Some Hyperoperational Aspects of Hypergraphs and Ring Theoretic Vertex Noetherian Path

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Abstract: In this paper some hyperoperational aspects involving hypergraphs are investigated. An introduction of new hyperoperational notions (one is with so called partial character) signifies non-commutative, self-reciprocating and index commuting self associative attributes. Furthermore, some hyperoperational characteristics applicable to a Noetherian ring on some sub-algebraic structures equipped with ideal hypergraphs are developed. Here, the study is mainly to perceive how far the new notions presented would take us in the algebraic hyperstructural area discussed. On the other way another innovative attempt is made to develop a hypergraphic exploration of some ring theoretic finiteness equivalent conditions that may be called some sort of chain conditional path.

Keywords: *Hypergraph; Recursive Hyperedge; Recursive Diameter; Ideal Maximal Condition.*

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I. INTRODUCTION

Hypergraph is being widely and deeply investigated since its inception and have been used as a successful tool to represent and model complex concepts and structures in various areas of computer science and discrete mathematics [1]. Algebraic hyperstructures appear as a natural extension of usual algebraic structures, introduced by the French mathematician Marty [12] in the year 1934. It is very well known that in classical algebraic structures, its attached composition dealing with two elements give rise to an element only. On the other hand, the composition available in an algebraic hyperstructure is responsible to give the output as a set. Some authors have studied hyperstructures in connection with cryptography, coding, automata, probability, geometry, graphs, hypergraphs etc. so as to suit their objectives. [4, 6, 8, 9, 10, 15].

Among the various research works done in this field it would be worth while to mention about Corsini's work in this regard [4, 5, 6, 7]. A special hypergroup, defined by Corsini has been named as Corsini hypergroup by authors like M. Al Tahan and B. Davvaz in their work "On Corsini hypergroups and their productional hypergroups" [15]. A range of work has been done based on the works of Corsini [11, 12, 13, 14]. Taking this as a motivation for our work we have tried to introduce two different definitions that proceed in a slightly different direction compared to the definitions that are at hand. The study is mainly to perceive how far these new notions will take us in this algebraic hyperstructural area. This would consist of the first part of the article. properties. Some noticeable results establish non-commutative, self-reciprocating and index

In the second section our attempt would be to develop a hypergraphic exploration of ring theoretic finiteness equivalent conditions leading to so called vertex Noetherian path.

II. PRELIMINARIES

In this section, the basic definitions and notations are introduced for the sake of the subsequent sections. If H is a non-empty set and $\mathcal{P}^*(H)$ is the set of all non-empty subsets of H, then a mapping $o: H \times H \to \mathcal{P}^*(H)$ is called a hyperoperation. We note that $x \quad o \quad y \subseteq H$ for all $x, y \in H$ and it is termed as the hyperproduct of x and y. If A and B are non-empty subsets of H, then by $A \quad o \quad B$, we mean.

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$$A \quad o \quad B = \bigcup_{x\in A, y\in B} x \quad o \quad y$$

An algebraic system (H, o) together with a hyperoperation is known as hypergroupoid.

A hypergraph [14] is a pair $\Gamma = (H, \mathcal{E})$, where H is a set of vertices and $\mathcal{E} = \{E_1, E_2, \dots\}$ is a collection of non-empty subsets of H (known as hyperedges) such that $\cup_{i=1}^{m} E_i = H$.

Here, we would like to note that the condition $\cup_{i=1}^{m} E_i = H$ is not obligatory in the definition of hypergraph in the theory of general hypergraphs.

- Definition 2.2 *A hyperoperation* $\ _n *_m$, *for all* $n, m \in \mathbb{N}$ *on a hypergraph* $\Gamma = (H, \mathcal{E})$ *is defined as, for all* $(x, y) \in H^2$,

$$x_n *_m y = E^n(x) \cap E^m(y),$$

Where $E^0(x) = x, E(x) = \cup_{x\in E_i} E_i, E(A) = \cup_{x\in A} E(x)$ for all non-empty subset A of H, and $E^n(x) = E^{n-1}(E(x))$.

It is important to note here that the hypergroupoid $(H, \ _n *_m)$ thus formed is a partial hypergroupoid.

- Definition 2.3 *If* A *and* B *are non-empty subsets of* H, *then by* $A \ _n *_m \ B$ *we mean,*

$$A \ _n *_m \ B = \bigcup_{x\in A, y\in B} x \ _n *_m \ y$$

And for $x \in H$, $x \ _n *_m \ A = \{x\} \ _n *_m \ A$ and $A \ _n *_m \ x = A \ _n *_m \ \{x\}$.

- Definition 2.4 *A partial hyperoperation* $\ _n\theta_m$, *for all* $n, m \in \mathbb{N}$ *on a hypergraph* $\Gamma = (H, \mathcal{E})$ *is defined as, for all* $(x, y) \in H^2$,

$$x_n\theta_m y = [E^n(x) \cup E^m(y)] \backslash [E^n(x) \cap E^m(y)],$$

 Where $E^0(x) = x, E(x) = \cup_{x\in E_i} E_i, E(A) = \cup_{x\in A} E(x)$ for all non-empty subset A of H, and $E^n(x) = E^{n-1}(E(x))$.

 It is important to note here that the hypergroupoid $(H, \ _n\theta_m)$ thus formed is a partial hypergroupoid.

- Definition 2.5 *If* A *and* B *are non-empty subsets of* H, *then by* $A \ _n\theta_m \ B$ *we mean,*

$$A \ _n\theta_m \ B = \bigcup_{x\in A, y\in B} x \ _n\theta_m \ y,$$

And for $x \in H$, $x \ _n\theta_m \ A = \{x\} \ _n\theta_m \ A$ and $A \ _n\theta_m \ x = A \ _n\theta_m \ \{x\}$.

- Note: The partial hypergroupoids $H_\Gamma = (H, \ _n *_m)$ and $H_\Gamma = (H, \ _n\theta_m)$ will be called *partial hypergraph hypergroupoid.*

- *Definition 2.6 A hyperoperation* $\ _n o_m$, *for all* $n, m \in \mathbb{N}$ *on a hypergraph* $\Gamma = (H, \mathcal{E})$ *is self reciprocating if for any* $(a, b) \in H^2$, $a \in b \ _n o_m \ b \Leftrightarrow b \in a \ _n o_m \ a$.
- *Definition 2.7 An* E *operator hyperoperation is a hyperoperation described with the help of* E *as already mentioned in Definition 2.2 and 2.4.*
- *Definition 2.8 In an* E*-operator hyperoperation, say,* $\ _n o_m$, m *and* n *are termed as indices.*
- *Definition 2.9 An* E*-operator hyperoperation* $\ _n o_m$ *is associative if for* $a, b, c \in H$,

$$(a \ _n o_m \ b) \ _n o_m \ c = a \ _n o_m \ (b \ _n o_m \ c).$$

- *Definition 2.10 An* E*-operator hyperoperation* $\ _n o_m$ *is commuting associative if for* $a, b, c \in H$,

$$(a \ _n o_m \ b) \ _n o_m \ c = a \ _n o_m \ (c \ _n o_m \ b).$$

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- *Definition 2.11 An E-operator hyperoperation ${}_n o_m$ is index commuting self-associative if for each $x \in H$,*

$$(x \;{}_n o_m\; x)\;{}_n o_m\; x = x \;{}_n o_m\; (x \;{}_m o_n\; x).$$

- *Theorem 2.1 For each $(x, y) \in H^2$ and $m, n \in \mathbb{N}$ the hypergroupoid $H_\Gamma = (H, {}_n *_m)$ satisfies the following:*

✓ $x \in x_n *_m x$.
✓ $x_n *_m y = y_n *_m x$. $[{}_n *_m$ is commutative.$]$
✓ $y \in x_n *_m x \Leftrightarrow x \in y_n *_m y$. $[{}_n *_m$ is self reciprocating.$]$
✓ $(x_n *_m x)_n *_m (x_n *_m x) \supseteq (x_n *_m x)_n *_m x$.

- *Proof.* Proof of (1) and (2) are straightforward.

$y \in x_n *_m x = E^n(x) \cap E^m(x) = E^m(x),\ n \geq m.$

$\Rightarrow y \in E^m(x).$

Then, $E(y) \cap E^{m-1}(x) \neq \phi$ and so $E^2(y) \cap E^{m-2}(x) \neq \phi, \ldots, E^{m-1}(y) \cap E(x) \neq \phi$

[Proof as given in [14]].

Therefore, $x \in E^m(y) = E^m(y) \cap E^n(y) = y_n *_m y$.

$\Rightarrow x \in y_n *_m y$.

Thus, $y \in x_n *_m x \Longrightarrow x \in y_n *_m y$.

Similarly, we can show, $x \in y_n *_m y \Longrightarrow y \in x_n *_m x$.

Hence, $y \in x_n *_m x \Longleftrightarrow x \in y_n *_m y$.

Or, ${}_n *_m$ is self reciprocating.

$$(x \;{}_m *_n\; x)\;{}_m *_n x = [E^m(x) \cap E^n(x)]\;{}_m *_n x, \qquad (n > m)$$

$$= E^m(x)\;{}_m *_n x$$

$$= \{x, x_1, x_2, \ldots, x_{p-1}\}\;{}_m *_n x, \qquad [Suppose\quad E^m(x) = \{x, x_1, x_2, \ldots, x_{p-1}\}]$$

$$= \underbrace{(x\;{}_m *_n x) \cup (x_1\;{}_m *_n x) \cup (x_2\;{}_m *_n x) \cup \ldots \cup (x_{p-1}\;{}_m *_n x)}_{\text{p components}}, \qquad \rightarrow (A)$$

Again,

$$(x\;{}_m *_n\; x)_m *_n (x\;{}_m *_n x) = [E^m(x) \cap E^n(x)]_m *_n [E^m(x) \cap E^n(x)]$$

$$= E^m(x)\;{}_m *_n E^m(x), \qquad (n > m)$$

$$= \{x, x_1, \ldots, x_{p-1}\}_m *_n \{x, x_1, \ldots, x_{p-1}\}$$

$$= (x\;{}_m *_n\; x) \cup (x\;{}_m *_n\; x_1) \cup \ldots (x\;{}_m *_n\; x_{p-1})$$

$$\cup (x_1\;{}_m *_n\; x) \cup (x_1\;{}_m *_n\; x_1) \cup \ldots \cup (x_1\;{}_m *_n\; x_{p-1})$$

$$\cup (x_{p-1}\;{}_m *_n x) \cup (x_{p-1}\;{}_m *_n x_1) \cup \ldots \cup (x_{p-1}\;{}_m *_n x_{p-1})$$

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$$= \{\underbrace{(x \; {}_m{*}_n \; x) \cup (x_1 \; {}_m{*}_n \; x) \cup \ldots \cup (x_{p-1} \; {}_m{*}_n \; x)}_{\text{p components}}\}$$

$$\cup\, (x \; {}_m{*}_n \; x_1) \cup (x_1 \; {}_m{*}_n \; x_1) \cup \ldots \cup (x_{p-1} \; {}_m{*}_n \; x_{p-1}), \qquad \longrightarrow (B)$$

From (A) and (B) it is easy to conclude that,

$$(x_n *_m x)_n *_m (x_n *_m x) \supseteq (x_n *_m x)_n *_m x$$

- *Example 2.1 Suppose $\Gamma = (H, E)$ is a hypergraph where $H = \{v_1, v_2, v_3, \ldots, v_8\}$ and $E = \{E_1 = \{v_1, v_2, v_3, v_7\}, E_2 = \{v_3, v_4, v_5, v_6, v_7\}, E_3 = \{v_6, v_8\}\}$.*

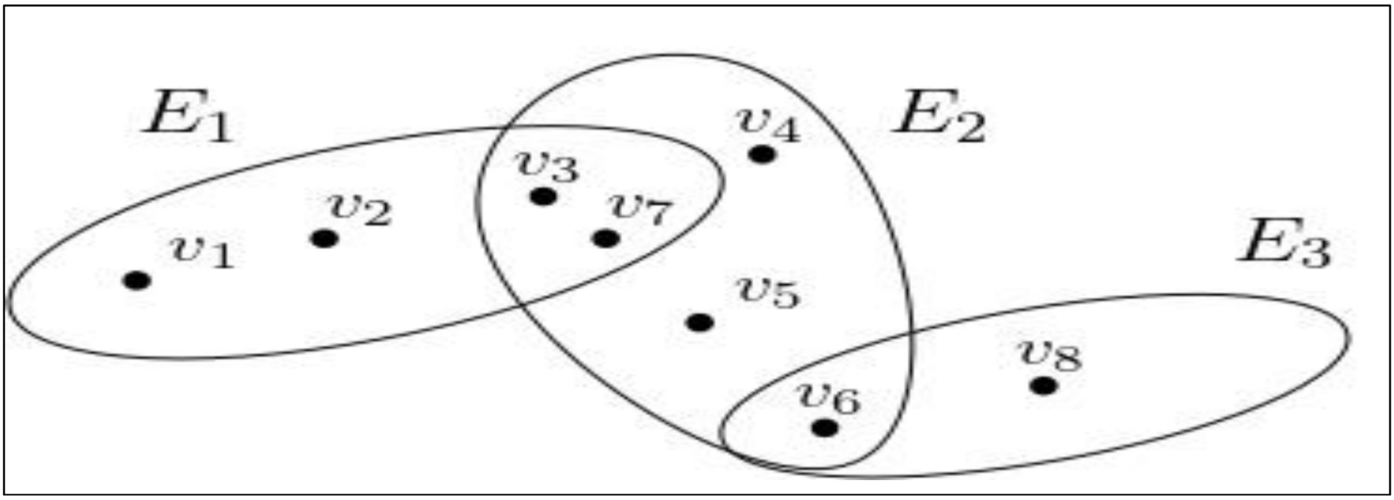

Fig 1 Hypergraph

Then,

$$(v_1 \; {}_2{*}_1 \; v_1) \; {}_2{*}_1 \; v_1 = [E^2(v_1) \cap E(v_1)] \; {}_2{*}_1 \; v_1$$

$$= [E(v_1)] \; {}_2{*}_1 \; v_1$$

$$= \{v_1, v_2, v_3, v_7\} \; {}_2{*}_1 \; v_1$$

$$= (v_1 \; {}_2{*}_1 \; v_1) \cup (v_2 \; {}_2{*}_1 \; v_1) \cup (v_3 \; {}_2{*}_1 \; v_1) \cup$$

$$(v_7 \; {}_2{*}_1 \; v_1)$$

$$= [E^2(v_1) \cap E(v_1)] \cup [E^2(v_2) \cap E(v_1)]$$

$$\cup\, [E^2(v_3) \cap E(v_1)] \cup [E^2(v_7) \cap E(v_1)]$$

$$= \{v_1, v_2, v_3, v_7\} \cup \{v_1, v_2, v_3, v_7\} \cup \{v_1, v_2, v_3, v_7\}$$

$$\cup\, \{v_1, v_2, v_3, v_7\}$$

$$= \{v_1, v_2, v_3, v_7\} \qquad \longrightarrow (i)$$

And,

$$(v_1 \; {}_2{*}_1 \; v_1) \; {}_2{*}_1 \; (v_1 \; {}_2{*}_1 \; v_1) = E(v_1) \; {}_2{*}_1 \; E(v_1), \qquad [Like \quad above]$$

$$= \{v_1, v_2, v_3, v_7\} \; {}_2{*}_1 \; \{v_1, v_2, v_3, v_7\}$$

$$= (v_1 \; {}_2{*}_1 \; v_1) \cup (v_1 \; {}_2{*}_1 \; v_2) \cup (v_1 \; {}_2{*}_1 \; v_3)$$

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$$\cup (v_1 \quad {}_2*_1 \quad v_7) \cup \ldots \cup (v_3 \quad {}_2*_1 \quad v_7)$$

$$= [E^2(v_1) \cap E(v_1)] \cup [E^2(v_1) \cap E(v_2)] \cup [E^2(v_1)$$

$$\cap E(v_3)] \cup \ldots \cup [E^2(v_3) \cap E(v_7)]$$

$$= \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}, \qquad \longrightarrow (ii)$$

From (i) and (ii) we get,

$$(v_1 \quad {}_2*_1 \quad v_1) \quad {}_2*_1 \quad v_1 \subseteq (v_1 \quad {}_2*_1 \quad v_1) \quad {}_2*_1 \quad (v_1 \quad {}_2*_1 \quad v_1)$$

Similarly, for any other $v_i \in H$ we can see that,

$$(v_i \quad {}_2*_1 \quad v_i) \quad {}_2*_1 \quad v_i \subseteq (v_i \quad {}_2*_1 \quad v_i) \quad {}_2*_1 \quad (v_i \quad {}_2*_1 \quad v_i), \qquad i \in \{1,2,\ldots,8\}.$$

- *Theorem 2.2 For each $(x, y) \in H^2$ and $m.n \in \mathbb{N}$ the partial hypergroupoid $H_\Gamma = (H,_n \theta_m)$ satisfies the following:*

- ✓ $x \in H \backslash (x_n \theta_m x)$.
- ✓ ${}_n\theta_m$ is non-commutative, [i.e. for some n, m, $x_n \theta_m y \neq y_n \theta_m x$.]
- ✓ $y \in x_n \theta_m x \Leftrightarrow x \in y_n \theta_m y$. [${}_n\theta_m$ is self reciprocating.]

Proof. Proof of (1) and (2) are straightforward.

$$y \in x_n \theta_m x \Longrightarrow y \in [E^n(x) \cap E^m(x)] \backslash [E^n(x) \cap E^m(x)]$$

$$= E^n(x) \backslash E^m(x), \qquad\qquad n \geq m.$$

$$So, y \in E^n(x) \backslash E^m(x) \;\; that \;\; gives, y \in E^n(x) \;\; but \;\; y \notin E^m(x).$$

When $y \in E^n(x)$, then we have $E(y) \cap E^{n-1}(x) \neq \phi$ and so, $E^2(y) \cap E^{n-2}(x) \neq \phi$, .

$E^{n-1}(y) \cap E(x) \neq \phi$.

Hence, $E^n(y) \cap x \neq \phi$. Therefore, $x \in E^n(y)$.

Next we have, $y \in E^m(x) \Leftrightarrow x \in E^m(y)$ [Proof as given [14].]

Also, we know that, $P \Longrightarrow Q$ gives $\sim Q \Longrightarrow \sim P$

So, combining the above both we have, $y \notin E^m(x) \Longrightarrow x \notin E^m(y)$.

Hence, we have,

$x \in E^n(y)$ but $x \notin E^m(y)$ that gives us

$x \in E^n(y) \backslash E^m(y)$

$= [E^n(y) \cup E^m(y)] \backslash [E^n(y) \cap E^m(y)]$

$= y_n \theta_m y$

So, $x \in y_n \theta_m y$.

Thus, we get,

$$y \in x_n \theta_m x \Longrightarrow x \in y_n \theta_m y$$

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Similarly, we can have,

$$x \in y_n\theta_m y \Longrightarrow y \in x_n\theta_m x$$

Thus,

$$y \in x_n\theta_m x \Leftarrow x \in y_n\theta_m y$$

Hence, ${}_n\theta_m$ is self reciprocating.

- *Example 2.2 Suppose* $\Gamma = (H, E)$ *is a hypergraph where* $H = \{v_1, v_2, v_3, \dots, v_8\}$ *and* $E = \{E_1 = \{v_1, v_2, v_3, v_7\}, E_2 = \{v_3, v_4, v_5, v_6, v_7\}, E_3 = \{v_6, v_8\}\}$ *then,*

$$(v_1 \quad {}_2\theta_1 \quad v_3) \quad {}_2\theta_1 \quad v_5 = [[E^2(v_1) \cup E(v_3)] \setminus [E^2(v_1) \cap E(v_3)]] \quad {}_2\theta_1 \quad v_5$$

$$= [\{v_1, v_2, \dots, v_8\} \setminus \{v_1, v_2, \dots, v_7\}] \quad {}_2\theta_1 \quad v_5$$

$$= \{v_8\} \quad {}_2\theta_1 \quad v_5$$

$$= \{v_8\}$$

And,

$$v_1 \quad {}_2\theta_1 \quad (v_3 \quad {}_2\theta_1 \quad v_5) = v_1 \quad {}_2\theta_1 \quad [[E^2(v_3) \cup E(v_5)] \setminus [E^2(v_3) \cap E(v_5)]]$$

$$= v_1 \quad {}_2\theta_1 \quad \{v_1, v_2, v_8\}$$

$$= (v_1 \quad {}_2\theta_1 \quad v_1) \cup (v_1 \quad {}_2\theta_1 \quad v_2) \cup (v_1 \quad {}_2\theta_1 \quad v_8)$$

$$= \{v_4, v_5, v_6, v_8\}$$

Also,

$$v_1 \quad {}_2\theta_1 \quad (v_5 \quad {}_2\theta_1 \quad v_3) = v_1 \quad {}_2\theta_1 \quad [[E^2(v_5) \cup E(v_3)] \setminus [E^2(v_5) \cap E(v_3)]]$$

$$= \{v_1, v_2, v_3, v_4, v_5, v_7\}$$

We observe here that,

$$(v_1 \quad {}_2\theta_1 \quad v_3) \quad {}_2\theta_1 \quad v_5 \neq v_1 \quad {}_2\theta_1 \quad (v_3 \quad {}_2\theta_1 \quad v_5),$$

And,

$$(v_1 \quad {}_2\theta_1 \quad v_3) \quad {}_2\theta_1 \quad v_5 \neq v_1 \quad {}_2\theta_1 \quad (v_5 \quad {}_2\theta_1 \quad v_3).$$

So, in general we can conclude that,

For the partial hypergroupoid $H_\Gamma = (H, {}_n\theta_m)$, where $a, b, c \in H$ and $m, n \in H$ the following,

$$(i) \quad (a \quad {}_n\theta_m \quad b) \quad {}_n\theta_m \quad c = a \quad {}_n\theta_m \quad (b \quad {}_n\theta_m \quad c)$$

$$(ii) \quad (a \quad {}_n\theta_m \quad b) \quad {}_n\theta_m \quad c = a \quad {}_n\theta_m \quad (c \quad {}_n\theta_m \quad b)$$

Are not true.

Furthermore, we observe that the partial hypergroupoid $H_\Gamma = (H, {}_n\theta_m)$ satisfies the index commuting self-associativity which is proved in the following theorem:

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- *Theorem 2.3 For any $x \in H$ and $m, n \in \mathbb{N}$ the partial hypergroupoid $H_\Gamma = (H,_n \theta_m)$ satisfies the index commutating self-associativity.*

Or,

$$(x_n\theta_m x)_m\theta_n x = x_n\theta_m(x_n\theta_m x).$$

Proof.

$$(x_n\theta_m x)_m\theta_n x = \bigcup_{z\in x_n\theta_m x} (z_m\theta_n x)$$

$$= \bigcup_{z\in x_n\theta_m x} [[E^m(z) \cup E^n(x)]\backslash[E^m(z) \cap E^n(x)]] \qquad \longrightarrow (C)$$

Again,

$$x_n\theta_m(x_n\theta_m x) = \bigcup_{z\in x_n\theta_m x} (x_n\theta_m z)$$

$$= \bigcup_{z\in x_n\theta_m x} [[E^n(x) \cup E^m(z)]\backslash[E^n(x) \cap E^m(z)]] \qquad \longrightarrow (D)$$

Therefore, from (C) and (D) we have,

$$(x_n\theta_m x)_m\theta_n x = x_n\theta_m(x_n\theta_m x)$$

- Example 2.3 *Suppose that $H = \{v_1, v_2, \ldots, v_{12}\}$ and $E = \{E_1 = \{v_1, v_2, v_3\}, E_2 = \{v_3, v_4, v_5, v_6\}, E_3 = \{v_6, v_7, v_8\}, E_4 = \{v_8, v_9, v_{10}\}, E_5 = \{v_{10}, v_{11}, v_{12}\}\}$.*

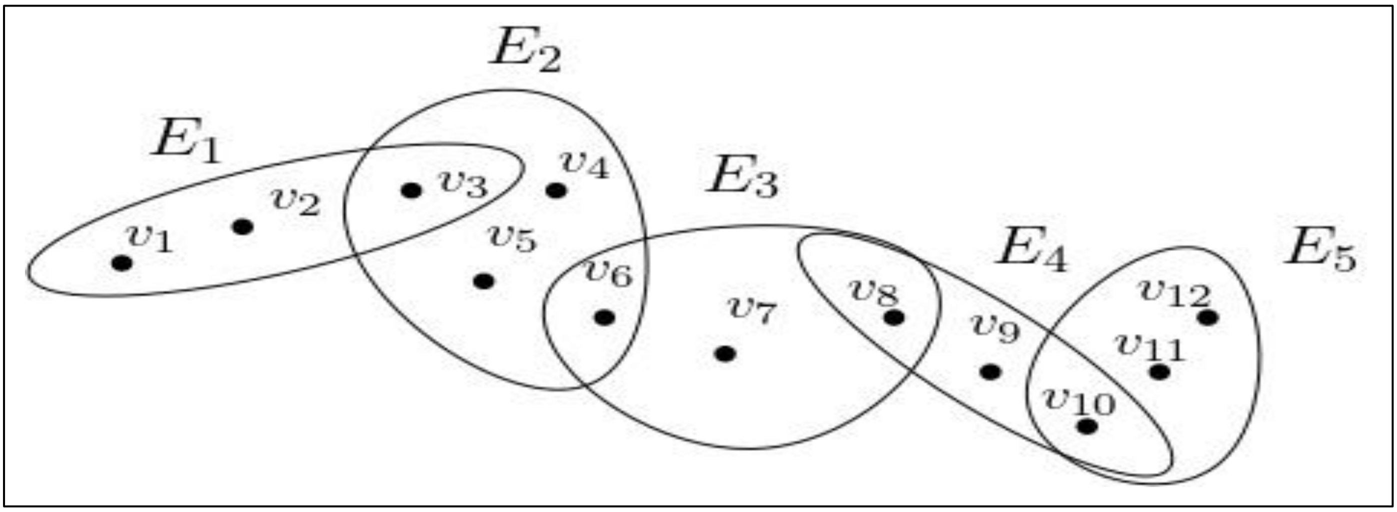

Fig 2 Hypergraph

Then,

$$(v_1 \ {}_2\theta_1 \ v_1)_1\theta_2 \ v_1 = [E^2(v_1) \cup E(v_1)]\backslash[E^2(v_1) \cap E(v_1)]_1\theta_2 v_1$$

$$= [E^2(v_1)\backslash E(v_1)]_1\theta_2 v_1$$

$$= [\{v_1, v_2, \ldots, v_6\}\backslash\{v_1, v_2, v_3\}]_1\theta_2 v_1$$

$$= \{v_4, v_5, v_6\}_1\theta_2 v_1$$

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$$= (v_4 \ \ {}_1\theta_2 \ \ v_1) \cup (v_5 \ \ {}_1\theta_2 \ \ v_1) \cup (v_6 \ \ {}_1\theta_2 \ \ v_1)$$

$$= [[E(v_4) \cup E^2(v_1)][E(v_4) \cap E^2(v_1)]] \cup [[E(v_5) \cup E^2(v_1)][E(v_5) \cap E^2(v_1)]]$$

$$\cup [[E(v_6) \cup E^2(v_1)]\backslash[E(v_6) \cap E^2(v_1)]]$$

$$= \{v_1, v_2\} \cup \{v_1, v_2\} \cup \{v_1, v_2, v_7, v_8\}$$

$$= \{v_1, v_2, v_7, v_8\}, \qquad \longrightarrow (iii)$$

Again,

$$v_1 \ \ {}_2\theta_1(v_1 \ \ {}_2\theta_1 v_1) = v_1 \ \ {}_2\theta_1\{v_4, v_5, v_6\}$$

$$= (v_1 \ \ {}_2\theta_1 \ \ v_4) \cup (v_1 \ \ {}_2\theta_1 \ \ v_5) \cup (v_1 \ \ {}_2\theta_1 \ \ v_6)$$

$$= [[E^2(v_1) \cup E(v_4)][E^2(v_1) \cap E(v_4)]] \cup [[E^2(v_1) \cup E(v_5)][E^2(v_1) \cap E(v_5)]]$$

$$\cup [[E^2(v_1) \cup E(v_6)]\backslash[E^2(v_1) \cap E(v_6)]]$$

$$= \{v_1, v_2, v_7, v_8\}, \qquad \longrightarrow (iv)$$

From (iii) and (iv) we get,

$$(v_1 \ \ {}_2\theta_1 \ \ v_1)_1\theta_2 \ \ v_1 = v_1 \ \ {}_2\theta_1(v_1 \ \ {}_2\theta_1 v_1)$$

Similarly, for any other $v_i \in H$ we can see that,

$$(v_i \ \ {}_2\theta_1 \ \ v_i)_1\theta_2 \ \ v_i = v_i \ \ {}_2\theta_1(v_i \ \ {}_2\theta_1 v_i), \qquad i \in \{1,2,\ldots,12\}$$

- *Corollary 2.4 For any $x \in H$ and $m, n \in \mathbb{N}$ the partial hypergroupoid $H_\Gamma = (H,_n \theta_m)$ satisfies,*

$$((x_n\theta_m x)_n\theta_m x)_m\theta_n x = x_n\theta_m(x_m\theta_n(x_n\theta_m x)).$$

Proof. (5).

$$((x_n\theta_m x)_n\theta_m x)_m\theta_n x = [\bigcup_{z \in x_n\theta_m x} z_n\theta_m x]_m\theta_n x$$

$$= \bigcup_{z \in x_n\theta_m x} [[E^n(z) \cup E^m(x)]\backslash[E^n(z) \cap E^m(z)]]_m\theta_n x$$

$$= R_m\theta_n x, \qquad where \qquad R = \bigcup_{z \in x_n\theta_m x} [[E^n(z) \cup E^m(x)]\backslash[E^n(z) \cap E^m(x)]]$$

$$= \bigcup_{u \in R} u_m\theta_n x$$

$$= \bigcup_{u \in R} [[E^m(u) \cup E^n(x)]\backslash[E^m(u) \cap E^n(x)]], \qquad \longrightarrow (E)$$

Again, we have,

$$x_n\theta_m(x_m\theta_n(x_n\theta_m x)) = x_n\theta_m(\bigcup_{z \in x_n\theta_m x} x_m\theta_n z)$$

$$= x_n\theta_m(\bigcup_{z \in x_n\theta_m x} [[E^m(x) \cup E^n(z)]\backslash[E^m(x) \cup E^m(z)]])$$

$$= x_n\theta_m R, \qquad where \qquad R = \bigcup_{z \in x_n\theta_m x} [[E^n(z) \cup E^m(x)]\backslash[E^n(z) \cap E^m(x)]]$$

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$$= x_n\theta_m R$$

$$= \bigcup_{u\in R} x_n\theta_m u$$

$$= \bigcup_{u\in R} [[E^n(x) \cup E^m(u)]\backslash[E^n(x) \cap E^m(u)]], \qquad \longrightarrow (F)$$

Therefore, from (E) and (F) we have,

$$((x_n\theta_m x)_n\theta_m x)_m\theta_n x = x_n\theta_m(x_m\theta_n(x_n\theta_m x))$$

Note: The above corollary appears valid for any finite number of composition.

III. HYPEROPERATION IN AN IDEAL HYPERGRAPH

In this section, we exhibit some hypergraph theoretic characteristics of some well known Noetherian properties available in ring theory. Keeping this in mind, we introduce here, the notions of recursive hyperedge, recursive diameter, infinite recursive diameter etc. together with vertex ideal maximal condition. All these would lead us to investigate the conditions that seem to be responsible for various types of finiteness characters in an ideal hypergraph.

- Definition 3.1 *An ideal hypergraph is a hypergraph $\Gamma = (H, \mathcal{E})$ where H is a ring (not necessarily commutative, finite or infinite) with unity and $\mathcal{E}$ is the collection of all the ideals of H. It is obviously a hypergraph as the ring $H \in \mathcal{E}$.*

If $I \subseteq H$, where H is a ring then the ideal generated by I is the smallest ideal containing I and is denoted by $< I >$. Thus, if $a \in H$ and $I = \{a\}$ then $< I >=< \{a\} >=< a > (= aH)$ the principal ideal generated by a.

- Definition 3.2 *A hyperoperation $*$ in an ideal hypergraph is a map of the type*

$$*: H \times H \to \mathcal{P}(H)$$

Such that, for, $a, b \in H$

$a * b =< \{a, b\} >= (a, b)$ [= the ideal generated by $\{a, b\}$].

If $a * b = I(= (a, b))$, then, for $c \in H$,

$$I * c = \bigcup_{s\in I}(s \quad * \quad c) = \sum\nolimits_{finite} r_i a + \sum\nolimits_{finite} s_i b + \sum\nolimits_{finite} t_i c = (a \quad * \quad b) \quad * \quad c =< a, b, c >$$

Where $r_i, s_i, t_i \in R$. Similarly, $c \ * \ I =< a, b, c >$. So, $I \ * \ c = c \ * \ I$.

Or, $(a \ * \ b) \ * \ c = a \ * \ (b \ * \ c)$.

If $a, b \in R$, then, $a \ * \ b$ stands for the ideal generated by $< \{a, b\} >$.

In general, we define, if $A, B \subseteq R$ then, $< A \cup B >=$ the ideal generated by $A \cup B$= the smallest ideal containing A and B and in symbol $< A \cup B >= A * B$.

For, $A, B, C \subseteq R$, it is known that, $(A \cup B) \cup C = A \cup (B \cup C) = A \cup B \cup C$ [by convention], and hence, it follows the associative property,

$A * (B * C) = (A * B) * C = A * B * C$ [by convention].

The term 'hypergraph' and 'family of sets' are used as synonyms, so that a hypergraph is a family $H = (\mathcal{E}_i | i \in M)$, in which the sets M and $\mathcal{E}_i$ may be finnite or infinite. If M is infinite then H is an infinite hypergraph.

In justification to the above definition we would like to mention in our context the ideal hypergraph $\Gamma = (H, \mathcal{E})$, with H, a ring (finite or infinite) and $\mathcal{E}$, the collection of all ideals of H.

- Definition 3.3 *A recursive path in an ideal hypergraph $\Gamma = (\mathcal{R}, \mathcal{E})$ is a path, where each hyperedge contains its preceeding hyperedge. Equivalently, each hyperedge is contained in its succeeding hyperedge.*

Thus, if $P_1 = xE_1E_2E_3 \ldots E_t y, E_i \cap E_{i+1} \neq \phi$ is a path of length t from x to y and if, $\mathcal{R}E_2 = E_1 * E_2, \mathcal{R}E_3 = E_1 * E_2 * E_3, \ldots, \mathcal{R}E_s = E_1 * E_2 * E_3 * \ldots * E_s, y \in \mathcal{R}E_s$ with $E_1 \subseteq \mathcal{R}E_2 \subseteq \mathcal{R}E_3 \subseteq \mathcal{R}E_4 \subseteq \ldots \subseteq \mathcal{R}E_s$, then, $x(\mathcal{R}E_1)(\mathcal{R}E_2) \ldots (\mathcal{R}E_s)y$ [$s \leq t$] is the recursive path corresponding to P_1.

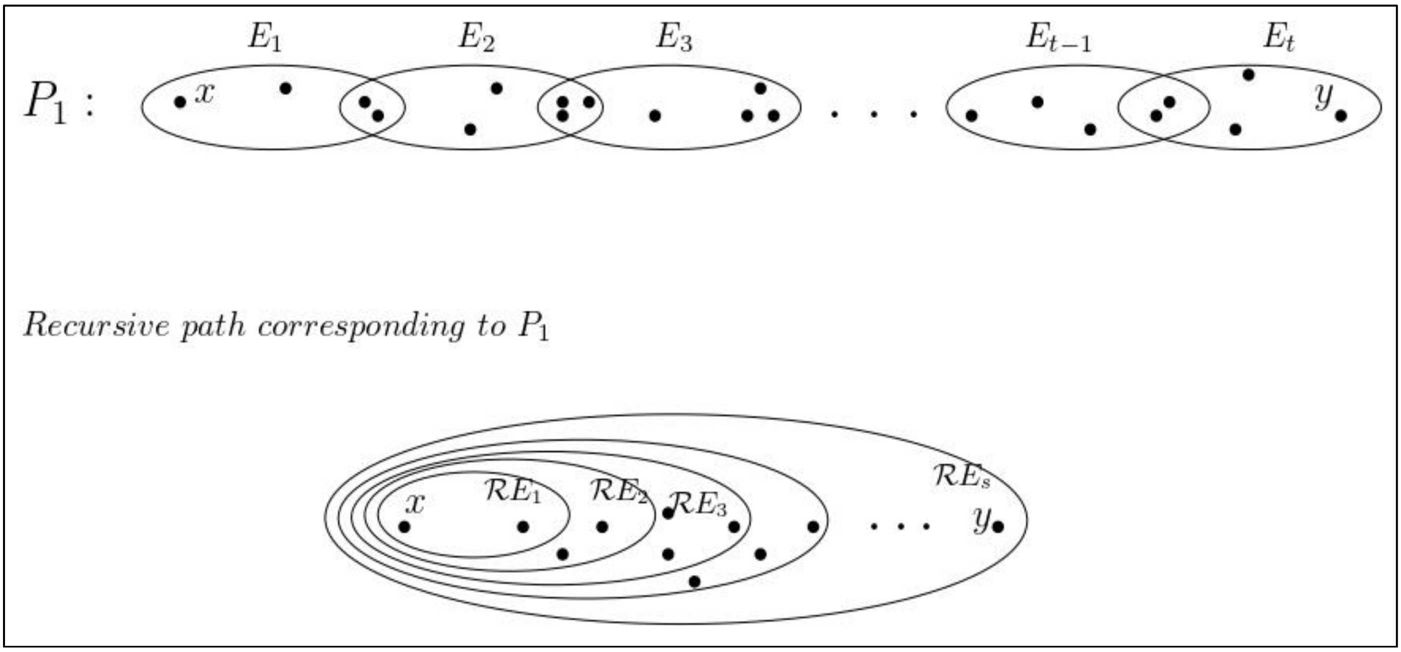

Fig 3 Recursive Path

We note that, intuitively, all such diameters of usual recursive paths are finite in nature. Now, we would like to introduce the notion of infinite recursive path. In other words assuming the existence of infinite recursive path that would be justified with the help of the following examples.

- Example 3.1 *For our purpose we will consider two rings* $Z[X_i|i = 1,2,\ldots]$ *where* $X_iX_j = X_jX_i$. *Here the chain of ideals* $< X_1 > \subsetneq < X_1, X_2 > \subseteq \ldots$ *is an ascending infinite chain and in our context this is nothing but an infinite recursive path [3].*
- Example 3.2 *Let* R *denote the collection of all finite subsets of* Z_+. *Then* $(R, \Delta, \cap)$ *is a commutative ring without identity (in fact,* R *is an ideal of the ring of sets* $P(Z_+)$*). If* $I_n = \{1,2,\ldots,n\}$, *then we get*

$$P(I_1) \subsetneq P(I_2) \subsetneq P(I_3) \subsetneq \ldots$$

For each n $I_n = \{1,2,\ldots,n\}$ and $P(I_n)$ =collection of all subsets of I_n and in the above ring each such $P(I)$ is an ideal and $P(I_{n-1}) \subsetneq P(I_n)$ $\forall$ n forms an increasing chain of ideals of R which terminates at no point [2].

- Definition 3.4 A path with initial point x of a hypergraph $\Gamma = (\mathrm{H}, \mathcal{E})$ is of vertex infinite recursive character if for any chosen recursive path $\mathrm{xK_1K_2\ldots K_ty}$, we have a path of the type $\mathrm{xK_1K_2\ldots K_tK_{t+1}z}$ whatever be t. If it is not of infinite recursive character, then it is of finite recursive character.
- Definition 3.5 A path $\mathrm{xK_1K_2\ldots K_ty}$ with initial point x of a hypergraph $\Gamma = (\mathrm{H}, \mathcal{E})$ is of vertex finite recursive character if there does not exist any $\mathrm{E_{k+1}}$ such that $\mathrm{xK_1K_2\ldots K_tK_{t+1}z}$ whatever be t.
- Definition 3.6 An ideal hypergraph $\Gamma = (\mathrm{H}, \mathcal{E})$ has vertex ideal maximal condition if for a class of ideal I, $x \in I$ contains a maximal element M with $x \in M$.

- Theorem 3.1 If an ideal hypergraph $\Gamma = (\mathrm{H}, \mathcal{E})$ is with vertex maximal condition, then it has a path of vertex finite recursive character.

Proof. If the hypregraph $\Gamma = (H, \mathcal{E})$ does not have a path of vertex finite recursive character, then we get a vertex recursive path of length d with initial vertex x as

$$xE_1 \subset E_2 \subset \ldots \subset E_d y,$$

And this gives another vertex recursive path of length $d+1, d+2, \ldots$ Thus, we get, an infinite vertex recursive path with initial vertex x, where

$$E_1 \subsetneq E_2 \subsetneq \ldots \subsetneq E_{d+1} \subsetneq \ldots$$

But as Γ is with vertex ideal maximum condition, we get a maximal vertex ideal E_t for some t and $E_i = E_t$ for all $i \geq t$, a contradiction. Thus, $\Gamma = (H, \mathcal{E})$ has a path of vertex recursive finite character.

- Theorem 3.2 If an ideal hypergraph $\Gamma = (\mathrm{H}, \mathcal{E})$ satisfies the vertex maximal condition, then each ideal is finitely generated.

Proof. Suppose an ideal I is not finitely generated. Then $a_1 \in I$ for some $a_1 \in I$ and $I \neq (a_1)$. So there exists $a_2 \in I$ with $a_1 \in (a_1) \subsetneq a_1 * a_2$ and since I in not finitely generated we have another $a_3 \in I$ such that

$$a_1 \in (a_1) \subsetneq a_1 * a_2 \subsetneq a_1 * a_2 * a_3.$$

Since I is not finitely generated, this process will continue infinitely. Thus, we get a collection $\mathcal{F} = \{(a_1), a_1 * a_2, a_1 * a_2 * a_3, \ldots\}$ each containing a_1 such that

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$$a_1 \in (a_1) \subsetneq a_1 * a_2 \subsetneq a_1 * a_2 * a_3 \subsetneq \ldots$$

Therefore, $\mathcal{F}$ is a collection of vertex ideals and it has no maximal element $M = a_1 * a_2 * \ldots * a_t$, a contradiction. Hence, I is finitely generated.

- Theorem 3.3 If an ideal hypergraph $\Gamma = (\mathrm{H}, \mathcal{E})$ has a path with vertex recursive finite character, then any ideal I is finitely generated.

Proof. Suppose an ideal I is not finitely generated. Then $a_1 \in I$ for some $a_1 \in I$ and $I \neq (a_1)$. So there exists $a_2 \in I$ with $a_1 \in (a_1) \subsetneq a_1 * a_2$ and since I in not finitely generated we have another $a_3 \in I$ such that

$$a_1 \in (a_1) \subsetneq a_1 * a_2 \subsetneq a_1 * a_2 * a_3.$$

Since I is not finitely generated, this process will continue infinitely. Thus, we get a recursive path of ideals $a_1 A_1 A_2 \ldots$, where $A_i = a_1 * a_2 \; * \ldots * a_i$ such that

$$a_1 \in (a_1) \subsetneq a_1 * a_2 \subsetneq a_1 * a_2 * a_3 \subsetneq \ldots$$

Since, $\Gamma = (H, \mathcal{E})$ has a path with vertex recursive finite character, there exist α such that $x_1 A_1 \ldots A_\alpha x_\alpha$ is a recursive path of length α and no other path exists of length greater than α. Thus, there exists $I = a_1 * a_2 * \ldots * a_\alpha$, a contradiction. Hence, I is finitely generated.

- Theorem 3.4 If any ideal I of an ideal hypergraph $\Gamma = (\mathrm{H}, \mathcal{E})$ is finitely generated, then it has a path with vertex finite recursive character.

Proof. If not, then suppose $\Gamma = (H, \mathcal{E})$ has a path with vertex infinite recursive character. So, for a vertex finite recursive path $xE_1E_2 \ldots E_k y$, there exists a vertex recursive path $xE_1E_2 \ldots E_k E_{k+1} z_k$, where $E_k \subseteq E_{k+1}$, whatever be k. Thus, we get an infinite collection $\mathcal{F} = \{E_i | E_i \subseteq E_{i+1}, \text{for each } i\}$, $x \in E_i$. Now, $E' = \cup E_i, E_i \in \mathcal{F}$ is an ideal with $x \in E'$. Thus, E' is a vertex maximal ideal with $xE_1 \ldots E' z$ with $z \in E'$, which is a vertex hyperpath with finite recursive character, a contradiction. Hence, $\Gamma = (H, \mathcal{E})$ has a path with vertex finite recursive character.

IV. CONCLUSION

The authors claim all the results presented here completely of their own together with the notions developed. There is ample scope to derive so many elegant results with far reaching affects to Hyperoperational aspects of Hypergraphic world. Moreover, there is a very interesting and serious aspects for developing some sort of decomposition of Hypergraphs into its substructures with prime characters in Algebraic sense that may be expected from what is presented in the second section with the idea of vertex Noetherian path.

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