Numerical Report on Thermal Behaviour of Magnetic Hybrid Nanofluid Over a Stretched Surface with Brownian Motion and Thermophoresis Effects

Sultana Begum[1]

[1]Associate Professor, Department of Mathematics
Govt First Grade College, Shahpur, Yadgir, Karnataka, India.

Publication Date: 2026/01/08

Abstract: Phonons are the main heat carriers in solids and Nickel based and Manganese based ferrite nano particles have very strong phonon crystal structures which makes them more effective for heat transfer applications. This current article explores the heat transfer in Nickel-zinc Ferrite ($Ni - ZnFe_2O_4$) , Manganese-zinc Ferrite ($Mn - ZnFe_2O_4$) nanoparticles with base fluid as Ethylene glycol ($C_2H_6O_2$).Mathematical model is solved by using shooting method and key factors friction at the surface and heat transfer is calculated and analysed with the help momentum, energy and concentration profiles. It is observed that for higher magnetic field strength velocity profile diminishes and friction coefficient decreases. Porosity parameter increases the temperature distribution.

Keywords: *Electromagnetohydrodynamic Forces, Incompressible Fluid Flow, Hybrid Nanofluid.*

How to Cite: Sultana Begum (2026) Numerical Report on Thermal Behaviour of Magnetic Hybrid Nanofluid Over a Stretched Surface with Brownian Motion and Thermophoresis Effects. *International Journal of Innovative Science and Research Technology*, 11(1), 110-117. https://doi.org/10.38124/ijisrt/26jan206

I. INTRODUCTION

Ferrite nanofluids are widely used in many engineering fields due to the unique thermal properties in them. Ferrite nanofluids improve thermal performance in systems where efficient heat removal is critical. The idea of improving heat transport in fluids with the help of solid particles is due to the experimentation by Choi and Eastman [1].Later many researchers initiated development correlations for thermal conductivity with respect to different physical configuration such as size of nano particle, volume fraction of nano particle, type of nano material etc. This pioneered research work created a new dimension to the convective heat transfer in fluids.(See [2], [3], [4]). Fluid flows over any moving surface have wide rnage of applications in many industries such as mechanical, aerospace, food processing, chemical engineering etc. Initially this type work was introduced by Sakiadis in his early works flow over axisymmetric geometry [5] , flow over a flat surface [6] and flow over a cylindrical vessel [7].This area turn out to be more favourable for heat transfer in nanofluids due to the fact of higher thermal conductivity in nanoparticles as compared to fluids. Abdullah et al.[8] studied about thermal characteristics of nano materials in improving exchange of heat in fluids. Acharya and Mabood [9] reported convective heat transfer in iron oxide nano martials in radiative fluid flow configuration. Afridi and Qasim[10] examined the properties of copper and alumina and also performed irreversibility analysis for the same. Later Adan Asghar et al.[11] extended this work to mixed diffusion scenario with copper alumina nanofluid. One of the important and crucial components in this nanofluid based heat exchange problems is shape and size of the nanoparticle. Ali et al.[12] conducted a finite element analysis on nanoparticle accumulation near surfaces during flows within the geometry and importance of surfactant to avoid aggregation of nano particles.Sulochana et al.[13] studied the significance of thermal conductivity models and role of distinct nanoparticle effects. Tirumala et al [14] reported importance of magnetic field in nanofluid flow for melting heat transfer applications. Ali et al. [15] discussed about nanofluid flow in porous media with two different heat source models. Some studies are reported with hybrid base fluid also.Sulochana[16] and Prasanna Kumar examined the hybrid base fluid model with water and ethylene glycol with equal ratio mixing formula. Pavithra et al [17] extended the hybrid model to trihybrid nanofluid model for enhanced thermal conductivity. Haider and Ahmed [18] investigated engine oil flow over stretching cylinder to replicate the heat transport

ISSN No:-2456-2165 https://doi.org/10.38124/ijisrt/26jan206

phenomena in automobile engineering.Prashar and Ojjela [19] reported nanofluid consisting of zinc oxide and ethylene glycol . In view of the above cited references, most of the researchers focused their studies on water-based metals or metal oxides nanofluid flow over stretching sheet, to the best of our knowledge no attempt has been made to analyse thermal properties of manganese and zinc with ethylene glycol as base fluid.

II. PROBLEM STATEMENT AND MATHEMATICAL MODEL

Let us consider incompressible boundary layer flow due to a nonlinear velocity stretching sheet. Figure-1 provides the detailed configuration and boundary conditions of the problem. Uniform transverse magnetic field $B_o(x)$ is applied to the flow region. Nickel-zinc Ferrite $(Ni - ZnFe_2O_4)$, Manganese-zinc Ferrite $(Mn - ZnFe_2O_4)$ nanoparticles with base fluid as Ethylene glycol $(C_2H_6O_2)$.Flow is assumed to be thermally equilibrium and there is no slip between the particles. The present configuration corresponds to a low Reynolds number flow; therefore, the influence of the induced magnetic field is negligible and is neglected. Momentum equation is considered with magnetic field term and porosity effect. Buingrno model is included for Brownian motion and thermophoresis effect.(See [20], [21], [22], [23])

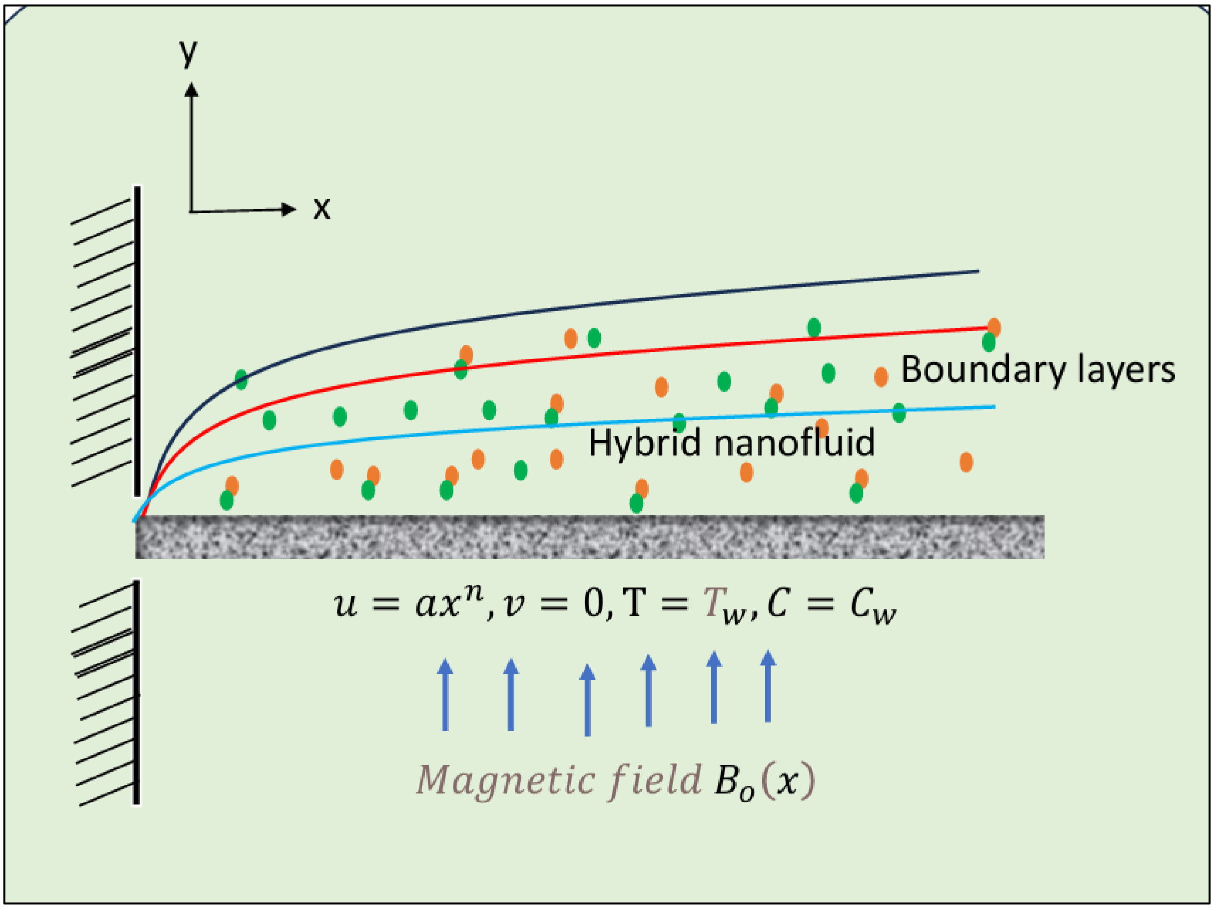

Fig 1: Model Diagram of Fluid Flow Configuration.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\rho_{hnf}\left(u\frac{\partial u}{\partial y} + v\frac{\partial u}{\partial y}\right) = \mu_{hnf}\left(\frac{\partial^2 u}{\partial y^2}\right) - \sigma_{hnf}\left(B_o{}^2(x)u\right) - \frac{\nu_{hnf}}{k}u \quad (2)$$

$$(\rho c_p)_{hnf}\left(u\frac{\partial T}{\partial y} + v\frac{\partial T}{\partial y}\right) = k_{hnf}\left(\frac{\partial^2 T}{\partial y^2}\right) + \tau\left[\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right] \quad (3)$$

$$\left(u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y}\right) = \frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial y^2}\right) + D_B\left(\frac{\partial^2 C}{\partial y^2}\right) \quad (4)$$

boundary conditions are: [24]

$$u = ax^n, v = 0, T = T_w, C = C_w \text{ at } y = 0$$
$$u \to 0, T \to T_\infty, C \to C_\infty \text{ as } y \to \infty \quad (5)$$

ISSN No:-2456-2165 https://doi.org/10.38124/ijisrt/26jan206

- *Similarity Variable and Transformed Equations:[16]*

$$u = ax^n f'(\eta), v = \sqrt{\left(\frac{a(n+1)}{2v}\right)} x^{(n-1)/2}\left(f(\eta) + \frac{n-1}{n+1}\eta f'(\eta)\right), \xi = y\sqrt{\left(\frac{a(n+1)}{2v}\right)} x^{(n-1)/2},$$

$$T = T_\infty + (T_w - T_\infty)\theta(\eta), C = C_\infty + (C_w - C_\infty)\phi(\eta) \quad (6)$$

non dimensionalised differential equations in similarity variable η are :

$$f'''(\eta) + A_1A_2\big(f(\eta)f''(\eta) - f'(\eta)f'(\eta)\big) - A_2A_3M\big(f'(\eta)\big) - \lambda f'(\eta) = 0 \quad (7)$$

$$\left(\frac{k_{hnf}}{k_f}\right)\theta''(\eta) + A_4 f(\eta)\theta'(\eta) + Nb\theta'(\eta)\phi'(\eta) + Nt\theta'(\eta)^2 = 0 \quad (8)$$

$$\phi''(\eta) + \frac{Nt}{Nb}\theta''(\eta) + Lef(\eta)\phi'(\eta) = 0 \quad (9)$$

boundary conditions in terms of similarity variable ξ are as follows:

near the surface $\eta = 0: f'(\eta) = 1, f(\eta) = 0, \theta(\eta) = 1, \phi(\eta) = 1$

at the free stream condition $\eta = \infty: f'(\eta) = 0, \theta(\eta) = 0, \phi(\eta) = 0$ (10)

where $A_1 = [(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}]$, $A_2 = \left[1 - \phi_2\left\{(1-\phi_1) + \phi_1\left(\frac{\rho_{s1}}{\rho_f}\right)\right\} + \phi_2\left(\frac{\rho_{s2}}{\rho_f}\right)\right]$

$$A_3 = \left[1 - \phi_2\left\{(1-\phi_1) + \phi_1\left(\frac{(\rho c_p)_{s1}}{(\rho c_p)_f}\right)\right\} + \phi_2\left(\frac{(\rho c_p)_{s2}}{(\rho c_p)_f}\right)\right]$$

- *Friction Coefficient and Heat Transfer Rate:*

Friction at the surface and rate heat transfer plays key Influence in boundary layer flow problems which are represented by skin friction coefficient and Nusselt number which can be written as $c_f = \frac{\tau_w}{\rho_f u_w^2}, Nu = \frac{xq_w}{k_f(T_w - T_\infty)}$ where $\tau_w = \mu_{hnf}\left(\frac{\partial u}{\partial y}\right)_{y=0}, q_w = k_{hnf}\left(\frac{\partial T}{\partial y}\right)$. After non dimensionalisation these two quantities are:

$$Re^{-1/2}Cf_x = \frac{\sqrt{\left(\frac{n+1}{2}\right)}\ f''(0)}{A_1}$$

$$Re_x^{1/2}Nu_x = -\sqrt{\left(\frac{n+1}{2}\right)}\left[\frac{k_{hnf}}{k_f}\right]\theta'(0)$$

III. RESULTS AND DISCUSSION

The mathematical model is solved by using shooting method and momentum, energy and concentration profiles plotted and analysed. In boundary layer flows, velocity, temperature, and concentration profiles are fundamental because they describe how momentum, heat, and mass are transported between a surface and the surrounding fluid. Figure 1 portrays the importance of magnetic field on velocity profile and from this figure it is clear that for higher magnetic parameter values velocity profile decreases. An externally applied transverse magnetic field generates a Lorentz force that acts opposite to the direction of fluid motion. This resistive force suppresses the fluid velocity, leading to a reduction in the velocity profile throughout the boundary layer. As the magnetic field strength (or magnetic parameter) increases, the momentum boundary layer becomes thinner, and the flow experiences enhanced damping. Physically, the magnetic field increases the effective resistance to flow, converting kinetic energy into thermal energy through Joule dissipation. Consequently, the peak velocity decreases, and the fluid approaches the free-stream velocity more slowly. This magnetic braking effect is particularly significant in electrically conducting fluids such as liquid metals, plasmas, and electrolytes. Figure 2 represents effect magnetic term on energy profile. An increase in the magnetic parameter intensifies the applied magnetic field, which strengthens the Lorentz force opposing the fluid motion. This resistance reduces the fluid velocity and weakens convective heat transport away from the surface. As a result, more heat accumulates within the boundary layer, leading to an increase in the fluid temperature and a thickening of the thermal boundary layer. Figure 3 shows the role of magnetic parameter on concentration profile and from this it is evident that concentration supresses due to Lorenz force.

The porosity term appears in the velocity profile when the flow occurs through or over a porous medium, and it represents the resistance offered by the porous matrix to fluid motion. Figure 4 indicates the effect of porosity on momentum profile. The porosity term acts as a retarding force, opposing the flow. Increasing porosity resistance leads to a reduction in fluid velocity throughout the boundary layer. The momentum boundary layer becomes thinner, and the peak velocity decreases. Figure 5 is plotted to know about the impact of temperature profile. The porosity term also influences the temperature profile in boundary layer flows over or through a porous medium, mainly through its indirect effect on fluid motion and heat transport. Lower velocity weakens convective heat transport, allowing heat to remain longer within the boundary layer. Figure 6is plotted for Nt and it shows increasing trend for thermophoresis parameter .

In boundary layer flows, the skin friction coefficient and the Nusselt number are required because they quantify the most important surface transport phenomena—momentum transfer and heat transfer between the fluid and the solid

ISSN No:-2456-2165 https://doi.org/10.38124/ijisrt/26jan206

boundary. Table-2 shows the computation of various physical quantities with respect to fluid flow parameters. It is observed that skin friction increases with magnetic and porosity parameters and it is not influenced by Brownian motion and thermophoresis parameter. Lewis number increase Sherwood number which basically tells us about mass transfer rate.

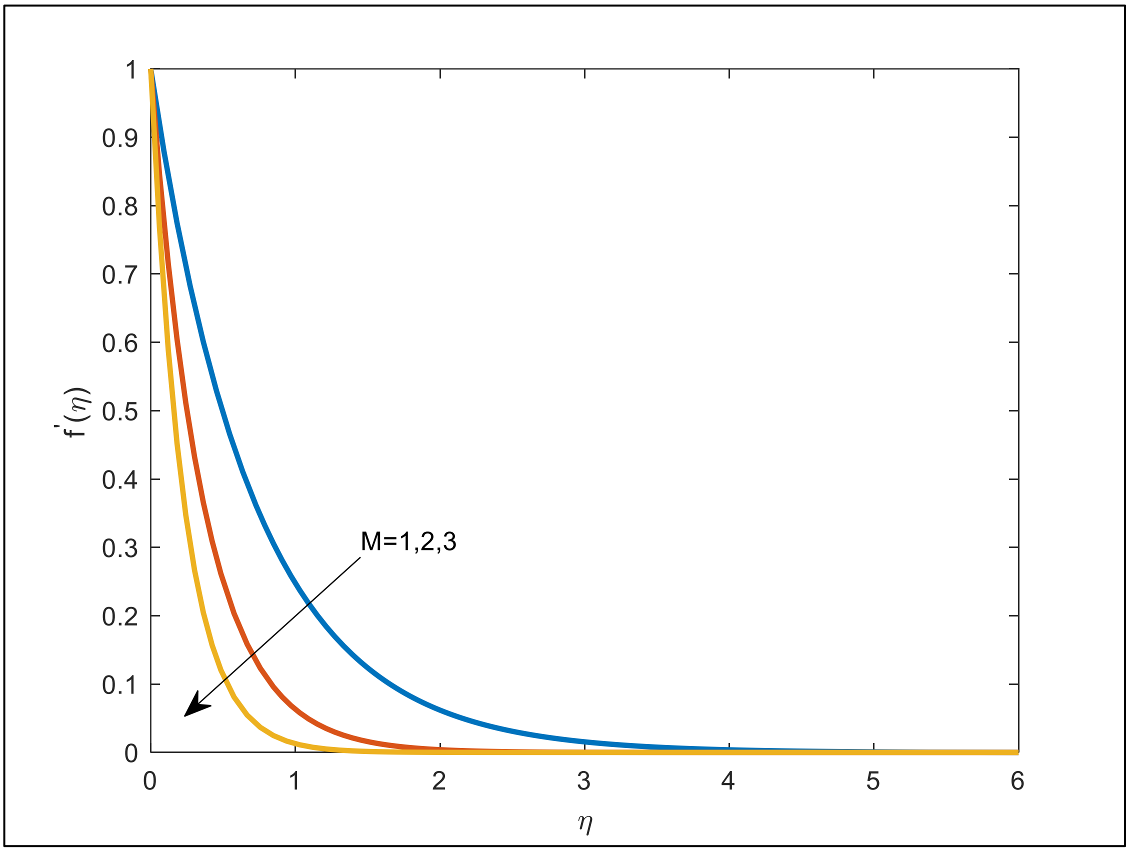

Fig 1: Effect of Magnetic Parameter on Velocity.

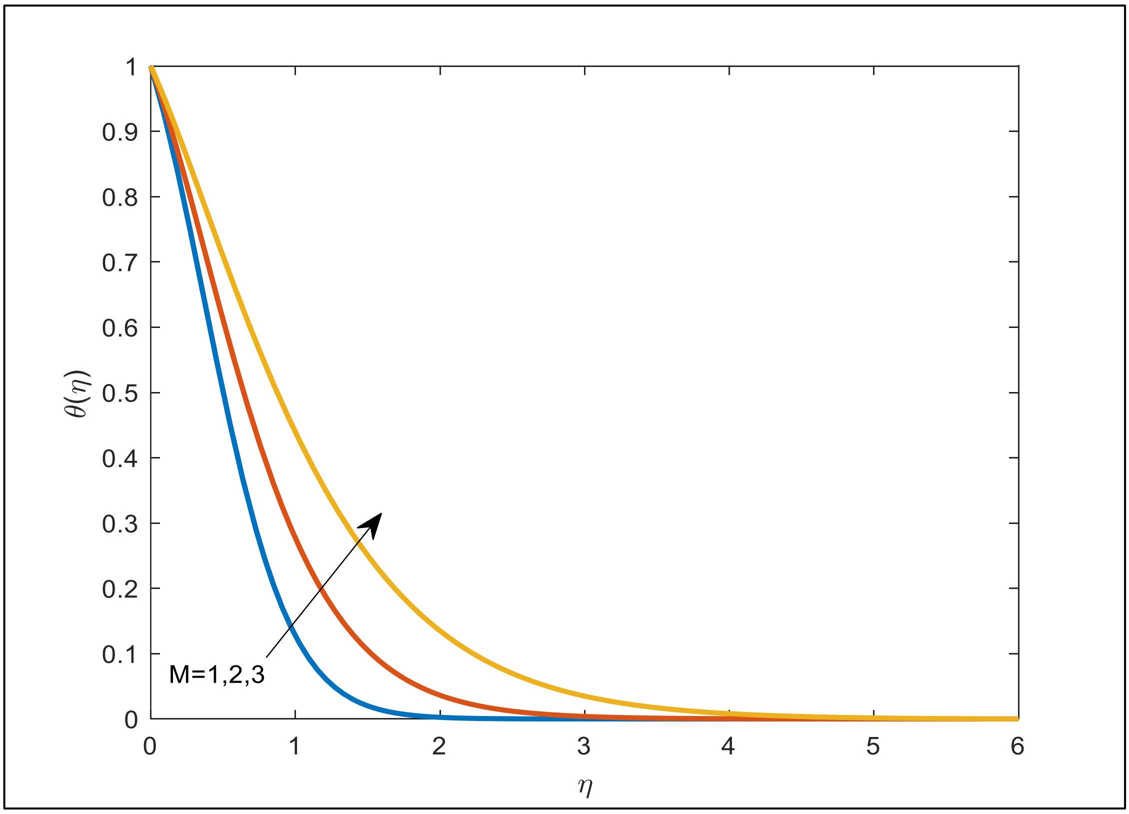

Fig 2: Effect of Magnetic Parameter on Temperature.

International Journal of Innovative Science and Research Technology
ISSN No:-2456-2165
https://doi.org/10.38124/ijisrt/26jan206

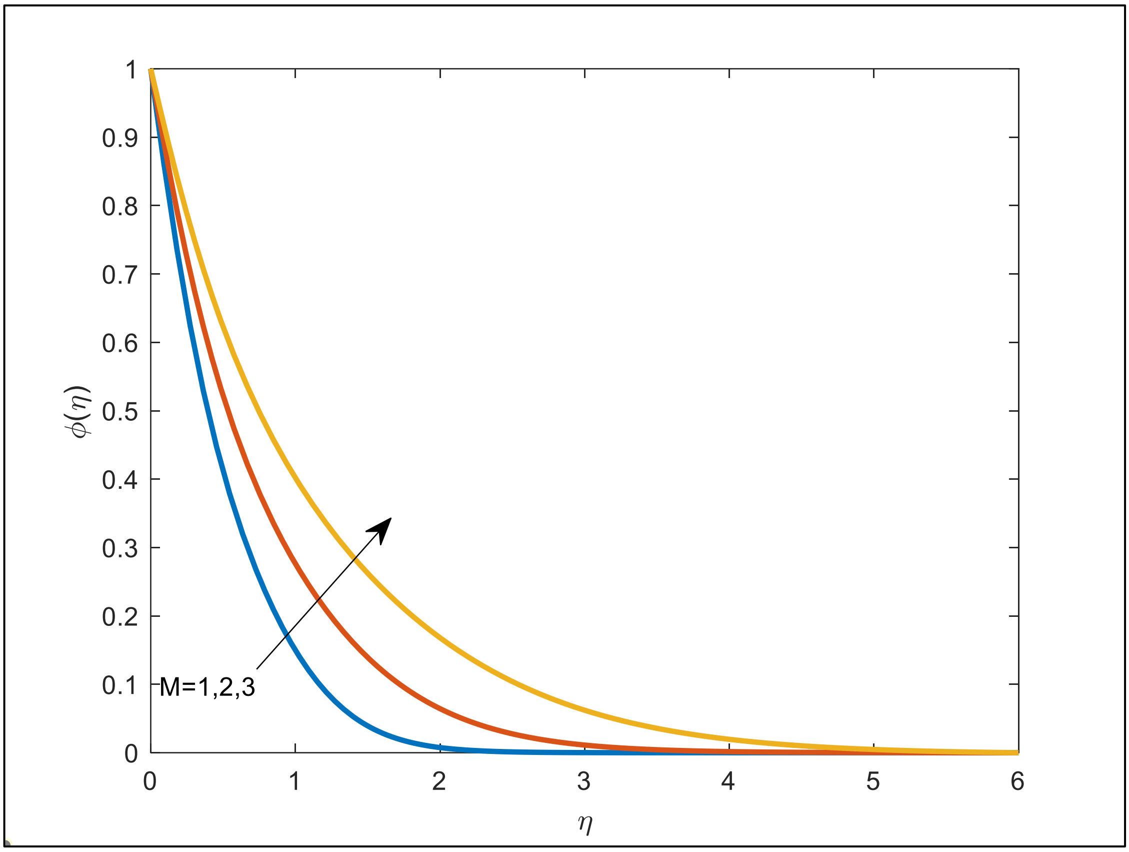

Fig 3: Effect of Magnetic Parameter on Concentration.

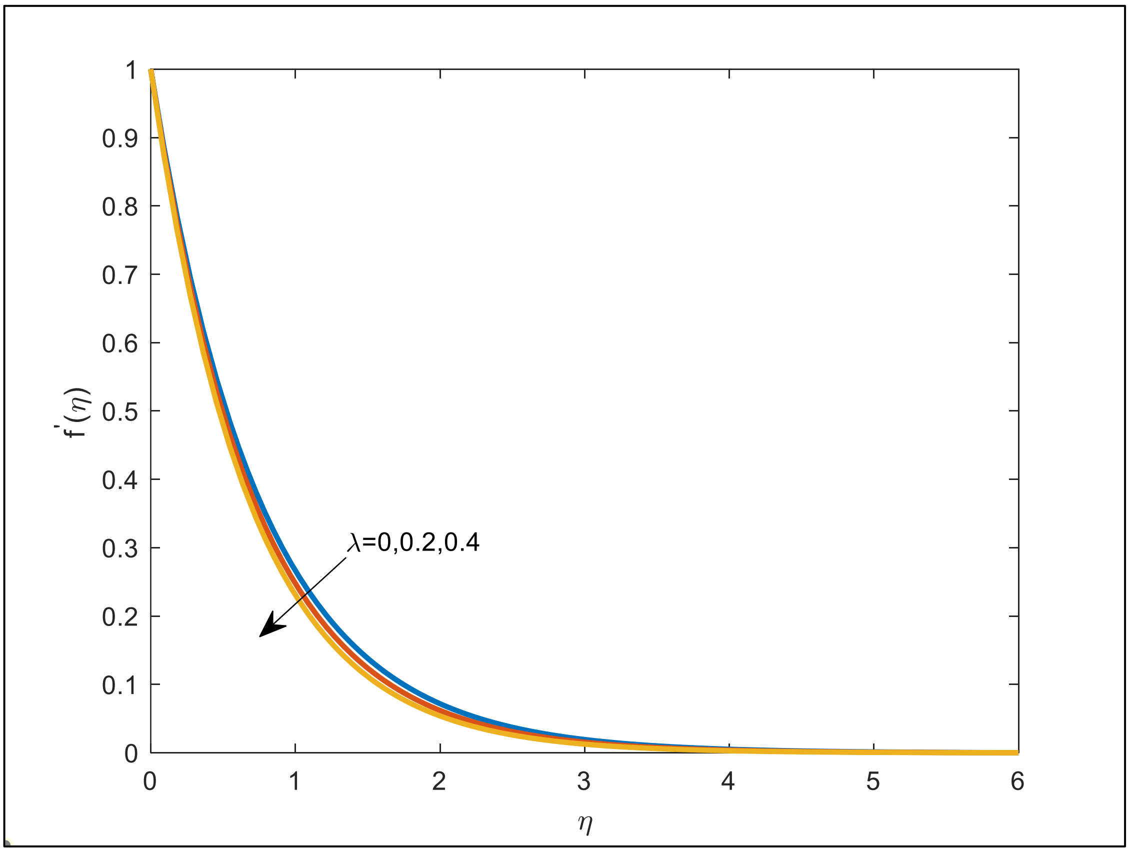

Fig 4: Effect of Porosity Parameter on Velocity.

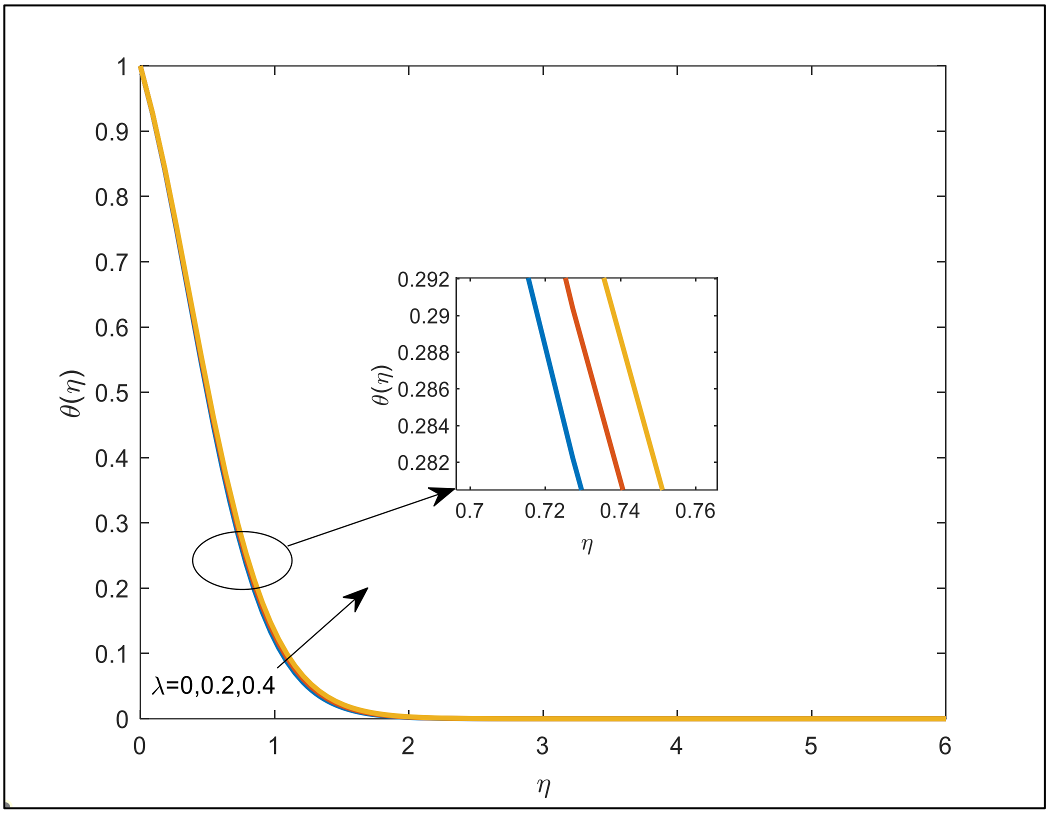

Fig 5: Effect of Porosity Parameter on Temperature.

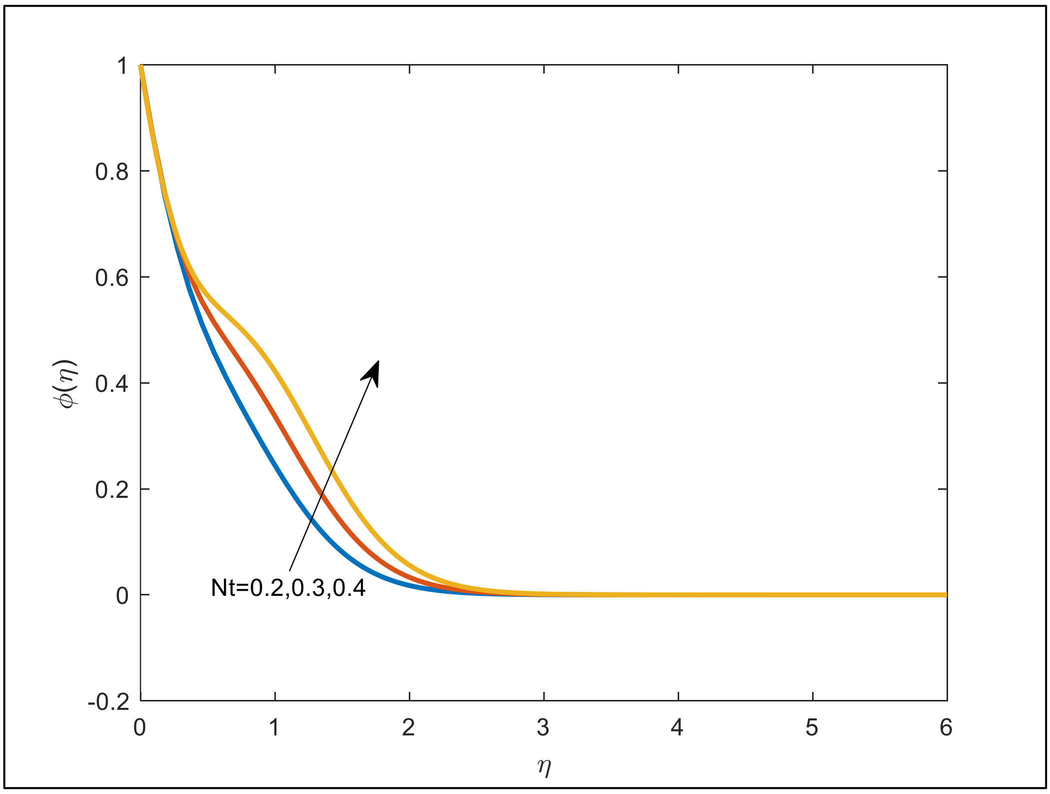

Fig 6: Effect of Thermophoresis Parameter on Concentration.

Table-1: Thermophysical Properties of Nano Particles and Base Fluid.[25]

Fluid Property	$Ni - ZnFe_2O_4$	$Mn - ZnFe_2O_4$	$C_2H_6O_2$
Density $\rho(kg/m^3)$	4800	4700	1116.6
Heat Capacity $C_p(J/kgK)$	710	1050	2382
Thermal conductivity k(W/mK)	6.3	3.9	0.249

Table-2: Computation of Skin Friction, Nusselt Number and Sherwood Numbers.

M	λ	Nb	Nt	Le	Cf_x	Nu_x	Sh_x
1					1.475907	0.758480	1.650502
3					2.906185	0.641948	1.282154
5					4.580278	0.517062	0.969278
	0.0				1.399327	0.764686	1.671763
	0.2				1.475907	0.758480	1.650502
	0.4				1.548696	0.752572	1.630358
		0.5			1.475907	0.230115	1.793808
		1.0			--------	0.022066	1.786740
		1.5			--------	0.001663	1.777075
			0.5		1.475907	0.626545	1.621162
			1.0		--------	0.523219	1.640984
			1.5		--------	0.441977	1.686443
				1	1.475907	0.758480	1.650502
				5	--------	0.530344	4.435639
				10	--------	0.485855	6.420351

IV. CONCLUDING REMARKS

This numerical study focuses on role of hybrid nanofluid on boundary layer flows embedded in porous media. Computational fluid dynamics model is solved by shooting method. Some of the important outcomes of this study are as follows.

- Momentum distribution drastically reduced for higher values of magnetic parameter.
- Skin friction coefficient is increased with high magnetic field strength.
- Brownian motion parameter depreciates heat and mass transfer rate.
- Thermophoresis parameter enhances the mass transfer rate.

REFERENCES

[1]. S. U. S. Choi and J. A. Eastman, “Enhancing thermal conductivity of fluids with nanoparticles,” Argonne National Lab. (ANL), Argonne, IL (United States), ANL/MSD/CP-84938; CONF-951135-29, Oct. 1995. Accessed: Sep. 09, 2023. [Online]. Available: https://www.osti.gov/biblio/196525
[2]. J. Buongiorno, “Convective Transport in Nanofluids,” Journal of Heat Transfer, vol. 128, no. 3, pp. 240–250, Aug. 2005, doi: 10.1115/1.2150834.
[3]. R. L. Hamilton and O. K. Crosser, “Thermal Conductivity of Heterogeneous Two-Component Systems,” Ind. Eng. Chem. Fund., vol. 1, no. 3, pp. 187–191, Aug. 1962, doi: 10.1021/i160003a005.
[4]. J. C. Maxwell, A Treatise on Electricity and Magnetism, vol. 2. in Cambridge Library Collection - Physical Sciences, vol. 2. Cambridge: Cambridge University Press, 2010. doi: 10.1017/CBO9780511709340.
[5]. B. C. Sakiadis, “Boundary-layer behavior on continuous solid surfaces: I. Boundary-layer equations for two-dimensional and axisymmetric flow,” AIChE Journal, vol. 7, no. 1, pp. 26–28, 1961, doi: 10.1002/aic.690070108.
[6]. B. C. Sakiadis, “Boundary-layer behavior on continuous solid surfaces: II. The boundary layer on a continuous flat surface,” AIChE Journal, vol. 7, no. 2, pp. 221–225, 1961, doi: 10.1002/aic.690070211.
[7]. B. C. Sakiadis, “Boundary-layer behavior on continuous solid surfaces: III. The boundary layer on a continuous cylindrical surface,” AIChE Journal, vol. 7, no. 3, pp. 467–472, 1961, doi: 10.1002/aic.690070325.
[8]. M. Z. Abdullah, K. H. Yu, H. Y. Loh, R. Kamarudin, P. Gunnasegaran, and A. Alkhwaji, “Influence of Nanoparticles on Thermophysical Properties of Hybrid Nanofluids of Different Volume Fractions,” Nanomaterials (Basel), vol. 12, no. 15, p. 2570, Jul. 2022, doi: 10.3390/nano12152570.
[9]. N. Acharya and F. Mabood, “On the hydrothermal features of radiative Fe3O4–graphene hybrid nanofluid flow over a slippery bended surface with heat source/sink,” J Therm Anal Calorim, vol. 143, no. 2, pp. 1273–1289, Jan. 2021, doi: 10.1007/s10973-020-09850-1.

[10]. M. I. Afridi and M. Qasim, “Comparative Study and Entropy Generation Analysis of Cu–H2O and Ag–H2O Nanofluids Flow Over a Slendering Stretching Surface,” Journal of Nanofluids, vol. 7, no. 4, pp. 783–790, Aug. 2018, doi: 10.1166/jon.2018.1488.

[11]. Adnan Asghar, Teh Yuan Ying, and Khairy Zaimi, “Two-Dimensional Mixed Convection and Radiative Al2O3-Cu/H2O Hybrid Nanofluid Flow over a Vertical Exponentially Shrinking Sheet with Partial Slip Conditions,” CFDL, vol. 14, no. 3, pp. 22–38, Apr. 2022, doi: 10.37934/cfdl.14.3.2238.

[12]. L. Ali, B. Ali, and T. Iqbal, “Finite element analysis of the impact of particles aggregation on the thermal conductivity of nanofluid under chemical reaction,” Waves in Random and Complex Media, vol. 0, no. 0, pp. 1–21, Feb. 2023, doi: 10.1080/17455030.2023.2172962.

[13]. S. C and T. P. Kumar, “Heat Transfer of SWCNT-MWCNT Based Hybrid Nanofluid Boundary Layer Flow with Modified Thermal Conductivity Model,” Journal of Advanced Research in Fluid Mechanics and Thermal Sciences, vol. 92, no. 2, Art. no. 2, Feb. 2022, doi: 10.37934/arfmts.92.2.1324.

[14]. T. P. Kumar and M. S. Uma, “MHD Casson nanofluid flow over a stretching surface with melting heat transfer condition,” Heat Trans, vol. 51, no. 8, pp. 7328–7347, Dec. 2022, doi: 10.1002/htj.22646.

[15]. R. Ali, M. R. Khan, A. Abidi, S. Rasheed, and A. M. Galal, “Application of PEST and PEHF in magneto-Williamson nanofluid depending on the suction/injection,” Case Studies in Thermal Engineering, vol. 27, p. 101329, Oct. 2021, doi: 10.1016/j.csite.2021.101329.

[16]. C. Sulochana and T. P. Kumar, “Enhancing heat transfer with 50%–50% water-ethylene glycol hybrid nanofluid flow over a nonlinear stretching sheet,” ZAMM - Journal of Applied Mathematics and Mechanics / Zeitschrift für Angewandte Mathematik und Mechanik, vol. 103, no. 12, p. e202200225, 2023, doi: 10.1002/zamm.202200225.

[17]. J. Pavithra, N. V. Raju, S. N. Sridhara, and T. Prasanna Kumar, “Insights of thermal characteristics with tri-hybrid nanofluid boundary layer flow past a thin needle,” Numerical Heat Transfer, Part A: Applications, vol. 0, no. 0, pp. 1–23, doi: 10.1080/10407782.2024.2364858.

[18]. G. Haider and N. Ahmed, “Unsteady Ohmic dissipative flow of ZnO-SAE50 nanofluid past a permeable shrinking cylinder,” Nanotechnology, vol. 34, no. 45, p. 455401, Aug. 2023, doi: 10.1088/1361-6528/aced57.

[19]. P. Prashar and O. Ojjela, “Numerical investigation of ZnO–MWCNTs/ethylene glycol hybrid nanofluid flow with activation energy,” Indian J Phys, vol. 96, no. 7, pp. 2079–2092, Jun. 2022, doi: 10.1007/s12648-021-02132-y.

[20]. C. Sulochana, S. Begum, and T. P. Kumar, “MHD mixed convective Non-Newtonian stagnation point flow over an inclined stretching sheet: numerical simulation,” Journal of Advanced Research in Fluid Mechanics and Thermal Sciences, vol. 102, no. 1, pp. 73–84, 2023.

[21]. N. Elboughdiri, K. Javid, P. Lakshminarayana, A. Abbasi, and Y. Benguerba, “Effects of Joule heating and viscous dissipation on EMHD boundary layer rheology of viscoelastic fluid over an inclined plate,” Case Studies in Thermal Engineering, vol. 60, p. 104602, Aug. 2024, doi: 10.1016/j.csite.2024.104602.

[22]. K. Gangadhar, E. Mary Victoria, and A. Wakif, “Irreversibility analysis for the EMHD flow of silver and magnesium oxide hybrid nanofluid due to nonlinear thermal radiation,” Mod. Phys. Lett. B, p. 2450337, Mar. 2024, doi: 10.1142/S0217984924503378.

[23]. K. Guedri et al., “Thermally Dissipative Flow and Entropy Analysis for Electromagnetic Trihybrid Nanofluid Flow Past a Stretching Surface,” ACS Omega, vol. 7, no. 37, pp. 33432–33442, Sep. 2022, doi: 10.1021/acsomega.2c04047.

[24]. F. Mabood, W. A. Khan, and A. I. M. Ismail, “MHD boundary layer flow and heat transfer of nanofluids over a nonlinear stretching sheet: A numerical study,” Journal of Magnetism and Magnetic Materials, vol. 374, pp. 569–576, Jan. 2015, doi: 10.1016/j.jmmm.2014.09.013.

[25]. N. Muhammad and S. Nadeem, “Ferrite nanoparticles Ni- ZnFe2O4 , Mn- ZnFe2O4 and Fe2O4 in the flow of ferromagnetic nanofluid,” Eur. Phys. J. Plus, vol. 132, no. 9, p. 377, Sep. 2017, doi: 10.1140/epjp/i2017-11650-2.