates. To avoid local minima, a robust multi-start optimization approach was implemented showing in Table 4. The estimations provide valuable information about the data's structure. The comparatively high dependence coefficient $\hat{\beta} \approx 0.71$ indicates substantial temporal persistence, while the value of $\hat{\alpha} > 1$ indicates a marked overdispersion (the variance of the counts exceeds the mean). This illustrates the inertia sometimes seen in epidemiological processes, as variations in polio deaths in one year typically have an impact on those in the subsequent year.

The null hypothesis that the residuals are independently distributed cannot be rejected because the p-value is higher than the 0.05 significance level. This proves that the model correctly reflects the temporal structure of the data by showing that the residuals have white noise properties showing in Table 5.

The ACF and PACF plots are then used to observe residual independence in figure 3 and 4 respectively. Overall, the diagnostic results show no discernible autocorrelation in either the ACF or PACF plots, confirming that the residuals of the fitted Delaporte DCINMA (1) model behave as white noise. This shows that no systematic pattern is left unexplained and that the model has effectively captured the dependence structure in the data. The suggested model's suitability and dependability for the U.S. polio mortality count series are therefore confirmed by the positive results of the Ljung–Box test and visual diagnostics.

IV. CONCLUSION

The Delaporte-DCINMA(q) model, a versatile integer-valued moving average procedure for overdispersed count time series, was effectively created in this study. The model successfully captures a broad range of variance patterns by utilizing Delaporte-distributed innovations, which get beyond the equidispersion limitation of its Poisson-based competitors. The model's fundamental statistical characteristics, such as moment structures, ergodicity, and stationarity, were determined. An effective GMM estimation method was developed and demonstrated to work well in simulations, with bias and RMSE declining with increasing sample size. An application of the model to polio incidence data showed its empirical utility by correctly identifying significant overdispersion and serial dependence, with diagnostic checks verifying a strong fit.

In summary, a useful and reliable paradigm for examining correlated overdispersed counts is offered by the Delaporte-DCINMA(q) model. It provides a strong tool for applications in a variety of sectors and is a significant addition to the DCINMA family. Future research might concentrate on creating likelihood-based estimate techniques or adding autoregressive elements.

REFERENCES

[1]. H. Steutel and K. van Harn, *Discrete Analogues of Self-Decomposability and Stability*, Ann. Probab., vol. 7, no. 5, pp. 893–899, 1979.

[2]. C. H. Weiß, *An Introduction to Discrete-Valued Time Series*, Weinheim, Germany: Wiley-VCH, 2018.

[3]. K. Yu and H. Wang, “A new overdispersed integer-valued moving average model with dependent counting series,” *Entropy*, vol. 23, no. 6, p. 706, 2021.

[4]. M. Al-Osh and A. Alzaid, “First-order integer-valued autoregressive (INAR(1)) process with generalized innovations,” *J. Time Series Anal.*, vol. 8, pp. 261–275, 1987.

[5]. R. A. Rigby, D. M. Stasinopoulos, and C. Akantziliotou, “A framework for modelling overdispersed count data, including the Poisson-shifted generalized inverse Gaussian distribution,” *Comput. Stat. Data Anal.*, vol. 53, no. 2, pp. 381–393, 2008.

[6]. A. Adler, “Delaporte Distribution Functions for R,” *CRAN R Package ‘Delaporte’*, 2017. [Online]. Available: https://cran.r-project.org/package=Delaporte

[7]. M. McKenzie, “Some simple models for discrete variate time series,” *Water Resour. Bull.*, vol. 21, no. 4, pp. 645–650, 1985.

[8]. H. Alzaid and M. Al-Osh, “An integer-valued moving average (INMA) process,” *J. Appl. Probab.*, vol. 25, pp. 314–324, 1988.

[9]. P. Consul and G. Jain, “A generalization of the Poisson distribution,” *Technometrics*, vol. 15, no. 4, pp. 791–799, 1973.

[10]. J. Fokianos, “Count time series models,” in *Handbook of Statistics*, vol. 41, Elsevier, 2019, pp. 403–430.

[11]. C. H. Weiß, “Thinning operations for modeling time series of counts—a survey,” *AStA Advances in Statistical Analysis*, vol. 95, pp. 319–341, 2011.

[12]. C. H. Weiß, “Serial dependence and overdispersion in count data: models, estimation, and diagnostics,” *Statistical Modelling*, vol. 19, no. 3, pp. 231–256, 2019.

[13]. K. Yu and H. Wang, “A dependent counting integer-valued time series model,” *Entropy*, vol. 24, no. 4, p. 546, 2022.

[14]. G. E. Taufer and J. C. R. Teixeira, “Parameter estimation for integer-valued autoregressive models using the GMM approach,” *Stat. Papers*, vol. 57, pp. 725–743, 2016.