Interaction Between a Non Linear Stokes Wave and Water Particles Motion at the Free Surface and the Bottom of a Wave Channel

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Abstract: The movement of currents and water particles in the oceans, both at the surface and at depth, plays an important role in ocean circulation and the description of surface waves. The movement of water particles is generally influenced by the passage of surface waves. This movement provides a better understanding of the influence of waves on the movements of water particles at the surface and at the bottom. In this study, the evolution of wave and particles movement can be determined by considering a channel with linearly varying bottom. The nonlinear Stokes theory equations to be solved in this case will allow us to determine the solutions for the hydrodynamic wave parameters. Because of the non-linearity of the equations, finite difference method and iterative method of Gauss-Siedel by using the Successive over relaxation (S.O.R.) are used to resolve numerically the nonlinear equations. Our results are obtained using the FORTRAN and MATLAB software to visualize the temporal propagation of the wave in the channel and its influence on the water motion at the free surface and at the bottom. Finally, we will also look at the influence of the linear bottom on the evolution of the wave and the movement of water particles.

Keywords: *Surface Waves, Channel, Movement of Water Particles, Nonlinear Equations, Finite Difference Method.*

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I. INTRODUCTION

The interaction between bottom swell and free surface swell is a natural physical phenomenon that is very important in coastal dynamics. This interaction causes many natural consequences such as coastal erosion, wave energy dissipation, wave refraction and even energy production. To understand this interaction phenomenon, it is necessary to understand the hydrodynamic aspect of waves. The movement of fluid particles is caused by the spread of the swell from the surface to the bottom [1]. Several natural factors can influence wave propagation and thus alter particle movement (bottom, barriers, etc.). To simulate wave propagation in the first instance, we use the Stokes wave theory model in a numerical wave channel. [2]

Wave's propagation and its hydrodynamic comportment have been experimenting by researchers with swell channels to reproduce the real phenomenon .for this mathematical and numerical theories have been developed to give approximate solutions. Molin et al. [3] and Rahman et al. [4] focused on the study of second-order wave interaction with a vertical square cylinder and a circular cylinder, respectively. Similarly, a time-domain method is used to analyze wave interactions with a group or an array of cylinders. The non-linear free surface boundary conditions are satisfied based on the perturbation method up to second order. The first- and second-order velocity potential problems at each time step are solved using a finite element method (FEM). Belibassakis, K.A.and Athanassoulis, G.A [5] present second-order Stokes theory has been extended to the case of a generally shaped bottom profile connecting two half-strips of constant (but possibly different) depths, initiating a method for generalizing the Stokes hierarchy of second- and higher-order wave theory, without the assumption of spatial periodicity. In modelling the wave–bottom interaction three partial problems arise: the first order, the unsteady second order and the steady second order. Ge Wei a, James T. Kirby b, Amar Sinha a [6] present a method for generating waves in Boussinesq-type wave models is described. The method employs a source term added to the governing equations, either in the form of a mass source in the continuity equation or an applied pressure forcing in the momentum equations. Assuming linearity, they derive a transfer function which relates source amplitude to surface wave characteristics. They then test the model for generation of desired incident waves, including regular and random waves, for both one and two dimensions.

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Our goal is to study the interaction between the propagation of the swell and the movement of particles at the free surface and at the bottom. By considering the swell as a Stokes wave, we will determine this interaction through the evolution of the potential field as a function of that of the wave. We use in our model a numerical wave channel with a fixed linear bottom whose entry is determined by a linear stokes wave. We determine at first a study of the hydrodynamic aspect buy using the equations governing the propagation of stokes waves to finally get the evolution of the free surface elevation and the distribution of the velocity potential in the wave channel.

We will observe the effect of the outlet condition of the channel, considered to be a permeable wall, and the slope on the interaction between the wave and the free surface and, finally, between the wave and the bottom

To do this we will first describe the physical model to be studied and describe the calculation through the mathematical formulation. The equations obtained being nonlinear, we will finally approach the numerical formulation to give the results.

II. FORMULATION OF THE PHYSICAL PROBLEM

We will expose our problem by proposing a numerical wave channel to simulate our physical model. We consider in our case and irrotational, a non-viscous and incompressible fluid that is on movement under the effect of a Stokes wave in a wave channel of length L, wavelength λ and amplitude A see Figure 2. The bottom condition is considered as a linear form (equation 1).the effect of the waves on particles motions from the free surface to the bottom are given in this study by the evolution of the potentiel field.

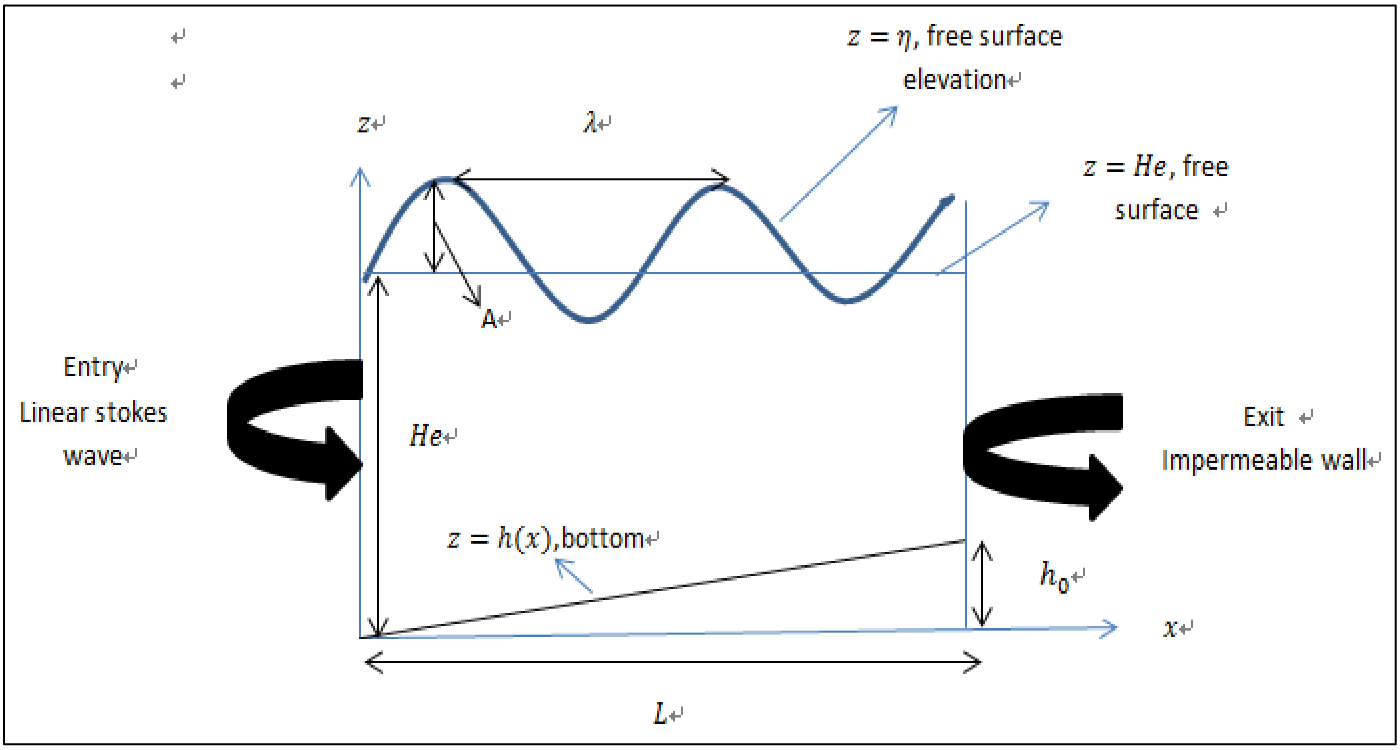

Fig 1 Physical Description of the Problem

$$h(x) = \frac{h_0}{L}x \qquad (1)$$

➢ *The Nonlinear System of Equations Representing Stokes Wave Theory are*

- *Laplace Equation:* $h(x) < z < \eta(x;t); 0 \leq x \leq L$

$$\Delta\phi = 0 \qquad (2)$$

- *Kinematic Free Surface Condition:* $z = \eta(x;t)\,; 0 \leq x \leq L$

$$\frac{\partial\eta}{\partial t} = -\frac{\partial\phi}{\partial x}\frac{\partial\eta}{\partial x} + \frac{\partial\phi}{\partial z} \qquad (3)$$

- *Dynamic Free Surface Condition:* $z = \eta(x;t); 0 \leq x \leq L$

$$\frac{\partial\phi}{\partial t} + \frac{1}{2}\left[\left(\left(\frac{\partial\phi}{\partial x}\right)^2 + \left(\frac{\partial\phi}{\partial z}\right)^2\right)\right] + g\eta = 0 \qquad (4)$$

- *Bottom Condition:* $z = h(x); 0 \leq x \leq L$

$$\frac{\partial\phi}{\partial z} - \frac{\partial\phi}{\partial x}\frac{\partial h}{\partial x} = 0 \qquad (5)$$

Considering a linear wave downstream the channel, the hydrodynamic conditions at the inlet will be determined by

$$\eta(x;t) = A\cos(kx-\omega t) \qquad (6)$$

$$\phi(x;z;t) = \frac{Ag}{\omega}\frac{\cosh[k(H_e+z)]}{\sinh(kH_e)}\sin(kx-\omega t) \qquad (7)$$

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Finally we can give the initial conditions and the edge conditions for the free surface elevation η and the velocity potential ϕ to complete the system of equations to solve.

- *Initial Condition At* $t = 0$

$$\eta(x; t = 0) = H_e + Acos(kx) \quad (8)$$

$$\phi(x; z; t = 0) = \frac{Ag}{\omega} \frac{\cosh[k(z + H_e)]}{\sinh(kH_e)} sin(kx) \quad (9)$$

- *Entry Condition:* $x = 0;\ h(0) < z < \eta(0; t)$

At $t > 0$

$$\eta(0; t) = Acos(\omega t) + H_e \quad (10)$$

$$\phi(0; z; t) = \frac{Ag}{\omega} \frac{\cosh[k(z + H_e)]}{\sinh(kH_e)} sin(-\omega t) \quad (11)$$

- *Exit Condition:* $x = L;\ h(L) < z < \eta(L; t)$

For the free surface elevation, we evaluate equation (6) for X=L and for the velocity potential we consider an impermeable wall

$$\eta(x = L; t) = Acos(kL - \omega t) + H_e \quad (12)$$

$$\frac{\partial \phi}{\partial x} = 0 \quad (13)$$

III. NON DIMENSION FORMULATION

It would be helpful to develop a method to reduce the number of parameters that interfere with the entire system of equations in our mathematical model. We can accomplish this by combining them into dimensionless groupings that have physical meaning and that enable us to learn about the answer before the problem is solved. This method is used to get dimensionless number to simulate nonlinear problem. For this we write the physical parameters as product of the non-dimension parameter and reference parameter.

$t = t^* \cdot t_0\ ;\ x = x^* \cdot L\ ;\ z = z^* \cdot h_0;\ \ \eta = \eta^* \cdot h_0;\ \ \phi = \phi^* \cdot \frac{L^2}{t_0};\ h^*(x^*) = x^*;\ A = A^*.h_0\ ;\ H_e = H_e{}^*.h_0\ ;\ k = k^* / L$

With t_0 , L and h_0 are respectively parameters of the time, the length along the x axis and the length along the z axis.

Two dimensionless numbers are used to simulate our model.

- *Froude Number*

$$1/_{Fr} = \frac{gh_0 t_0^2}{L^2} = \frac{gh_0}{L^2\omega^2}$$

- *Parameter of the Geometry of the Problem*

$$\beta = \frac{L}{h_0}$$

IV. TRANSFORMATION

To make the control of the boundaries easier, we use a curvilinear coordinate system that allows us to account for the irregularity of the fond and to always adopt the form of the free surface. This change makes it easier to write conditions to limits on irregular borders and in evolving domains. This transformation transforms the domain of our model into a mathematical domain (as shown in Figure 2) at each dimensionless instant t^* is defined by.

$$\begin{cases} T: D \to Dt \\ (x^*, z^*) \mapsto (\chi^*, \xi^*) \end{cases}$$

With

$$\chi^* = x^* \qquad \zeta^* = \frac{z^* - h^*(x^*)}{\eta^*(x^*; t^*) - h^*(x^*)} \qquad t^* = t^*$$

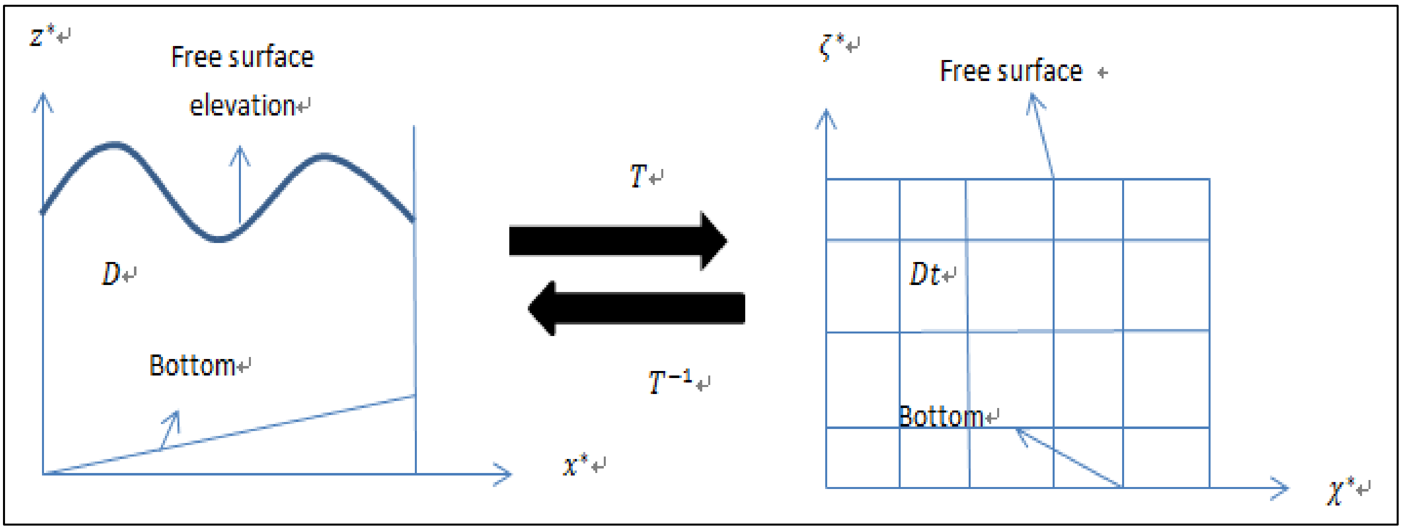

Fig 2 Physical Domain Transformation

The dimensionless equations in rectangular domain will be solved. The hydrodynamic parameters are obtained, respectively, by solving the coupled kinematic and Laplace equations in mathematical domain.

V. NUMERICAL METHOD

The algebraic equations obtained from our study are coupled and nonlinear. We therefore need to use a numerical method in order to be able to solve them. In our case, we will use the finite difference method since the equations are essentially composed of partial derivatives. The finite difference method allows us to provide an approximation of the algebraic solutions with discrete solutions. As illustrated in figure 4, nodes selected from grid that partitions the domain are used to approximate discrete places in space. At each grid node, the discretization process approximates the spatial derivatives of the flow variables included in the differential equation.

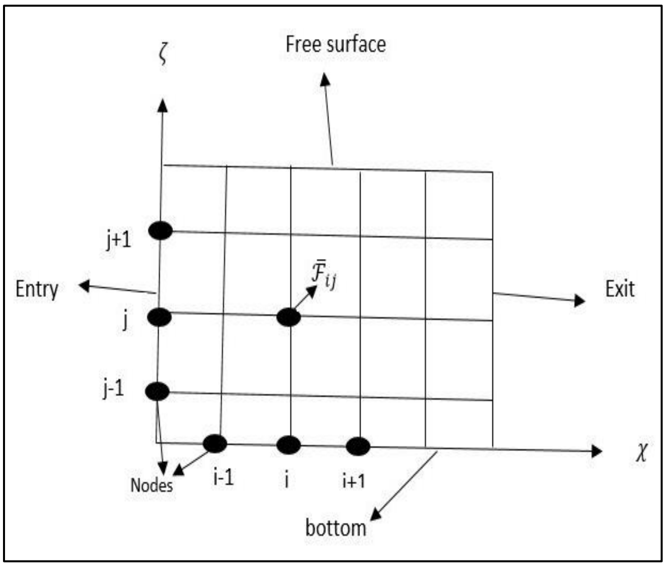

Fig 3 Meshing of the Rectangular Domain

Classical finite difference schemes [7] of order 2 will be used to approximate the spatial partial derivatives derived from the system's equations and an implicit method with to approximate time derivative.The discretization computation will clarify the decision between the upstream and downstream off-center diagrams and provide justification based on the geometry of our situation. This decision was made in order to guarantee that the fluid's movement stays inside the computation domain.

VI. NUMERICAL TECHNIQUE

After discretizing the equations, we obtained a system of linear algebraic equations.To solve this system, several numerical solution techniques exist in the literature.

We will use the iterative method of relaxation line by line of Gauss-Siedel [8] by using the Successive Over relaxation (S.O.R.) [9]. The principle of iterative methods consists in seeking the solution of the system using a series of successive approximations. From an arbitrary vector of components $(f_i)^k$, we can find $(f_i)^{k+1}$ at the next iteration. The process is stopped when the following convergence criterion is met by considering equation (14)

$$\frac{\sum_{i=1}^{i=i_m}|(f_i)^k - (f_i)^{k+1}|}{\sum_{i=1}^{i=i_m}|(f_i)^{k+1}|} \leq \varepsilon_f \quad (14)$$

With ε_f a fixed error criterion for iterative calculation.

Since the problem is unsteady, it is also necessary to stop the time iterative process when the difference between the values at two successive times begins to become constant. To do this, we consider equation (15), which stops the iterative process.

$$max\{e_\eta ; e_\phi\} \leq e_t \quad (15)$$

With e_t is the fixed error criterion for the iterative calculation such that

$$e_f = \frac{\sum_{i=1}^{i=i_m}|(f_i)^{n+1} - (f_i)^n|}{\sum_{i=1}^{i=i_m}|(f_i)^{n+1}|} \quad (16)$$

VII. RESULTS

The simulations will be carried out using FORTRAN and MATLAB software. The results obtained from these simulations show us the evolution of the free surface at early and later times for different slope values. We will then present the influence of wave propagation on water motion inside the channel through the evolution of the potential field for different depths. For that we study a numerical waves channel with an amplitude $A = 0.5$ a wavelength $\lambda = 0.25$ and an average height He=10 as parameters for an linear incident wave. The first results presented for a fixed slope $\beta = 100$, the error criteria for the convergence of the iterative calculations for the free surface elevation and the velocity potential is set to 1.10^3.

VIII. WAVE PROFILE FOR DIFFERENT VALUES OF βAT THE FIRST AND LONGER TIMES

A linear evolution of the wave at the initial time for t=0.0025 is observed for each value of β in the channel. This evolution is characterised by a peak height equal to the trough depth (Fig. 4 left). At long times, the wave is non-linear (Fig. 4 right). There is a decrease in peak height relative to an increase in trough depth when beta increase

An increase in the value of β accounts for the decrease in slope, resulting in an increase in depth. In fact, there is a slight decrease in crest height as depth increases.

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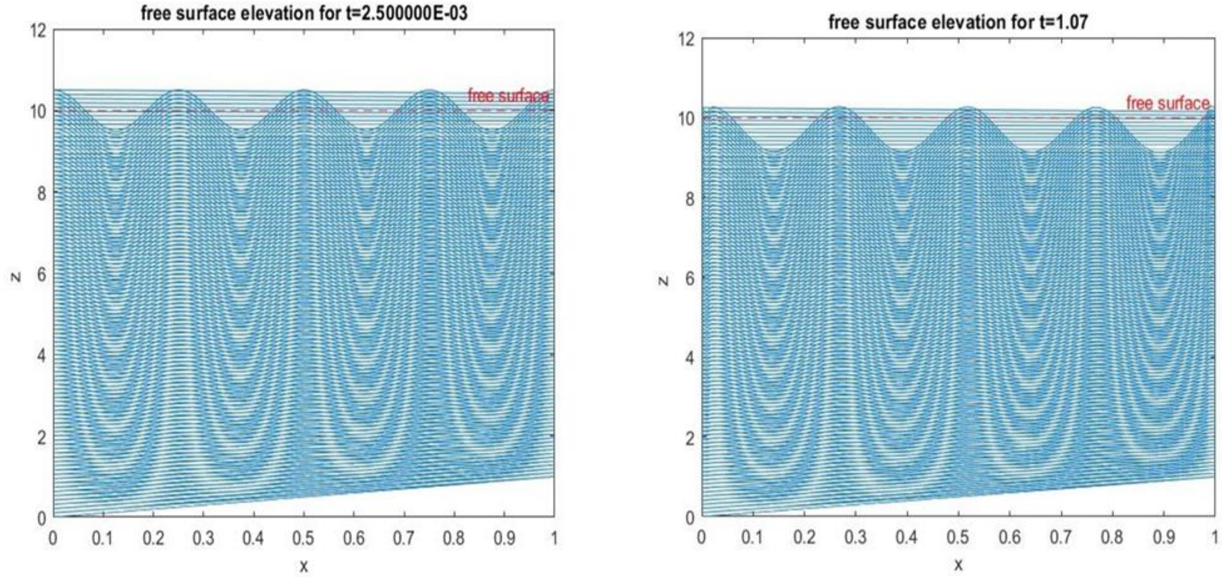
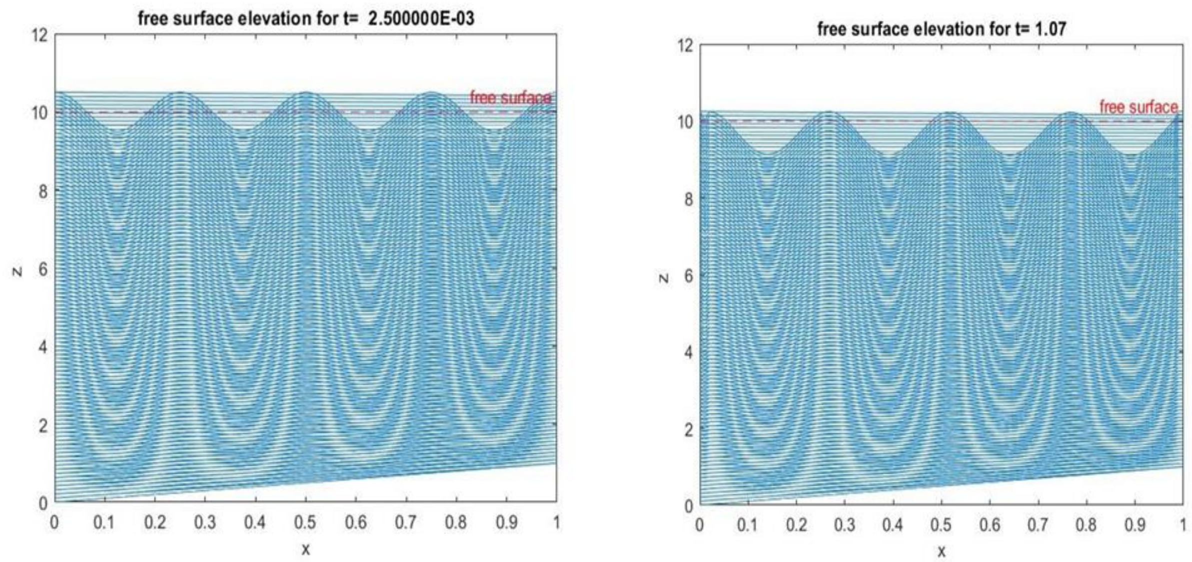
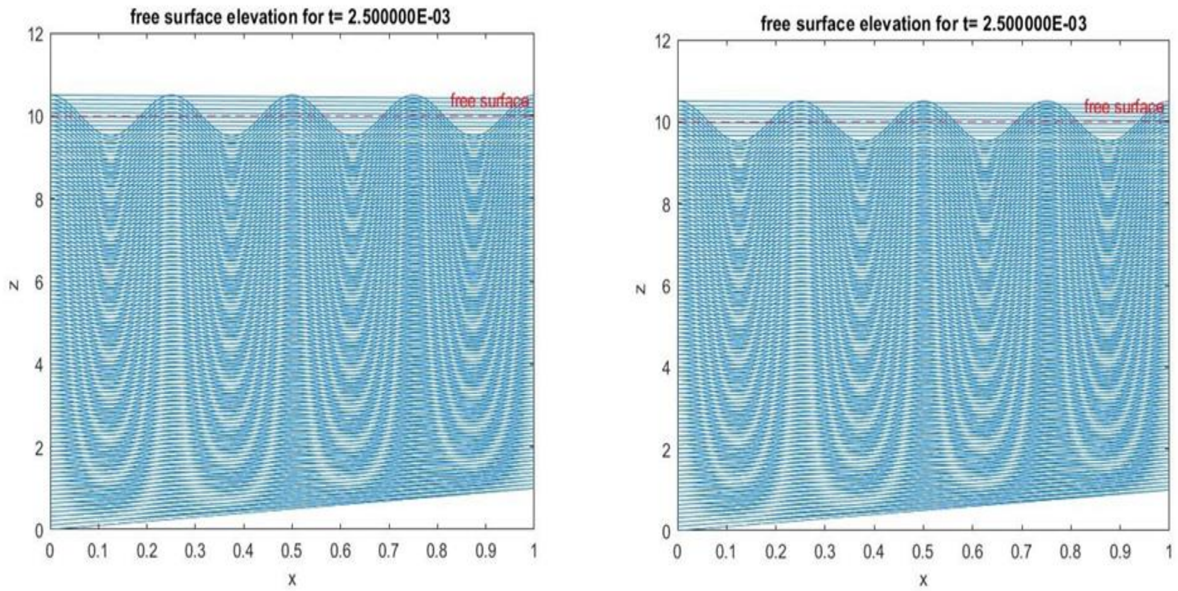


a) $\beta = 100$



b) $\beta = 105$



c) $\beta = 110$

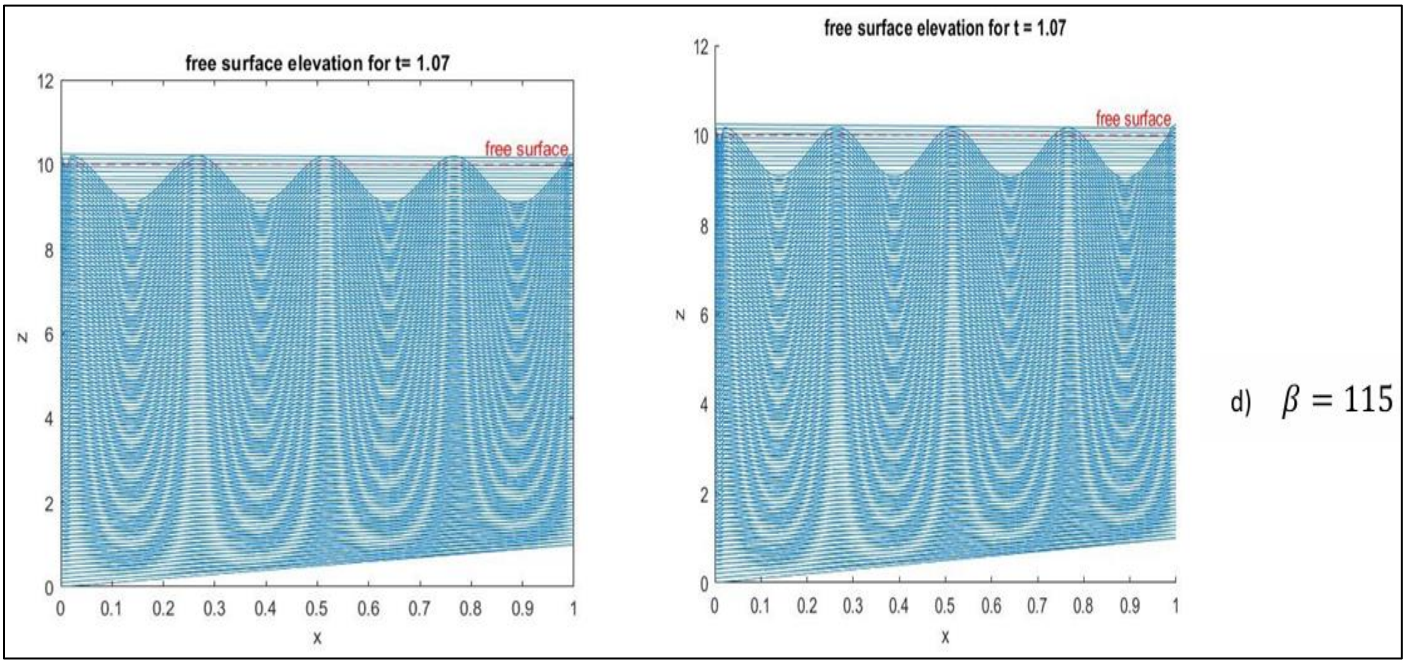
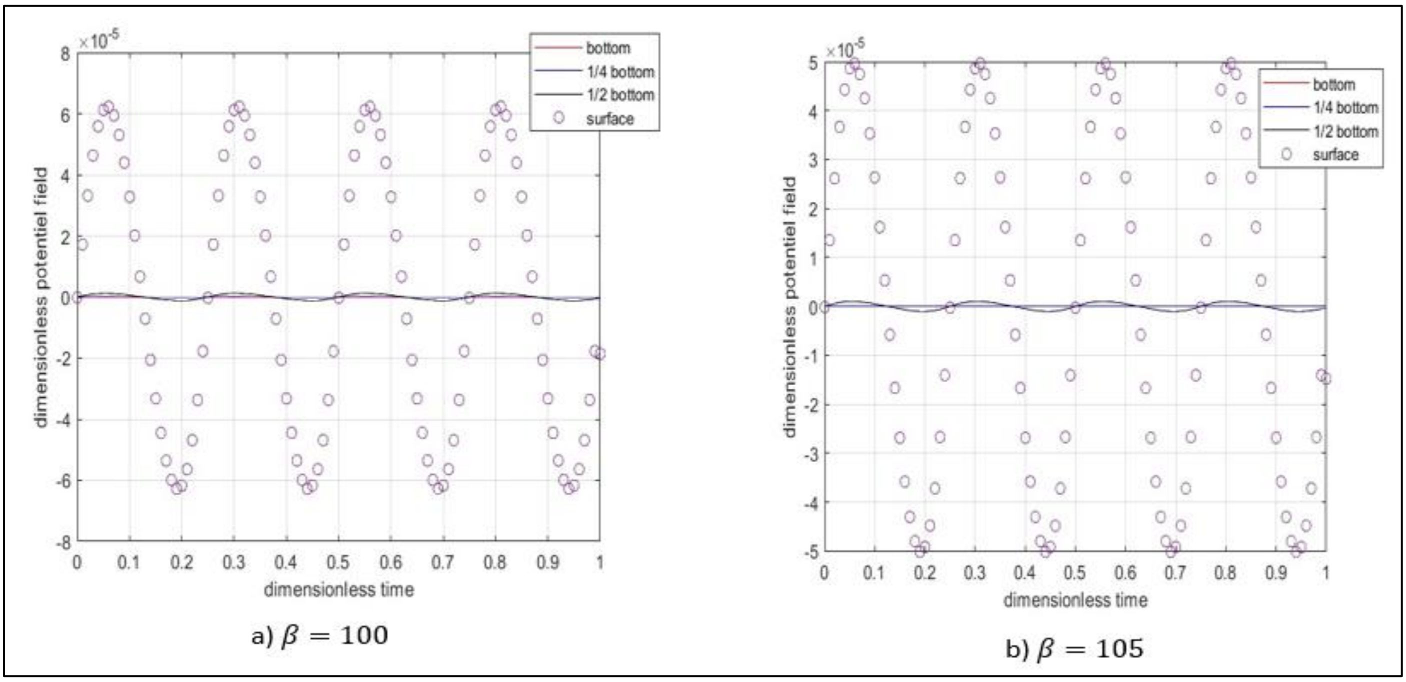


Fig 4 Longitudinal Wave Profile for t=0.025 (Left) and t=1.07 (Right) for Different Values of β

IX. VARIATION IN THE POTENTIAL FIELD FOR DIFFERENT VALUES OF β AT T=0.0025

This evolution is accompanied by movement of the fluid particles in the water.in the early phase (t=0.0025) and for each value of β a particle movement is observed with maximum intensity at the surface of the channel below the crests and troughs. This intensity decreases until it is cancelled out as we get closer to the bottom. The positive intensity is relative to movement in the direction of the wave and the negative intensity in the opposite direction. As the wave increases, the slope decreases, i.e. as the depth increases, the intensity of movement in the proximity of the surface decrease slightly

Initially, there is no interaction between the wave and the bottom, since there is no movement near the bottom for all values of β (Fig 5).



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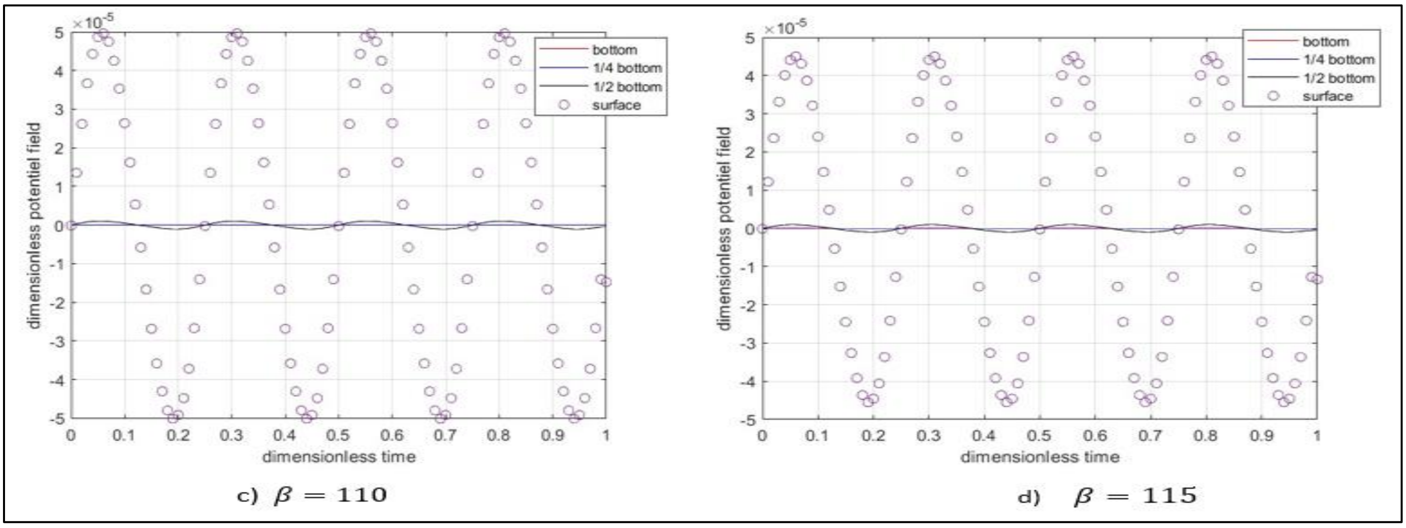


Fig 5 Particle Motion for Different Depths and Different Values of β for t=0.025

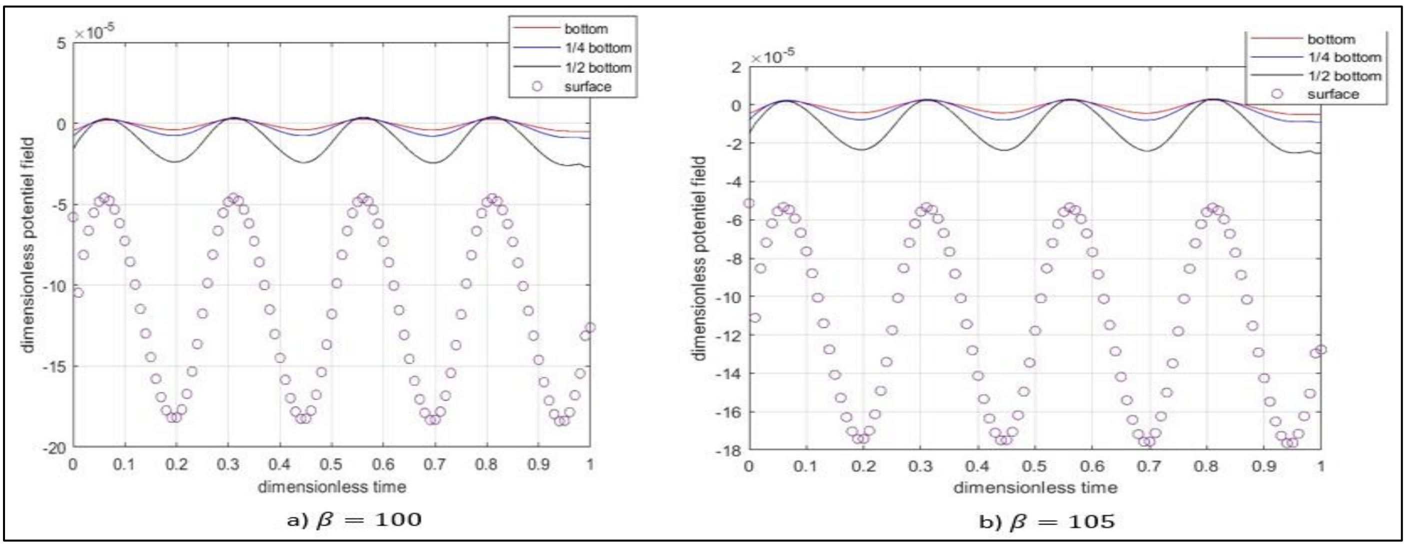
X. VARIATION IN THE POTENTIAL FIELD FOR DIFFERENT VALUES OF β AT T=1.07

At long times, the intensity of the movement is always highest near the free surface. However, the movement only occurs under the troughs (in the opposite direction to the wave). This is due to the nature of the non-linear wave at long times, where we have low crest heights compared to large trough depths (Fig. 4 left).When the slope decreases (increase in depth), we notice that the intensity of the movement decrease near the surface of the channel, as in the early stages.We also notice an interaction at long times between the non-linear wave and the bottom, since small particle movements are felt near the bottom (Fig. 5). These movements are induced by the increase in trough depth (Fig. 4 left). These movements therefore occur in the opposite direction to the wave, as shown in Fig. 5.When the depth increases, there is virtually no variation in the intensity of the movement near the bottom.

XI. CONCLUSION

A numerical wave channel was used in our simulation to observe the effect of the stokes wave on particules movement from free surface to bottom. To do this, we used a mathematical method based on Stokes' theory to determine the free surface elevation and the potential Field. We observed a simulation using the nonlinear stokes theory to describe the calculation in a hydrodynamic approach using a channel with a linear bottom and an exit condition taken as an impermeable wall. The numerical hydrodynamic results are obtained by using the finite difference method and the iterative method of relaxation line by line of Gauss-Siedel .For the first times, there is only movement in the proximity of the free surface up to a certain depth, but little movement begins to occur in the proximity of the bottom at greater times. This is because the impermeable wall taken as the exit condition reflects the wave, creating an interaction between it and the bottom. However, the interaction is significant in the proximity of the free surface, since this is where the velocity potential field is strongest. The wave has a greater influence on movement at the surface than at the bottom.

Finally, we also observe that there is no interaction between the wave and the bottom as depth increases.



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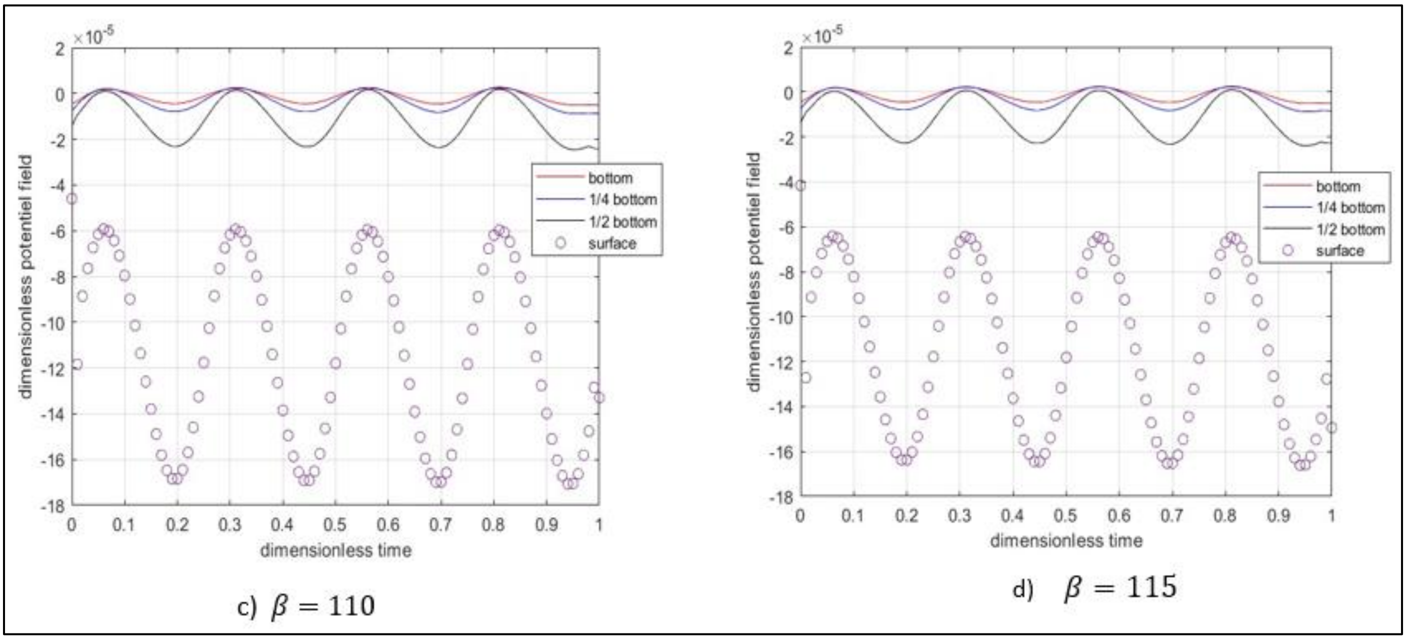

Fig 6 Particle Motion for Different Depths and Different Values of β for t=1.07

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