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Derivation of the Quadratic Arithmetic Sequence and its Applications Using the Arithmetic Sequence in the Kuwait National Curriculum for Grade 10

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Abstract: Arithmetic sequences are fundamental and important mathematical concepts included in the Grade 10 curriculum in the State of Kuwait. A new formula for the quadratic arithmetic sequence has been derived based on the fundamental principles of arithmetic sequences. This quadratic sequence is characterized by having constant second differences between its terms. A quadratic arithmetic sequence is defined as a numerical sequence where the difference between consecutive terms is not constant, but the difference between these differences (i.e., the second differences) remains constant, making it a mathematical model that represents quadratic patterns.The results and conclusions were analyzed and verified using MATLAB**, which enabled precise and efficient computations, as well as effective visual data representation.**

Keywords: *Quadratic Arithmetic Sequences, Second Differences, Kuwait National Curriculum, Grade 10, MATLAB Software, Quadratic Models, Applied Mathematics.*

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I. INTRODUCTION

Mathematics education at the secondary level plays a vital role in developing students' analytical and abstract thinking skills. One of the main topics included in the Grade 10 curriculum in the State of Kuwait is the arithmetic sequence, which is characterized by a constant first difference between consecutive terms. Although the national curriculum primarily addresses the linear pattern represented by the arithmetic sequence, this research aims to extend the foundational knowledge it provides to explore more complex mathematical relationships—specifically, the quadratic arithmetic sequence.

A quadratic arithmetic sequence is defined as a numerical pattern in which the second differences between terms remain constant, even though the first differences vary. This sequence represents the discrete counterpart of the continuous quadratic function $f(m) = am^2 + bm$, graphically represented by a parabola or conic section known as the paraboloid. These relationships are important not only in pure mathematics but also appear in fields such as physics, engineering, and economics, where nonlinear growth and motion are common.

In this research, the general formula for quadratic arithmetic sequences was derived using the conceptual framework established in the Grade 10 textbook of Kuwait's national curriculum (Ministry of Education, 2023). By analyzing term-to-term differences and employing the method of finite differences, the structure of quadratic sequences was revealed and linked to their algebraic and graphical counterparts. Moreover, MATLAB was utilized to perform precise computations, generate sequence plots, and verify the derived formulas. The integration of computational tools enhances mathematical understanding and promotes digital literacy in line with modern educational practices.

Recent studies (Yang, 2025; Sebsibe & Abdella, 2025) emphasize the importance of visual and application-based approaches in teaching quadratic concepts. Through technology-enhanced learning and the use of real-world problems, students gain a deeper understanding of abstract mathematical ideas. By extending the topic of arithmetic sequences into the quadratic domain, this study aligns with Kuwait's educational vision of preparing a generation capable of critical thinking, problem-solving, and keeping pace with rapid scientific and technological advancements (Al-Shaye, 2019; UNESCO, 2021).

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This research contributes not only to the local educational context but also adds value to the global discussion on curriculum development, especially regarding the transition from linear to nonlinear mathematical thinking. The findings are supported by both classical references and recent scholarly contributions up to the year 2025.

II. ARITHMETIC PROGRESSION FORMULA

An arithmetic sequence is a list of numbers that follow a definitive pattern. Each term in an arithmetic sequence is added or subtracted from the previous term. For example, in the sequence 2,4,6,… six is added to each previous term. This consistent value of change is called the common difference. A sequence can be increasing if the common dfference is positive and decreasing if the common difference is negative. For example, in the sequence 6,4,2,..the common difference is -2 .

We can mention the formulas used to find the relationship between the elements of the arithmetic sequence and sum of them if the first term of an arithmetic progression is a_1 , a_{n-1} second last term where the common difference of successive me mbers is d then the nth term of the sequence (a_n) is given by:

$$d = a_n - a_{n-1} \tag{1}$$

$$a_n = a_1 + (n-1)d \; \forall \in Z^{+} \tag{2}$$

An arithmetic series is the sum of an arithmetic sequence. We find the sum by adding the first, a_1 and last term, a_n, divide by 2 in order to get the mean of the two values and then multiply by the number of values n:

$$S_n = \frac{n}{2}(a_1 + a_n) \tag{3}$$

To derive the above formula, begin by expressing the arithmetic series in two different ways :-

$$\begin{aligned} S_n &= a_1 + (a_1 + d) + (a_1 + 2d) + \cdots + (a_1 + (n-2)d) + (a_1 + (n-1)d) \\ S_n &= (a_n - (n-1)d) + (a_n - (n-2)d) + \cdots + (a_n - 2d) + (a_n - d) + a_n \end{aligned} \tag{4}$$

Adding both sides of the two equations, all terms involving d cancel:

$$2S_n = n(a_1 + a_n) \tag{5}$$

Dividing both sides by 2 produces a common formula of the equation:-

$$S_n = \frac{n}{2}(a_1 + a_n) \tag{6}$$

Alternative arithmetic series formula:-

$$S_n = \frac{n}{2}(a_1 + a_1 + (n-1)d) \tag{7}$$

$$S_n = \frac{n}{2}(2a_1 + (n-1)d) \tag{8}$$

Furthermore, the mean value of the arthimatic sequance can be calculated via S_n / n :

$$\bar{a} = \frac{a_1 + a_n}{2} \tag{9}$$

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➢ *Explicit Formula for Arithmetic Sequences:*

An explicit formula is a mathematical formula for the value of any successive term without having to calculate the previous terms. It is specified by the general term formula. $a_n = a_1 + (n-1)d$, where a_n is the n^{th} term, a_1 is the first term, and d is the common difference.

➢ *Recursive Formula for Arithmetic Sequences:*

The recursive formula for an arithmetic sequence is a complete assignment of the sequence to the previous term. It is represented by the formula $a_n = a_{n-1} + d$, where a_n is the n^{th} term, a_{n-1} is the previous (n-1) term, and d is the common difference.

III. MAIN RESULT

➢ *Theorem 1:*

Aquadratic arithmetic sequence is a sequence whose m^{th} term is defined as:

$$f(m) = am^2 + bm \quad \forall m \in \textit{Real number}$$

$$a = d/2$$
$$b = a_1 - a$$

Also We can use the alternative arithmetic series formula (8)to deduce the quadratic arthimatic sequence $f(m) = am^2 + bm$ and solve it.

➢ *Example (1):-*

Find the second –degree function(quadratic function) if we have the arithmetic sequence 6, 12, 18, 24, . . . with a common difference is 6 and first term is 6.

Where:

a is the coffiencient of the quadratic term,

b is the coffiencient of the linear term,

➢ *Definition:*

Aquadratic arithmetic sequence is a sequence of numbers in which the difference between consecutive terms is not constant, but the second difference is constant.

➢ *Proof:*

We deduced that the relationship between of the quadratic arthimatic sequence and arithmetic sequences as follow:-

We can use the a_1 (the first term) and d (the "common difference" between terms) to obtain the quadratic arthimatic sequence $f(m) = am^2 + bm$ at these :-

(10)

• *The Solve:-*

From proposition (10) we can investigated these function from arithmetic sequances as:

$$a = d/2 = 6/2 = 3 \tag{11}$$

$$b = \frac{f_m - am^2}{m}, b = a_1 - a = 6 - 3 = 3 \tag{12}$$

Table 1 let a= 3, b=3

$f(m) = 3m^2 + 3m$, $m = 0,1,2,\ldots$	$S_n = \frac{n}{2}(2a_1 + (n-1)d)$ $n = 1,2,\ldots$	$S_n = \frac{n}{2}(a_1 + a_n)$ $n = 1,2,\ldots$
$f(0) = 3\times(0) + 3\times(0) = 0$		
$f(1) = 3\times(1) + 3\times(1) = 6$	$S_1 = \frac{1}{2}(2\times6 + (1-1)\times6) = 6$	$S_1 = \frac{1}{2}(6+6) = 6$
$f(2) = 3\times(4) + 3\times(2) = 18$	$S_2 = \frac{2}{2}(2\times6 + (2-1)\times6) = 18$	$S_2 = \frac{2}{2}(6+12) = 18$
$f(3) = 3\times(9) + 3\times(3) = 36$	$S_3 = \frac{3}{2}(2\times6 + (3-1)\times6) = 36$	$S_3 = \frac{3}{2}(6+18) = 36$
$f(4) = 3\times(16) + 3\times(4) = 60$	$S_4 = \frac{4}{2}(2\times6 + (4-1)\times6) = 60$	$S_4 = \frac{4}{2}(6+24) = 60$

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$f(5)=3\times(25)+3\times(5)=90$	$S_5=\frac{5}{2}(2\times 6+(5-1)\times 6)=90$	$S_5=\frac{5}{2}(6+30)=90$
$f(6)=3\times(36)+3\times(6)=126$	$S_6=\frac{6}{2}(2\times 6+(6-1)\times 6)=126$	$S_5=\frac{6}{2}(6+36)=126$

From table (1) we can deduce an infinite number of terms from equation of the second degree with given a_1(the first term) and d (the common difference) also, from our point of view to get the values of the quadratic arthimatic sequance , we observe that the sum have same value with second-degree equation and formula in Eqs. (6-8) of the law of the sum of the arithmetic sequence through the conclusions in table (1) we proof the theorem (1) as:-

$$f(1)=S_1=a_1 \tag{13}$$

$$f(m)=am^2+bm=S_n=\frac{n}{2}(2a_1+(n-1)d) \tag{14}$$

$$f(m)=am^2+bm=S_n=\frac{n}{2}(a_1+a_n) \tag{15}$$

- *Therom (2):*

The sum of the first terms of quadratic arthimatic sequence can be write as:

$$F_{m=}=a_1+\frac{1}{2}\left[\left(\left(\frac{m(m+1)}{2}-1\right)2a_1\right)+\left(\left(\frac{(m-1)m(m+1)}{3}\right)2\ a\right)\right]$$

- *The Proof:*

From the formula (8) we can investigated the sum of the first terms of quadratic arithmetic sequence as follow:-

$$F_{1=}S_1 \tag{16}$$

$$F_{2=}S_1+\frac{2}{2}(2a_1+(2-1)d) \tag{17}$$

$$F_{3=}S_1+\frac{2}{2}(2a_1+(2-1)d)+\frac{3}{2}(2a_1+(3-2)d) \tag{18}$$

$$F_{4=}S_1+\frac{2}{2}(2a_1+(2-1)d)+\frac{3}{2}(2a_1+(3-2)d)+\frac{4}{2}(2a_1+(4-2)d) \tag{19}$$

$$F_{m=}S_{m=}S_1+\frac{1}{2}(1\times 2)a_1+(2\times 2)a_1+(3\times 2)a_1+(4\times 2)a_1+\ldots+(m\times 2)a_1 \\ +(2\times 1)d+(3\times 2)d+(4\times 3)d+\ldots+(m+1)m\,d \tag{20}$$

$$F_{m=}=a_1+\frac{1}{2}\left[\left(\left(\frac{m(m+1)}{2}-1\right)2a_1\right)+\left(\left(\frac{(m-1)m(m+1)}{3}\right)d\right)\right] \tag{21}$$

$$F_{m=}=S_1+\frac{1}{2}\left[\left(\left(\frac{m(m+1)}{2}-1\right)2a_1\right)+\left(\left(\frac{(m-1)m(m+1)}{3}\right)2\ a\right)\right] \tag{22}$$

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Formula (21) represent the sum of m terms of quadratic arthimetic sequence by know the common difference d and the first term of arthimatic sequence and formula (22) by know the coffiencient of the quadratic term a and first term of quadratic arthimatic sequance . represent $\left(\frac{m(m+1)}{2}\right)$ the m[th] partial sum of triangular number[21] and called sum of natural numbers 1 to m and $\left(\frac{(m-1)m(m+1)}{3}\right)$ represent to product of two consecutive numbers.

➢ *Example (2) : -*

Find the sum of 6[th] in quadratic arthimatic sequence if a_1 =6 (the first term of arthimetic sequence) and d =6(the common difference between terms).

- *The Solve:-*

let $a = 3, b = S_1 - a = 6 - 3 = 3$,from formula (21) we can obtain the sum of 6^{th} terms of sum second function as follow:

Table 2 Sum Second Function

Second Function Degree	Low the sum of the first m terms of an arthimetic quadratic sequence
$f(m) = am^2 + bm$	$F_{m=} = S_1 + \frac{1}{2}\left[\left(\left(\frac{m(m+1)}{2} - 1\right)2S_1\right) + \left(\left(\frac{(m-1)m(m+1)}{3}\right)2a\right)\right]$
$f(1) = 6$	$F_1 = 6 + \frac{1}{2}\left(((\frac{1\times(1+1)}{2}) - 1)\times 2\times 6) + ((\frac{(1-1)\times 1\times(1+1)}{3})\times 6)\right) = 6$
$\sum_{m=1}^{2} f(m) = 6 + 18 = 24$	$F_2 = 6 + \frac{1}{2}\left(((\frac{2\times(2+1)}{2}) - 1)\times 2\times 6) + ((\frac{(2-1)\times 2\times(2+1)}{3})\times 6)\right) = 24$
$\sum_{m=1}^{3} f(m) = 6 + 18 + 36 = 60$	$F_3 = 6 + \frac{1}{2}\left(((\frac{3\times(3+1)}{2}) - 1)\times 2\times 6) + ((\frac{(3-1)\times 3\times(3+1)}{3})\times 6)\right) = 60$
$\sum_{m=1}^{5} f(m) = 6 + 18 + 36 + 60 + 90 = 210$	$F_5 = 6 + \frac{1}{2}\left(((\frac{5\times(5+1)}{2}) - 1)\times 2\times 6) + ((\frac{(5-1)\times 5\times(5+1)}{3})\times 6)\right) = 210$
$\sum_{m=1}^{6} f(m) = 6 + 18 + 36 + 60 + 90 + 126 = 336$	$F_6 = 6 + \frac{1}{2}\left(((\frac{6\times(6+1)}{2}) - 1)\times 2\times 6) + ((\frac{(6-1)\times 6\times(6+1)}{3})\times 6)\right) = 336$

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- *Lemma.1: Arithmetic Mean in Quadratic Arthimatic Sequence can be Write as:*

- *First Method:*
  Let S_1, S_2, S_3 an terms of sum arithmetic sequence S_1, S_2, S_3 are elements belonging to the real numbers

$$S_2 - S_1 = S_3 - S_2 - 2a$$

$$S_2 = \frac{S_3 + S_1}{2} - a \tag{23}$$

a represent the factor of m^2 in $f(m) = am^2 + bm$.

We can get $a = \frac{d}{2}$ from Lemma (10)

- *Examble(4):-*
  Find arithmetic means between 18, 60 when S_1=6 and S_3=36

The solve:

$$S_1 = 6 = a + b$$

$$S_3 = 36 = 9a + 3b \rightarrow 12 = 3a + b \tag{24}$$

From (36and 37) we find a=3

$$S_2 = \frac{6+36}{2} - 3 = 18 \tag{25}$$

- *Second Method:*
  Let f_1, f_2, $f_3$an terms of second function and f_1, f_2, f_3 are elements belonging to the real numbers

$$f_2 - f_1 = f_3 - f_2 - 2a \tag{26}$$

$$f_2 = \frac{f_3 + f_1}{2} - a \tag{27}$$

a represent the factor m^2 in $f(m) = am^2 + bm$.

- *Third Method:*
  From the sum of sequence if we let

$$S_n = \frac{n}{2}(2a_1 + (n-1)d) \text{ and } S_v = \frac{v}{2}(2a_1 + (v-1)d) \tag{28}$$

$$S_m - S_v = \frac{1}{2}\left(2a_1(m-v) + m(m-1)d - v(v-1)d\right) \tag{29}$$

$$d = \frac{2(S_m - S_v) - 2 \times a_1 \times (m-v)}{m(m-1) - v(v-1)} \tag{30}$$

In these formula we need provided that the value of the first term of an arithmetic sequence(a_1) is known.

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- *Examble(5):-*

Enter 3 arithmetic means for the sum of the quadratic arthimetic sequence consider that 90 is the 5[th] and 3[th] in the sum of arithmetic sequence.

The solve:

To find the means between (6,90) we first find d (common difference)

By using law in equation(43)

$$d=\frac{2(90-36)-2\times6\times(5-3)}{5(5-1)-3(3-1)}=\frac{84}{14}=6 \quad (31)$$

In these law provided that the value of the first term of an arithmetic sequence(a_1) is known ,($a=d/2$),(b = S_1 – a) when a represent the factor m^2 and b represent factor m in $f(x)=am^2+bm$. and S_1=a_1 ($a_1$is the first term in arithmetic sequences and S_1 is the first term quadratic arthimetic sequence).

Note: all example agree with findi value d (common difference)

To find the arithmetic mean of numbers from quadratic arthimetic sequence, we need to determine the rank of the terms, for example, from the first term to the third, or from the third term to the sixth…etc, then we can find the value of d (common difference).

- *Four Method:*

These method depend on the quadratic arthimetic sequence by know the value of these quadratic arthimetic sequence and the rank of this sequence .

$$f(m)=am^2+bm \quad (32)$$

$$f(x)=ax^2+bx \quad (33)$$

By substract (46) from(45) we can deduce that :

$$f(m)-f(x)=a(m^2-x^2)+b(m-x)$$

$$a=\frac{f(m)-f(x)-b(m-x)}{(m^2-x^2)}$$

Thise formula depend on the quadratic arthimetic sequence by know the value of these quadratic arthimetic sequence and ranke and value of b.

if give $S_1=a+b\rightarrow b=S_1-a$, then we can write

$$f(m)-f(x)=a(m^2-x^2)+(S_1-a)(m-x) \quad (34)$$

$$f(m)-f(x)=a[(m^2-x^2)-(m-x)]+S_1(m-x)$$

$$f(m)-f(x)-S_1(m-x)=a[(m^2-x^2)-(m-x)] \quad (35)$$

$$a=\frac{f(m)-f(x)-S_1(m-x)}{(m^2-x^2)-(m-x)} \quad (36)$$

IV. APPLICATION OF FORMULA QUADRATIC ARTHIMETIC SEQUENCE

- *Examble(6) :-*

Find $\sum_{m=1}^{10} m^2 = 1 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2$

The solve:

$$\sum_{m=1}^{10} m^2 = 1 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2 = 385$$

We can use

$$F_m = S_1 + \frac{1}{2}\left(((\frac{m(m+1)}{2}) - 1) \times 2 \times S_1) + ((\frac{(m-1)m(m+1)}{3}) \times d) \right)$$

$$F_m = 1 + \frac{1}{2}\left[\frac{10 \times 11}{2} - 1) \times 2 + \frac{9 \times 10 \times 7}{3} \times 2 \right] = 1 + 384 = 385$$

- *Examble(7):-*

Find $\sum_{m=1}^{17} 100 \times m^2$

The solve:

$$\sum_{m}^{7} 100 \times m^2 = 100 \times (1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2) = 14000$$

$$S_7 = 100 + \frac{1}{2}\left[\left(\frac{7 \times 8}{2} - 1 \right) \times 2 \times 100 + \frac{6 \times 7 \times 8}{3} \times 200 \right] = 14000$$

- *Lemma.2: The First Explicit Formula for the Quadratic Arthimatic Sequence can be Write as:*

We can define a quadratic arithmetic sequence as a real arithmetic sequence whose domain is the set of positive integers or a su bset of them arranged in the form (1,2,3,...) and whose corresponding domain is the set of real numbers.It is written as a quadratic function as follows:

$$f(m) = a m^2 + b m \qquad (37)$$

Thise called the first explicit formula for the quadratic arthimatic sequence Using this formula, you can get the value of any term without knowing the term that precedes it.

If each term of a quadratic arithmetic sequence is subtracted from the term immediately preceding it, we will have an arithmetic sequence whose first term is equal to the first term of the quadratic arithmetic sequence and $b = \frac{f_m - am^2}{m}$ in this arithmetic sequence, the difference between each term and the term immediately following it is equal to twice the square coefficient of the quadratic arithmetic sequence.

- *Lemma.3: The Second Explicit Formula for the Quadratic Arthimatic Sequence can be Write as:*

The n[th] term of a quadratic arithmetic sequence is equal to the sum of the n[th] terms of an arithmetic sequence, the first term being known, and the difference between each term and the immediately preceding it and it can be written in the form the second explicit formula for the quadratic arthimatic sequence.

$S_n = \frac{n}{2}(2a_1 + (n-1)d)$, a_1, d , represents the first term and common difference of an arithmetic sequence. Or

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we can write another explicit formula of quadratic arthimatic sequence as $S_n = \frac{n}{2}(2a_1 + (n-1)d)$, a_n the n^{th} term.

V. APPLICATIONS OF QUADRATIC SEQUENCES

A quadratic arithmetic sequence is represented by a parabola, A parabola is a type of conic section. It is a two-dimensional curve that is geometrically defined as the set of points that are equal in distance to a fixed point (the focus) and equal in distance to a fixed straight line (guid).

➢ *Equation of a Parabola:*

Axis the equation of a parabola depends on its location and orientation, but in general, it can be written in the following:

$$f(m) = am^2 + bm \pm c = S_n = \frac{n}{2}(2a_1 + (n-1)d) \pm c) \quad (38)$$

Suppose the vertex of the parabola (h,K), we can be written the equation of parpola in standard form as follows:

$$(m-h) = 4p(f(m) - K) \quad (39)$$

$$a = \frac{1}{4p}, b = \frac{-h}{2p} \quad (40)$$

$$h = \frac{-b}{2a}, K = \frac{4ac - b^2}{4a}, c = \frac{h^2}{4a} + K \quad (41)$$

➢ *Definition and Characteristics:*

- Focus: A fixed point inside a parabola.

$$(h, K + \frac{1}{4a}) = (\frac{-b}{2a}, \frac{(4ac-(b)^2}{4a} + \frac{1}{4a}) = (\frac{a1-(\frac{d}{2})}{d}, \frac{(2dc-\left(a1-\left(\frac{d}{2}\right)\right)^2 + 1}{2d}) \quad (42)$$

- Focal length : $\left|\frac{1}{4a}\right| = \left|\frac{1}{2d}\right|$ and the focal chord= $\left|\frac{1}{a}\right| = \left|\frac{1}{(1/2)d}\right|$

- Guide: A straight line outside the parabola.

$$f(m) = \frac{(2dc-\left(a1-\left(\frac{d}{2}\right)\right)^2 - 1}{2d} \quad (43)$$

- Axis of symmetry: is the straight line that passes through the focus and is perpendicular to the directrix .

$$h = -\frac{b}{2a} = \frac{a_{1-(\frac{d}{2})}}{d} \quad (44)$$

- Vertex: The point where the parabola intersects the parabolic axis.

$$(h, K) = (\frac{-b}{2a}, \frac{(4ac-(b)^2}{4a}) = (\frac{a1-(\frac{d}{2})}{d}, \frac{(2dc-\left(a1-\left(\frac{d}{2}\right)\right)^2}{2d}) \quad (45)$$

The tangent at apoint on the y axis has the equation

$$y = px + b_1 \quad (46)$$

Is the straight line that passes through the focus and is perpendicular to the directrix.

VI. PROPERTIES OF QUADRATIC ARTHIMATIC SEQUENCES

- *First Property:*

By subtracting each term from the term immediately preceding it, we obtain arithmetic sequences whose base is twice the coefficient of the quadratic term, and whose first term is the same as the first term of the quadratic sequence.

For example, let's take the quadratic sequence (1, 4,9, 16, …). If we subtract each term from the term immediately before it, we get the arthimatic sequence (1, 3, 5, …) see table (3).

Table 3 m=1:4, $f(m)=m^2$, a1=1, d=2.

m	quadratic arhimatic sequence m^2	The first difference (Arthimatic sequence)	The second difference
1	1	1	3-1=2
2	4	4-1=3	5-3=2
3	9	9-4=5	7-5=2
4	16	16-9=7	

- *Second Property:*

If we add or subtract a constant number to each term of a quadratic sequence, a new quadratic sequence is produced. An arithmetic sequence is produced as a result of subtracting each term of this quadratic sequence from the term immediately preceding it. The following table (4,5) explains this to us:

Table 4 m=1:4, $f(m)=m^2$, d=2, c=1.

m=1	Quadratic Sequence m^2	Athematic sequence	Quadratic Sequance+1 (m^2+1)	Athematic sequence of (m^2+1)	Common difference
1	1	1	2	2	3-2=1
2	4	4-1=3	5	5-2=3	5-3=2
3	9	9-4=5	10	10-5=5	7-5=2
4	16	16-9=7	17	17-10=7	

Table 5 m=1:4, $f(m)=m^2$, d=2, c=-1.

m=1	Quadratic sequence m^2	Athematic sequence	Quadratic Sequence (m^2-1)	Athematic sequence of (m^2-1)	Common difference
1	1	1	0	0	3-0=3
2	4	4-1=3	3	3-0=3	5-3=2
3	9	9-4=5	8	8-3=5	7-5=2
4	16	16-9=7	15	15-8=7	

From table(4) and table (5) we observe that if we have a quadratic sequence in which a constant number has been added or subtracted, we can call it the constant of the quadratic function. The value of this constant term is given by the first term of the resulting arithmetic sequence, the (column) representing the sequence resulting from adding 1 or the (column) representing the sequence resulting from subtracting 1. In both cases, to obtain the first number of the resulting arithmetic sequence, the second term is subtracted from its common difference. The relationship between the quadratic constant and the common difference can be explained as follows:

From table (4) we investigated that:

$a_2=a_1+(n-1)$, Let d=2 , $3= a_1+(2-1)$, $a_1=3-2=1$

The constant of quadratic arthimatic sequence= $S_1-a_1=2-1=1$and by another method $a_1=a_2-d=3-2=1$.

From table (7) we investigated that the constant of the quadratic function being subtracted can be obtained as:

$a_2=a_1+(n-1)$, Let d=2 , $3= a_1+(2-1)$, $a_1=3-2=1$.

The constant of quadratic arthimatic sequence= $S_1-a_1=0-1=-1$ and another method can write as:

$a_2=a_1+(n-1)$, $a_1=a_2-d=3-2=1$

From table (4) and table (5) , we see that the common difference of arthimatic sequence it is constant in the basic sequence and the resultant after adding or subtracting a constant term. and the constant of a quadratic function can be obtained from the first term of the 54resulting arithmetic sequence, because the first term it does not give a sequence with its second term or this term has a different common difference from the rest of the terms and the quadratic arthimatic sequence don't change because the quadratic coefficient don't change if the value of the constant term

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added to or subtracted from the original quadratic sequence is specified.

We can rewrite the some formula which determind the law of quadratic arthimatic sequence as dividing both sides by 2 produces a common formula of the equation:-

Alternative Arithmetic Sequence Formula:-

$$S_n = \frac{n}{2}(2a_1 + (n-1)d) \pm c \tag{47}$$

$$\sum_{m=1}^{m} f(m) = F_m = (a_1) + \frac{1}{2}\left(\begin{array}{l} ((\frac{m(m+1)}{2}) - 1) \times 2 \times (a_1)) \\ +((\frac{(m-1)m(m+1)}{3}) \times d) \end{array} \right) \pm mc \tag{48}$$

$$f(1) = S_1 = a_1 \tag{49}$$

$$f(m) = am^2 + bm \pm c = S_n = (\frac{n}{2}(2a_1 + (n-1)d) \pm c) \tag{50}$$

$$f(m) = am^2 + bm \pm c = S_n = \frac{n}{2}(a_1 + a_n) \pm c \tag{51}$$

- *Third Property:*

If we multiply each term of quadratic arithmetic sequence by a constant, we produce a new quadratic and arithmetic sequence with a different common difference than the common difference of the original sequence.

Table 6 m=1:4, a=1, b=3, d=2, f(m)=m^2+3m.

m=1	Quadratic sequence m^2+3m	Athematic sequence	Quadratic Sequence 3(m^2+3m)	Athematic Sequence 3(m^2+3m)
1	4	4	12	12
2	10	10-4=6	30	30-12=18
3	18	18-10=8	54	54-30=24
4	28	28-18=10	84	84-54=30

We obseserve from table(6) the common difference of arthimatic sequence which produce from *f(m)*= m^2+3m is equal 2 and in colmme (5) the common difference of arthimatic sequence which produce from (3(m^2+3m)) is equal common difference of the original sequence(colmm3) multiplied by the constant 3 and the quadratic arithmetic sequence will change because the value of the square coefficient is equal to the square coefficient of the basic sequence multiplied by the value of this constant.

- *Four Property:-*

A new quadratic sequence is produced if each term of the quadratic sequence is divided by a constant term. The resulting arithmetic sequence, formed by the difference of each term of the new quadratic sequence, has a common difference equal to the common difference of the original sequence divided by this constant term.

The quadratic arithmetic sequence varies so that each of its terms is divided by this constant. The following table shows this.

Table 7 m=1:4, a=$\frac{1}{4}$, b=3, d=$\frac{1}{2}$, f(m)= 0.5(.05m^2+m).

m	Quadratic sequence 0.5(.05m^2+)	Athematic sequence	Quadratic Sequence 0.5(0.5m^2+m)	Athematic sequence of 0.5(0.5m^2+m)	Common difference
1	1.5	1.5	0.75	0.75	1.25-0.75=0.5
2	4	4-1.5=2.5	2	2-0.75=1.25	1.75-1.25=0.5
3	7.5	7.5-4=3.5	3.75	3.75-2=1.75	2.25-1.75=0.5
4	12	12-7.5=4.5	6	6-3.75=2.25	

- *Five Property:*

A quadratic arthimatic sequence (quadratic function) becomes arthimatic sequence (linear function) if a constant term is added from it, and its value is equal to twice the coefficient of the quadratic term. In this case, the coefficient of the quadratic term is zero, and we have an arithmetic sequence whose common difference is equal to the value of the coefficient of the linear function, as follows:

Table 8 m=1:4, a=3, d=6, b=1, c=1, f(m)= $3m^2+m+6$.

m	Quadratic sequence $3m^2$+m+6	Athematic sequence	Quadratic Sequence 3(m^2+3m)	Athematic Sequence 3(m^2+3m)	Common difference
1	10	10	12	12	18-12=6
2	20	20-10=10	30	30-12=18	24-18=6
3	36	36-20=16	54	54-30=24	30-24=6
4	58	58-36=22	84	84-54=30	

From table(8) we can investigated that from colmm(3) $a_1=a_2$ then the constant c=common difference in quadratic arthimatic sequence, a=$\frac{d}{2}=\frac{6}{2}$,=3,at $,m=1, b=10-3-6=1$, that case when quadratic arthimatic sequence take the formula$f(m)=am^2+bm+c$.

- *Six Property:*

The relationship between an arithmetic sequence, its corresponding linear function, and the slope of the first derivative of the quadratic sequence:

Let (2,4.6,..) even arthimatic have (d=2), (a_1=2) we can write

The formula for the nt^h term of an arithmetic sequence

$$a_n=2+(n-1)\times 2 \quad (52)$$

the linear function of which corresponds to the arthmatic sequence as:

$$y^*=p_1x+b_1=2x \quad (53)$$

y_1= 2 at x=1 , y_2 =4 at x=2

p is he slope of a straight line is $p_1 = (y_2 - y_1) / (x_2 - x_1)=2$, where ($x_1$, y_1) and (x_2, y_2) are the coordinates of two points on the straight line. The slope (p_1) represents the gradient of the line, which is the vertical change (difference of the y) divided by the horizontal change (difference of the x) .from equ (53) we can say the slop p_1 equals the coefficient of x, $b_1=a_1-d=2-2=0$

By comparison eq(52) and eq(53) we get:

The slope of an arithmetic sequence is equal to the slope of a straight line. This slope in each of them is equal to the common difference of the arithmetic sequence.

P_1=d=2

We can write the quadratic sequence of even arthimatic sequence as:

Let $a=\frac{d}{2}=1$, b=a_1 - $\frac{d}{2}$=2-1=1

$$f(m) = am^2+bm=m^2+m \quad (54)$$

$$f(\grave{m})=2am+b=2m+1 \quad (55)$$

b=a_1- $\frac{d}{2}$ = a_1-a=2-1=1

From eq(55) we find the slope = 2 and we can investeqated that the quadratic coffecient as:

$$a=b+b_1=1+0=1 \quad (56)$$

We can investigated that the slope of the straight line resulting from the first derivative of a quadratic sequence which can be obtained from the product of its common difference and the order of the term + the coefficient of the linear term of the quadratic sequence, which is equal to (the first term of the arithmetic sequence - its common difference / 2). This quadratic sequence expresses the sum of the terms of this arithmetic sequence.

VII. RESULTS AND DISCUSSION

Arithmetic and quadratic sequences were solved, and the imposed formulas were verified and graphed using MATLAB.

Fig. 1 and 2 we notic that the arthimatic sequence,the linear function that corresponds to it, and the first derivative of the quadratic arithmetic sequence are represpesented by astraight line with the same slope, which is equal to the common difference of arthimatic sequence , when $1\geq n\geq 5$. The straight line of arthimatic sequence is rises upward and slope of arthimatic sequence,the linear function that corresponds to it, and the first derivative of the quadratic arithmetic sequence is positive. Fig.2 clear that if$-5\geq n\geq 5$ and $-5\geq m\geq 5$,$d=-2$ (negative) the papola of the quadratic sequence, it opens downwards, the slope of the line resulting from its first derivative is negative, and the slope of the arithmetic sequence and the corresponding linear function of thise arithmetic sequence are also negative. From the drawing in Fig.3, we notice that the parabola of the quadratic function and the quadratic arithmetic sequence are identical, and that the sum of the terms of a quadratic arithmetic sequence is represented by the curve of third-degree function.

When the common difference (d=2, 4,6) and the first term (a_1=1,2, 3) is increase the quadratic coffecient (a=1,2,3) and the first term of quadratice arthimatic sequanceses(s_1=1,2, 3) increases and parpola is curvature in

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another hand at common difference decreases the parpola becomes flatter and the quadratic coffecient decreases and the line of symmetry passes through the vertices of the parabolas this note is illustrated in Fig.4 . in fig.5 parabola of quadratic function($3m^2$+3m) and the quadtatic arthimatic sequance((2x6+(n-1)x6)) have the same line of symmetry $\frac{-b}{d} = 0$, the same vertex(0,0), and open upward because $d > 0$. We notice from Fig.6 that a constant term can be added to the quadratic arithmetic sequence and that this term represents the shift upward if it is $c > 0$ or downward if it is $c < 0$on the vertical line.and at $a_1 = 6, a = 3, d = 6, b = 3, c = 30$ we observe that the quadtatic arthimatic sequance((2x6+(n-1)x6) +30)and quadratic function($3m^2$+3m+30) have same vertex ($\frac{-3}{6}$,$\frac{-9}{12}$)and have the same properties of parpola.and at $a_1 = 6, a = 3, d = 6, b = 3, c = 30$ we see axis of symmetry($\frac{-3}{6}$)is the line perpendicular to the guid and passing through the focus (the line that divides the parabola in half).effects of constant c on sn and quadadratic function correspond appar in effects the displacement of the parabola (upward if positive, downward if negative). It also affects the properties of the parabola, such as the vertex ($\frac{-3}{6}$, $\frac{117}{4}$) it the point of intersection of a parabola with its axis of symmetry , and is the point where the parabola is at its maximum curvature. The position of the guide (K-$\frac{1}{4a}$) =($\frac{175}{6}$)is affected by the vertical displacement represented by the constant = 30 it is a line that lies at a fixed distance from each point on the parabola, where this distance is equal to the distance from the same point to the focus (a fixed point within the parabola). The parabola opens downward when an = $\left(-\frac{1}{6}\right) + (n-1) \times \left(-\frac{1}{3}\right)$ and sn = $\frac{n}{2} \times (2 \times \left(-\frac{1}{6}\right) + (n-1) \times \left(\frac{-1}{3}\right))$ and$f(m) = -\frac{1}{6}m^2$. we note that the parpola of quadratic arthimatic sequance applies to the quadratic function which corresponds to it , the properties of a parabola such as vertex (0,0), focus (0,$-\frac{3}{2}$), and guid=$\frac{3}{2}$, the focal chord= $\left|\frac{1}{a}\right| = \left|\frac{1}{(1/2)d}\right| = |6|$is the straight line segment passing through the focus of the parabola and perpendicular to the axis of symmetry and it is equal to twice the distance between the vertex and the focus.,, focal length $\left|\frac{1}{4a}\right| = \left|\frac{1}{2d}\right|$ is the distance from the vertex of a parabola to its focus and it is also equal to the distance between the focus and theguid . These properties can be observed through the parabola in the fig.7

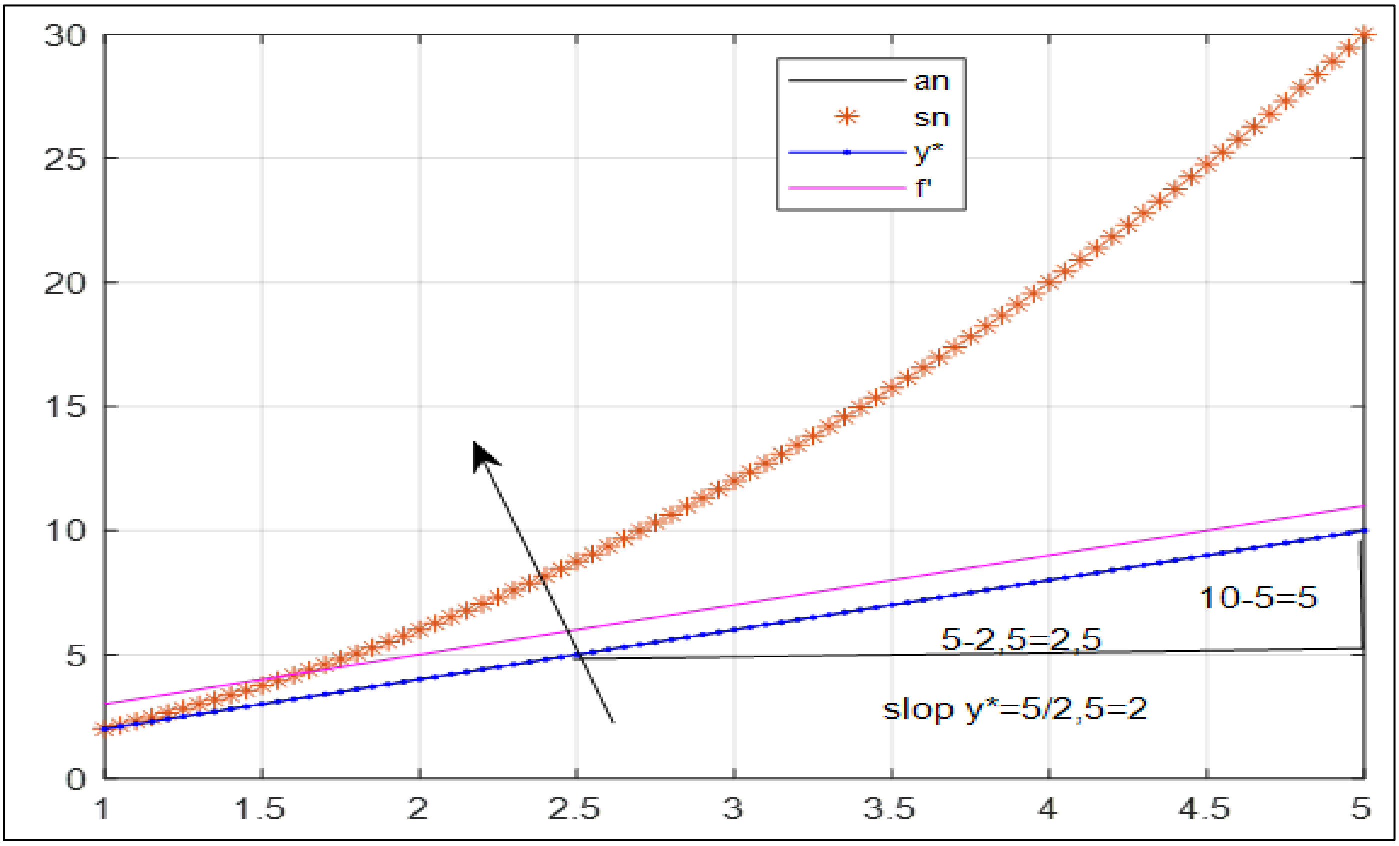

Fig 1 Effects of d on a_n and s_n ,y*, f ' at d=2, a_1=2,b_1=0, p_1=2

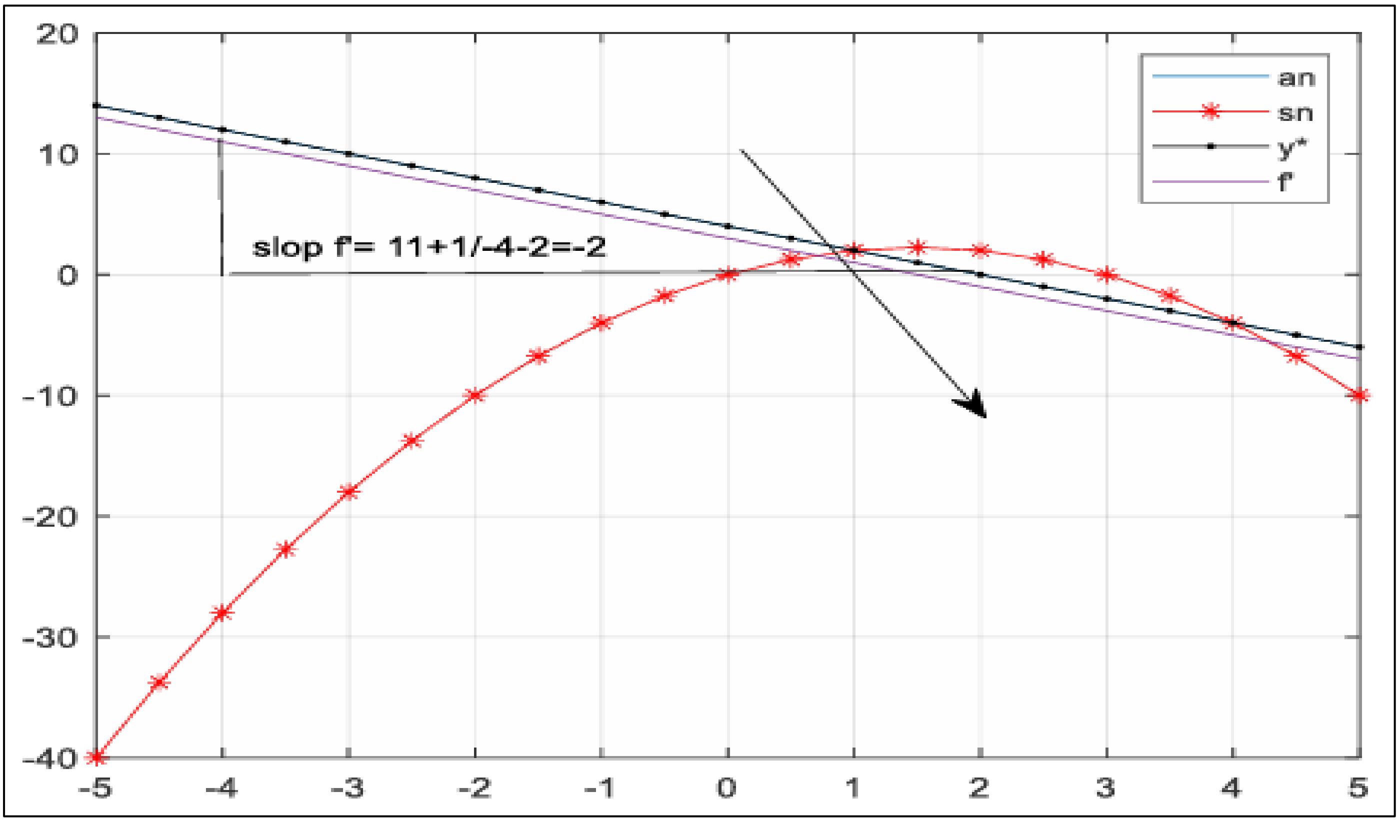

Fig 2 Effects of d on a_n and s_n ,y*, f ' at d=-2, a_1=2,b_1=4, p_1=-2.

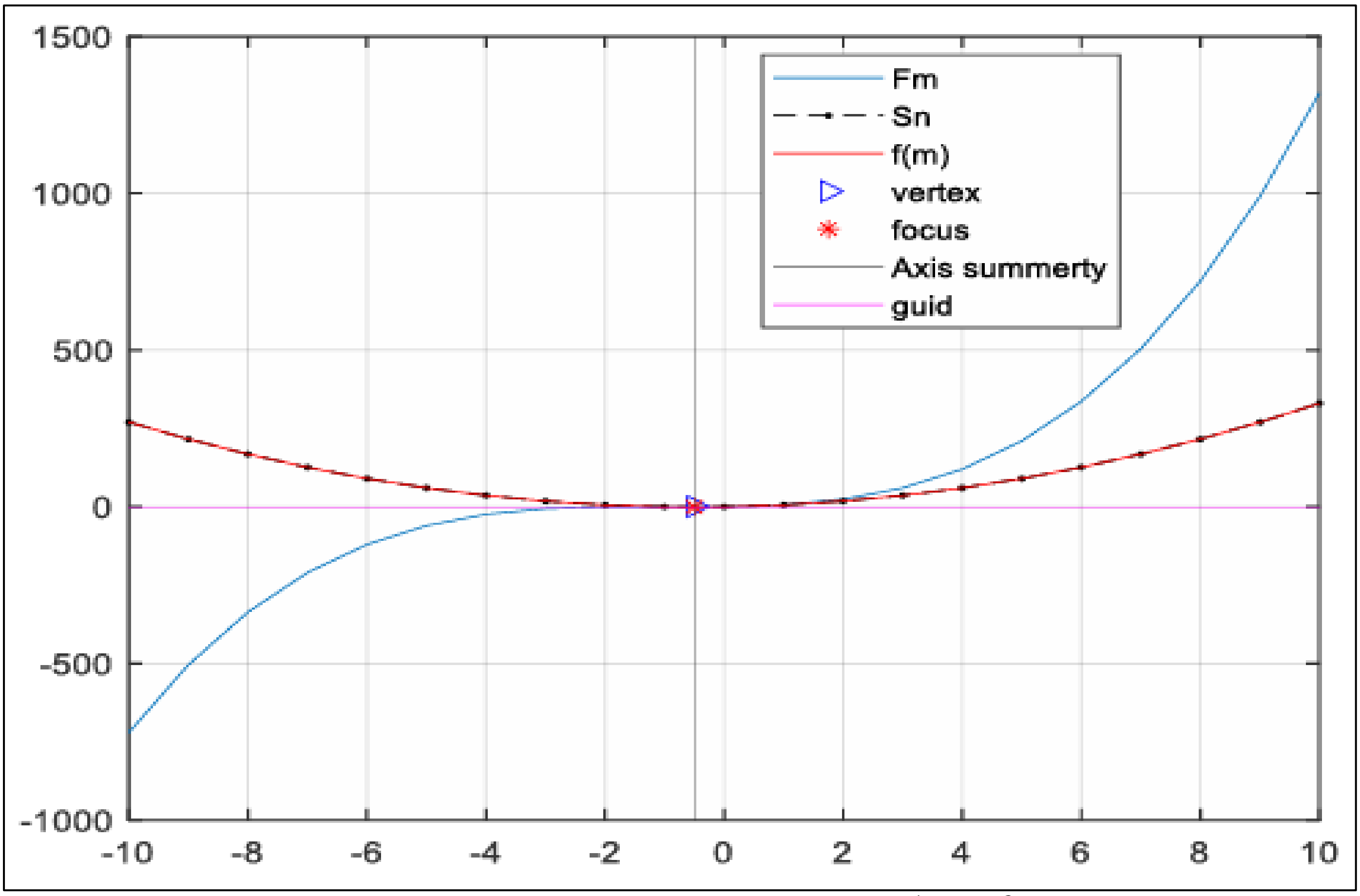

Fig 3 Effects of d on F_m and s_n ,f(m), at d=3,a=3, a_1=6, h=$\frac{-1}{2}$, $K=\frac{-3}{4}$,c=0.

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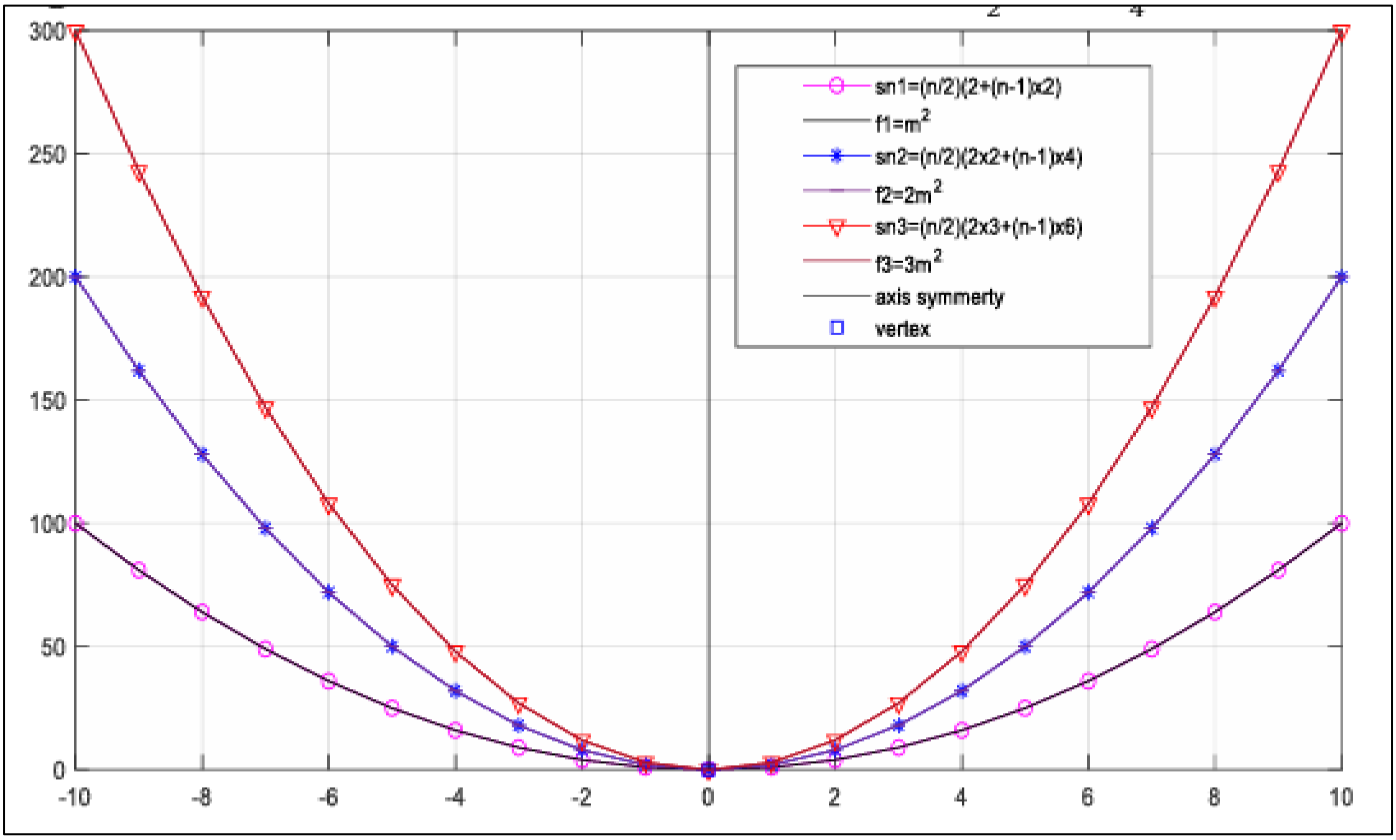

Fig 4 Effects of d on Quadratic Sequence and Quadratic Function at b=c=0.

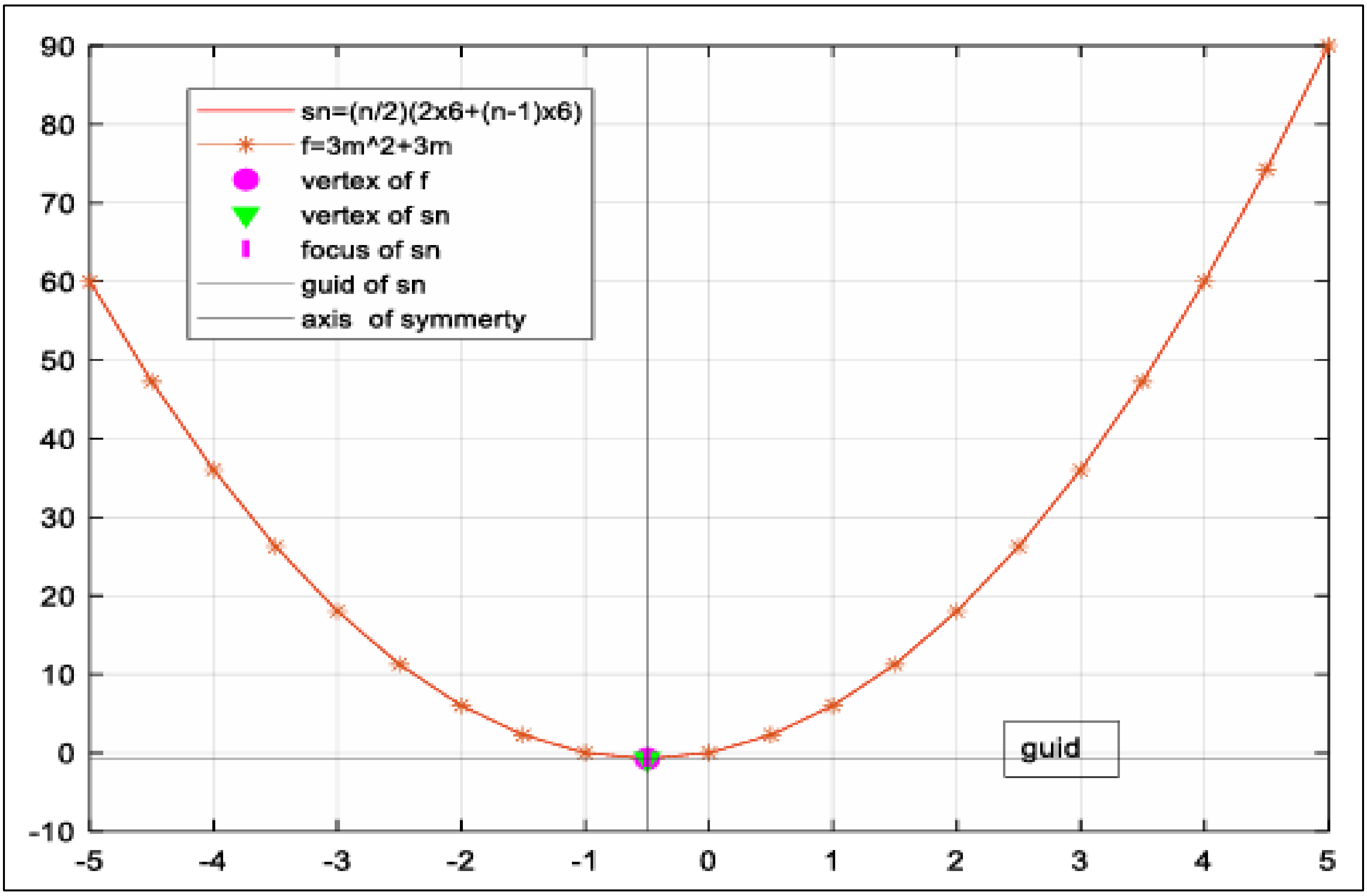

Fig 5 Effects of Constant c on Quadratic Sequence and Quadratic Function at b=3, c=0, a_1=3, d=6.

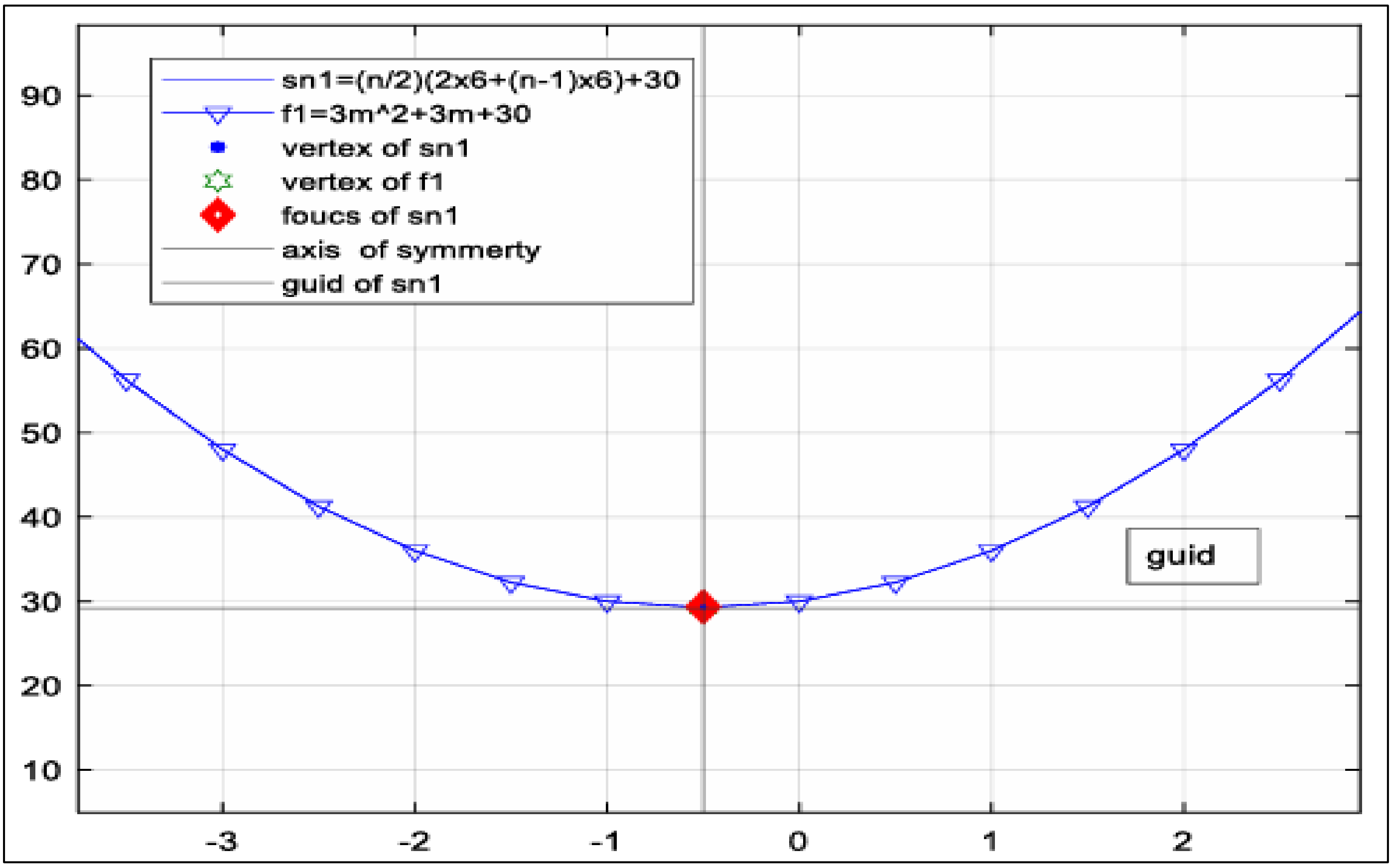

Fig 6 Effects of Constant c on Quadratic Sequence and Quadratic Function at b=3,

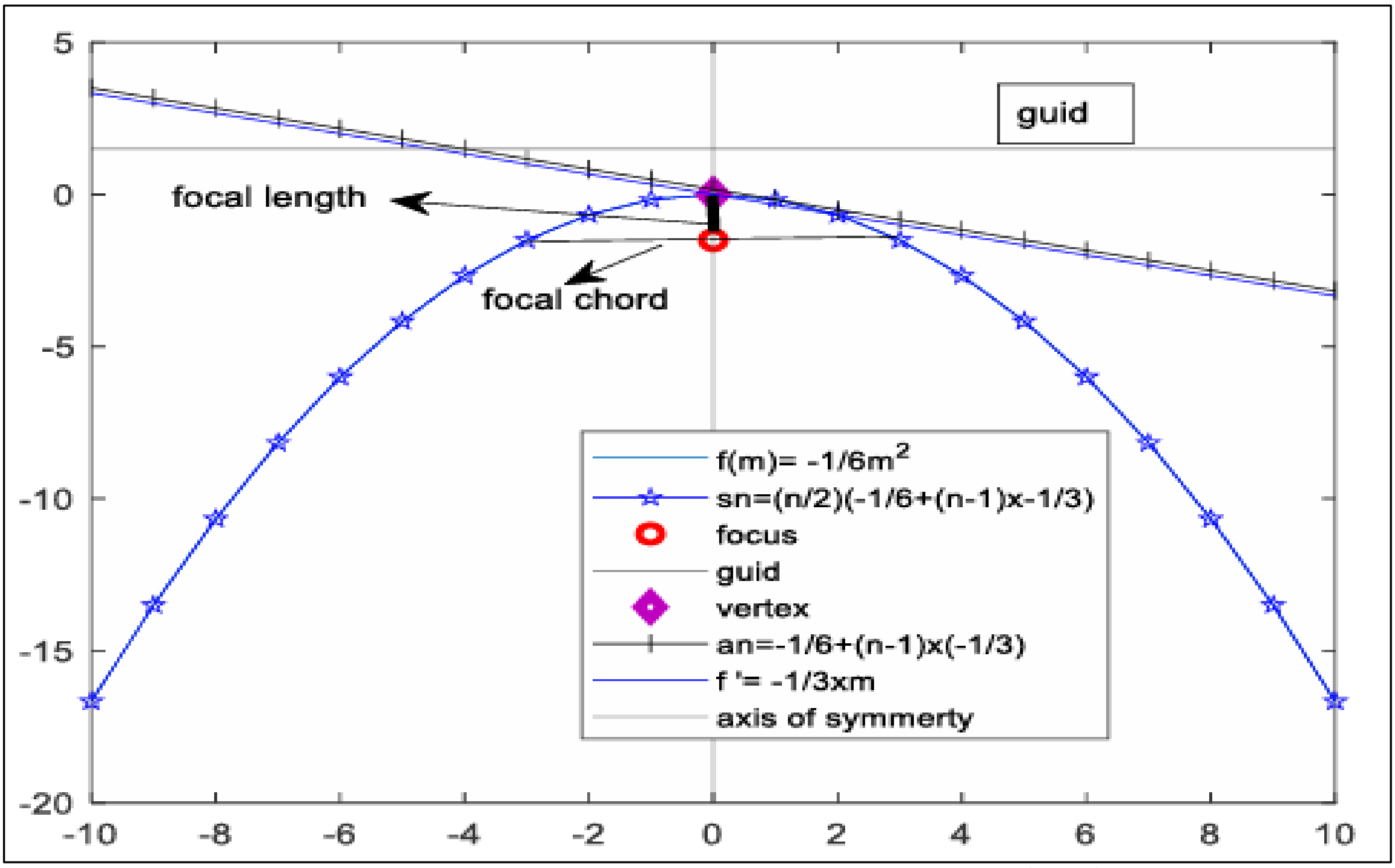

Fig 7 Effects of d on Focal Length and Focal Chord at b=0, c=0, $a_1=\frac{-1}{6}$, $d=\frac{-1}{3}$.

VIII. CONCLUSIONS

This study showed a new type of mathematical patterns of arithmetic sequences, which is the quadratic arithmetic sequence. The effect of the basis of the arithmetic sequence on the shape of the parabola, the focal length, and also its focus was studied. A quadratic arithmetic sequence was deduced that matches the quadratic function, and it was obtained that the sum of n terms of an arithmetic sequence equals one term of the quadratic arithmetic sequence. Formulas arrived at and represented graphically using *bvp4c* function in MATLAB package.

➢ *Most Important Significant Results of this Study are as Follow:*

- The sum of n terms of an arithmetic sequence equals the m term of a quadratic sequence and value of second function which have same first term and square coefficient of the quadratic function equal halfe base of arthimatic sequance.
- The base of an arithmetic sequence is equal to twice the square coefficient of the quadratic function.
- The first term of an arithmetic sequence is equal to the value of the first term of a quadratic equation.
- The quadratic arithmetic sequence is represented by a parabola that opens upward $d > 0$ or opens downward $d < 0$.
- The base of the arithmetic sequence increases, the parabola narrows and the focal length decreases, and vice versa. When the base decreases, the parabola becomes wider and the focal length increases.
- As the arithmetic sequence increases, the slope of the arithmetic sequence, the slope of the corresponding linear function, and the slope of the first derivative of the quadratic arithmetic sequence increase.
- An arithmetic sequence is graphed using a straight line, a quadratic sequence is graphed using a parabolic section, and their sum is graphed using a cubic function.

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