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Combined Effects of Steady Variable Viscosity and Thermal Conductivity on Hydrodynamic Electro-Osmotic Flows of a Reactive Fluid

J. O. Ajilore[1*]; S. O. Salawu[2]; R. A. Kareem[1]; A. A. Abdurasid[1]; T. O. Ogunjare[3]; V. O. Iluebe[4]; S. O. Sogunro[1]; Y. O. Anthonio[1]

[1]Department of Mathematical Science, Lagos State University of Science and Technology, Ikorodu, Nigeria
[2]Department of Mathematics, Federal University of Technology and Environmental Sciences, Iyin-Ekiti, Nigeria
[3]Department of Mathematics, Yaba College of Technology, Yaba, Lagos, Nigeria
[4]Department of Mathematics, Anchor University, Ayobo, Lagos.

Corresponding Author: J. O. Ajilore[1*]

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Abstract: The effects of steady electro-osmotic flow (EOF) and magnetohydrodynamics (MHD) with variable viscosity and the reactive fluid flow thermal conductivity are investigated with varying exponential temperature-dependent properties. The dimensionless variables are used to transform the governing equations of the flow to an invariant model. Thus, steady variable viscosity, thermal conductivity momentum and energy-coupled nonlinear equations are solved using the Weighted Residual method and a collocation integrating scheme. The graphical representation of results is done to effectively study the effects of the thermophysical behaviour of the model. The influence of electro-osmotic and magnetic fields on the fluid flow was significant, as Lorentz force retarded the flow while thermal conductivity dampened the fluid flow. Viscosity enhanced the temperature field due to the thickness of the thermal boundary layer as the parameter increased. Also, in conclusion, variable viscosity and thermal conductivity increased the velocity and temperature profiles for steady EOF-MHD flow. This information will be helpful in the chemical processing industry, combustion industry and allied engineering.

Keywords: *Electro-Osmotic Flow; Magnetohydrodynamic Fluid; Variable Viscosity; Variable Thermal Conductivity.*

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I. INTRODUCTION

Newtonian fluids are fluids that obey Newton's law of viscosity and for which μ has a constant value. More precisely, a fluid is Newtonian only if the tensors that describe the viscous stress and strain rate are related by a constant viscosity tensor that does not depend on the stress state and velocity of the flow. Most common liquids and gases, such as water and air, can be assumed to be Newtonian for practical calculations under ordinary conditions. Zeta potential is the electrical potential at the shear plane. Application of an electric field along the length of the micro–plates cause an electrical body force to be exerted on the mobile ions in the diffuse layer. Then the ions move under the influence of the electrical field and move the liquid by viscous forces. This type of flow is called electro-osmotic flow (EOF) [2,5,10].

Magneto hydrodynamic (MHD) studies the magnetic properties of electrically conducting fluids such as Plasmas, liquid metals and saltwater or electrolytes. The fundamental concept behind MHD is that magnetic fields can induce currents in a moving conductive fluid, which in turn polarizes the fluid and reciprocally changes the magnetic fluid itself. More so, reactive fluid flow has received increasing attention for studies of contaminant transport, groundwater quality, waste disposal, acid mine drainage, remediation, mineral deposits, and sedimentary just to mention a few. However, a liquid is said to be viscous if its viscosity is substantially great. Joule heating is the process by which the passage of an electric current through a

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conductor releases heat. The amount of heat released is proportional to the square of the current, such that $Q \propto l^2$. This is caused by interactions between the moving particles that form the current (but not always electrons) and the atomic ions that make up the body of the conductor [13-15].

This fluid has wide applications in many branches of science and engineering, with most focus on the thermal behaviours of fluids whose viscosity changes with temperature and the flow is accompanied by a simultaneous transfer of mass, energy and momentum in the system due to reaction occurring between the fluids. The ability to adequately describe such a system is necessary for the prediction of its thermal stability, among others. Hence, Efforts have been devoted to the study of heat transfer and thermal stability of reacting Newtonian fluids, which is of extreme importance not to compromise on the safety of life and materials during handling and processing of such fluids and for quality control purposes in many manufacturing and processing industries. An improvement in thermal recovery and utilization during the conventional flow in any fluids is one of the fundamental thermal integrations of such systems that provides better materials processing, energy conservation and a more environmentally being process.

The possibility of the existence of considerable resistance to heat transfer between the reacting fluids and system as a result of low-conducting fluids or highly conductive vessel walls, resulting in a significant temperature gradient, as was reported by Frank-Kamanetskii [1]. In recent times, the mathematical formulation of thermally critical systems mainly focuses on the determination of the critical regions separating the regions of explosivity and non-explosivity. Various works on the stability of flows were examined by Billingham [2]. Yihao Zhery et al. [3] investigated the kinetic behaviour and hydrodynamics of pressure-driven Poiseuille flow. Makinde [4] studied the thermal stability of a reactive third – grade liquid flowing steadily between two parallel plates with symmetrical convective cooling at the walls. Hence, the study of electro-osmotic, magneto-hydrodynamics with variable viscosity and thermal conductivity in reactive flow is significant for practical reasons.

The present study investigates combined steady electro-osmotic and magneto-hydrodynamic with variable viscosity and thermal conductivity in reactive fluid flow and determines the effect of fluid parameters on velocity profile and temperature profile for a steady, variable viscosity and thermal conductivity in a reactive fluid flow using the Weighted Residual method and a collocation integrating scheme.

II. MATHEMATICAL FORMULATION

An incompressible viscous fluid flow of the combined effects of steady electro-osmotic and magnetohydrodynamics with variable viscosity and thermal conductivity in a reactive fluid flow between two parallel plates was investigated**.** The equations governing the motion of the fluid are momentum, energy and electrical potential written as:

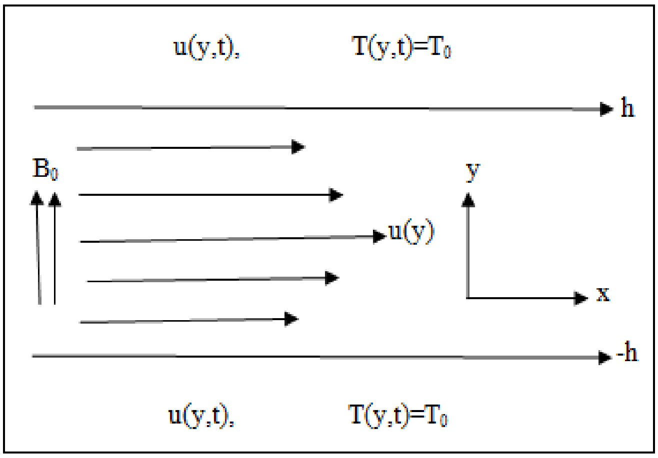

Fig 1 Flow Geometry

$$\rho\left(\frac{\partial \phi}{\partial \bar{t}} + V_o \frac{\partial \phi}{\partial \bar{y}}\right) = \frac{\partial}{\partial \bar{y}}\left(\mu(T)\frac{\partial \phi}{\partial \bar{y}}\right) + E_X \rho_e - \frac{\partial \bar{p}}{\partial \bar{x}} - B_0^2 \sigma \phi \tag{1}$$

$$\rho C_p\left(\frac{\partial T}{\partial \bar{t}} + V_o \frac{\partial T}{\partial \bar{y}}\right) = \frac{\partial}{\partial \bar{y}}\left(k(T)\frac{\partial T}{\partial \bar{y}}\right) + \mu(T)\left(\frac{\partial \phi}{\partial \bar{y}}\right)^2 + \beta_0^2 \sigma \phi^2 + QC_0 A \exp\left[\frac{-E}{RT}\right] \tag{2}$$

$$\frac{d^2\psi}{dy^2} = \frac{\rho_e}{B} \tag{3}$$

The boundary and the initial conditions of the flow are as follows:

$$\phi(\bar{y}, 0) = 0, \; T(\bar{y}, 0) = T_o$$

$$\phi(-h, \bar{t}) = 0, \; \phi(h, \bar{t}) = 0, \tag{4}$$

$$T(-h, \bar{t}) = T_o, \quad T(h, \bar{t}) = T_o$$

$$\frac{d^2\psi}{d^2y}(0) = 0, \; \psi(1) = \varsigma$$

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Where ρ is the density, μ the viscosity, C_p heat capacity with constant pressure, ϕ and V_o velocity components along x and y axis respectively, T the temperature, $\overline{p}$ is pressure, k the thermal conductivity, x the coordinate in the direction of flow, E the activation energy, R the universal gas constant, Q heat released per unit mass during reactions.

σ the electric field conductivity, ψ the electrical potential, ρ_e the net electric charge density, ε the dielectric constant, B_o the magnetic field strength, C_0 the constant pressure gradient, E_X the Electrical field and A is the rate of heat reaction. A temperature-dependent viscosity is assumed to be $\mu(T)=\mu_0\exp(\alpha[T-T_0])$ and the following non–dimensional variables were introduced.

Where t_o, μ_o are references to time and viscosity, respectively:

$$u=\frac{\phi}{v_0},\ y=\frac{\overline{y}}{h},\ x=\frac{\overline{x}}{h},\ p=\frac{\overline{p}h}{\mu_0 v_0},\ V_0=\frac{v_0}{h}\quad t=\frac{\overline{t}v_0}{h},\ \rho_e = -2\left(z1\,\eta_o\sinh\left(\frac{z1\,\psi^*}{k_b\theta^*}\right)\right) for\ \theta<1\,(i.e\quad \sin\theta=\theta)$$

$$\Rightarrow \sinh\theta=\theta,\ hence\quad \rho_e=\frac{-2Z^2 1^2\,\eta_0\psi^*}{K_b\theta^*},\ k(T) = k_0\exp a\left(T-T_0\right),\ \theta=\frac{E\left(T-T_0\right)}{RT^2{}_0},\ T=T_0+\frac{RT^2{}_0\theta}{E}, \tag{5}$$

Where η_o is bulk ionic concentration and z is valence of type –i ions

Substituting (4) into (1) to (3) with the initial and boundary conditions gives;

$$\frac{\partial u}{\partial t}+\frac{\partial u}{\partial y}=e^{\lambda\theta}\frac{\partial^2 u}{\partial y^2}+\lambda e^{\lambda\theta}\frac{\partial u}{\partial y}\frac{\partial\theta}{\partial y}+NK^2\psi+P-Mu \tag{6}$$

$$\Pr\left(\frac{\partial\theta}{\partial t}+\frac{\partial\theta}{\partial y}\right)=\exp(\beta\theta)\frac{\partial^2\theta}{\partial y^2}+\beta\exp(\beta\theta)\left(\frac{\partial\theta}{\partial y}\right)^2+Br\exp(\lambda\theta)\left(\frac{\partial u}{\partial y}\right)^2+BrMu^2+F\exp\left(\frac{\theta}{1+\varepsilon\theta}\right) \tag{7}$$

$$\frac{d^2\psi}{dy^2}=\frac{K^2\psi}{B} \tag{8}$$

The corresponding initial and boundary conditions are:

$$u(y,0)=0,\qquad \theta(y,0)=0$$

$$u(-1,t)=0,\qquad u(1,t)=0 \tag{9}$$

$$\theta(-1,t)=0,\qquad \theta(1,t)=0$$

$$\frac{d^2\psi}{d^2y}(0)=0,\ \psi(1)=\varsigma$$

Where $K = Z\ell\sqrt{\frac{2\eta_0}{k_b\theta}}$, $M=\frac{h^2B_0^2\sigma}{\mu}, P=-\frac{\partial p}{\partial x}, N=\frac{t_oE_X}{\mu}, \mu=\frac{\mu(T)}{\mu_0}, k=\frac{k(T)}{k_0},$,

$$Br=\frac{\mu v^2{}_0E}{kRT_0^2},\ F=\frac{QC_0A\exp\left[{}^{-1}/_{\varepsilon}\right]E}{RT_0^2k},\ \beta=\frac{aRT^2{}_0}{E},\ \lambda=\frac{\alpha RT^2{}_0}{E},\ Pr=\frac{\mu C_p}{k}$$

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K is the Debye Huckel parameter, and 1/K is the Debye characteristics thickness of EDL (electric double layer), p pressure gradient. Pr Prandtl number, Br Brinkman number, β variable conductivity, λ variable viscosity, M defines the magnetic term, N electro-Osmotic, μ viscosity, F Frank-Kamenetskii, K conductivity, u Dimensionless velocity, θ dimensionless temperature.

For a steady case, $\psi(y)=\frac{\varsigma\cos h\,ky}{\cos h\,k}$, equations (5) and (6) become

$$\frac{du}{dy}=e^{\lambda\theta}\frac{d^2u}{dy^2}+\lambda e^{\lambda\theta}\frac{du}{dy}\frac{d\theta}{dy}+\frac{NK^2\varsigma\cos h\,ky}{\cos h\,k},+P-Mu \tag{10}$$

$$\Pr\left(\frac{d\theta}{dy}\right)=\exp(\beta\theta)\frac{d^2\theta}{dy^2}+\beta\exp(\beta\theta)\left(\frac{d\theta}{dy}\right)^2+Br\exp(\lambda\theta)\left(\frac{du}{dy}\right)^2+BrMu^2+F\exp\left(\frac{\theta}{1+\varepsilon\theta}\right) \tag{11}$$

Corresponding boundary conditions are:

u(y,0)= 0, θ(y,0)= 0

$$u(-1,t)=0,\ u(1,t)=0,\ \text{and}\ \theta(-1,t)=0,\ \theta(1,t)=0 \tag{12}$$

$$\frac{d^2\psi}{dy^2}(0)=0,\ \psi(1)=\varsigma$$

III. WEIGHTED RESIDUAL (COLLOCATION) METHOD (WRCM)

In this method, the weight functions are chosen to be Dirac delta functions:

$$W_m(x)=\delta(x-x_m) \tag{13}$$

Such that the error is zero at the chosen nodes x_m. The Weighted Residual (Collocation) method was used to solve the steady state using the collocation method to handle the integration. These are a set of n-order linear equations which must be solved to obtain all the a_j and b_j coefficients in the assumed solution or trial solution.

Substituting into equation (8) and simplifying the result into:

$$\frac{du}{dy}=e^{\lambda\theta}\frac{d^2u}{dy^2}+\lambda e^{\lambda\theta}\frac{du}{dy}\frac{d\theta}{dy}+\frac{NK^2\varsigma\cos h\,ky}{\cos h\,k},+P-Mu \tag{14}$$

Simplifying (12) resulted in;

$$\frac{du}{dy}=e^{\lambda\theta}\frac{d^2u}{dy^2}+\lambda e^{\lambda\theta}\frac{du}{dy}\frac{d\theta}{dy}+\frac{NK^2\varsigma\cos h\,ky}{\cos h\,k},+P-Mu \tag{15}$$

Let $u(y)=y^5a_5+y^4a_4+y^3a_3+y^2a_2+y^1a_1+y^0a_0$

$$\frac{du}{dy}=5y^4a_5+4y^3a_4+3y^2a_3+2ya_2+a_1 \tag{16}$$

$$\frac{d^2u}{dy^2}=20y^3a_5+12y^2a_4+6ya_3+2a_2$$

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Similarly

$$\Pr\left(\frac{d\theta}{dy}\right) = \exp(\beta\theta)\frac{d^2\theta}{dy^2} + \beta\exp(\beta\theta)\left(\frac{d\theta}{dy}\right)^2 + Br\exp(\lambda\theta)\left(\frac{du}{dy}\right)^2 + BrMu^2 + F\exp\left(\frac{\theta}{1+\varepsilon\theta}\right) \tag{17}$$

Let $\theta(y) = b_5y^5 + b_4y^4 + b_3y^3 + b_2y^2 + b_1y^1 + b_0y^0$

$$\frac{d\theta}{dy} = 5b_5y^4 + 4b_4y^3 + 3b_3y^2 + 2b_2y^1$$

$$\frac{d^2\theta}{dy^2} = 20b_5y^3 + 12b_4y^2 + 6b_3y^1 + 2b_2$$

The residual of (12) and (13) are given as;

$$ur = \left(\begin{array}{l} 5a_5y^4 + 4a_4y^3 + 3a_3y^2 + 2a_2y + a_1 \\ -g\lambda e^{\lambda(b_5y^5+b_4y^4+b_3y^3+b_2y^2+b_1y^1+b_0y^0)}(5a_5y^4 + 4a_4y^3 + 3a_3y^2 + 2a_2y + a_1) \\ -g\lambda e^{\lambda(b_5y^5+b_4y^4+b_3y^3+b_2y^2+b_1y^1+b_0y^0)}(20a_5y^3 + 12a_4y^2 + 6a_3y + 2a_2) \\ -\dfrac{NK^2Z\cosh(ky)}{\cosh(k)} - p + M(a_5y^5 + a_4y^4 + a_3y^3 + a_2y^2 + a_1y + a_0y^0) \end{array}\right) \tag{18}$$

$$\theta r = \left(\begin{array}{l} 5a_5y^4 + 4a_4y^3 + 3a_3y^2 + 2a_2y + a_1 \\ -q\lambda e^{\lambda(b_5y^5+b_4y^4+b_3y^3+b_2y^2+b_1y+b_0y^0)}(5b_5y^4 + 4b_4y^3 + 3b_3y^2 + 2b_2y^1 + b_1) \\ -qe^{\lambda(b_5y^5+b_4y^4+b_3y^3+b_2y^2+b_1y+b_0y^0)}(20b_5y^3 + 12b_4y^2 + 6b_3y + 2b_2) \\ +gBr(5a_5y^4 + 4a_4y^3 + 3a_3y^2 + 2a_2y + a_1)^2 - MBr(a_5y^5 + a_4y^4 + a_3y^3 + a_2y^2 \\ +y\,a_1 + a_0)^2 - A(a_5y^5 + a_4y^4 + a_3y^3 + a_2y^2 + y\,a_1 + 1) \end{array}\right) \tag{19}$$

Substituting the initial conditions into the assumed solutions result into the equations;

$$\begin{aligned} eq1 &= a_0 - a_1 + a_2 + a_4 - a_5 \\ eq2 &= a_0 + a_1 + a_2 + a_4 + a_5 \\ eq3 &= b_0 - b_1 + b_2 + b_4 - b_5 \\ eq4 &= b_0 + b_1 + b_2 + b_4 + b_5 \end{aligned} \tag{20}$$

Similarly, collocating between the boundary conditions and substituting the values of the various parameters $(Z:=1:F:=0:N:=0.1:\lambda:=1:p:=1:L:=1:g:=1:d:=1:k:=2:J:=0.1:G:=1)$.

Equating equation (16) and other collocation equations to zero and solving the equations using Maple 18 software resulted in;

$$\left\{\begin{array}{llll} a_0 = 0.3449863200, & a_1 = -0.006457611095, & a_2 = -0.3110467470, & a_3 = 0.007680423592 \\ a_4 = -0.03393957301, & a_5 = -0.001222812497, & b_0 = 0.08830913625, & \mathrm{b}_1 = -0.001603406266 \\ b_2 = -0.05379214633, & b_3 = 0.0003815605281, & b_4 = -0.03451698992, & \mathrm{b}_5 = 0.001221845738 \end{array}\right\}$$

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For $p=1$,

$$u(y)=\begin{pmatrix}-0.001222812497y^5-0.03393957301y^4+0.007680423592y^3\\-0.3110467470y^2-0.006457611095y+0.3449863200\end{pmatrix}$$

Repeating the same process for the various parameters except for $p=1.3$ and $p=1.6$ to obtain;

$$\begin{Bmatrix}a_0=0.4382113601,\ \ a_1=-0.01040840718,\ \ a_2=-0.3961495033,\ \ a_3=0.01209071852\\ a_4=-0.04206185680,\ \ a_5=-0.001682311342,\ \ b_0=0.1106978034,\ \ \mathrm{b}_1=-0.002375313789\\ b_2=-0.05596956500,\ \ b_3=-0.0003339962896,\ \ b_4=-0.05472823838,\ \ \mathrm{b}_5=0.002709310079\end{Bmatrix}$$

For $p=1.3$,

$$u(y)=\begin{pmatrix}-0.001682311342y^5-0.04206185680y^4+0.01209071852y^3\\-0.3961495033y^2-0.01040840718y+0.4382113601\end{pmatrix}$$

Moreover,

$$\begin{Bmatrix}a_0=0.5274359163,\ \ a_1=-0.01544222567,\ \ a_2=-0.4774075761,\ \ a_3=0.01763914678\\ a_4=-0.05002834018,\ \ a_5=-0.002196921112,\ \ b_0=0.1360267697,\ \ \mathrm{b}_1=-0.003383250716\\ b_2=-0.05833680453,\ \ b_3=-0.001533705001,\ \ b_4=-0.07768996518,\ \ \mathrm{b}_5=0.004916955717\end{Bmatrix}$$

For $p=1.6$,

$$u(y)=\begin{pmatrix}-0.002196921112y^5-0.05002834018y^4+0.01763914678y^3\\-0.4774075761y^2-0.01544222567y+0.5274359163\end{pmatrix} \tag{3.77}$$

Follow the same procedure for all other thermophysical parameters for both the velocity and Temperature profiles. Having obtained the constants of the trial functions as above, values were substituted for thermodynamic parameters p(pressure), L (magnetic), G(viscosity), N (electro-osmotic), K (electro-kinetic), ζ (specific internal energy) into the velocity profile function and these thermo-physical parameters, values were varied, which resulted in Figs. 1-6. The same was done for thermo physical parameters of the temperature profile function, and resulted in Figs. 7-10.

IV. RESULTS AND DISCUSSION

The transport phenomena of magnetohydrodynamic (MHD) and electro-osmotic flow (EOF) have attracted close attention as a result of their wide usefulness in energy systems, biomedical engineering, chemical processing, and microfluidic devices. As such, the interaction of magnetic and electric fields with electrically conducting reactive fluid meaningly changes the thermal and flow characteristics. Combining with reactive species and thermal-dependent fluid properties, these impacts is complex and require careful theoretical analysis. Variations in thermal and viscosity conductivity play an essential role in precisely describing fluids under high reactive conditions and temperature gradients. Thus, to improve the insight into the study, the graphical results are presented with the default values taken from related studies.

Figure 2 depicts the effect of increasing values of Debye-Huckel parameter on the flow velocity. The Debye-Huckel term demonstrates the electric double layer thickness, with high values conforming to the thinner electric double layer close to the device surface. As noticed, a rise in the Debye-Huckel term causes a significant increase in the reactive flow velocity along the channel. The increase in the Debye-Huckel term compresses the electric double layer, leading to the channel walls concentration of the electric charge density. This inspires the fluid electro-osmotic force due to the applied electric field, which strengthens the fluid momentum transfer along the flow domain. Figure 3 illustrates the effect of variation in the electro-osmosis term on the velocity field. The electro-osmosis term denotes the electric field strength and the related electro-kinetic driven force acting on the reactive charged fluid in the electric double layer. As noticed, the rising electro-osmosis term gives rise to a significant

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increase in the fluid velocity along the channel. The electro-osmosis force domination reduces the pressure gradient relative influence and viscous resistance; this thereby boosts the flow in the presence of reactive and magnetic effects. This impact demonstrates the electro-osmosis effectiveness as a fluid low control dynamics in electro-kinetic and nanofluidic systems for accurate manipulation of velocity with magnetic field strength. Therefore, the rising electro-osmosis term encourages the adjacent ions' mobility to the device surface. The ions, in turn, resist the viscous interaction, causing an increase in the flow velocity axis. Figure 4 gives the magnetic field influence on the combined effects of steady variable viscosity and thermal conductivity of hydrodynamic electro-osmotic reactive fluid flow. As found in the figure, boosting the magnetic field term prompts a momentous decrease in the fluid velocity all over the flow regime. Thus, the magnetic field induces a Lorentz force that restricts the flow motion of the electrically conducting fluid. The flow momentum is reduced with the rising resistivity force, converting the kinetic energy to a magnetic suppressing force, thereby declining the flow.

Figure 5 depicts the pressure gradient impact on the reactive fluid flow velocity distribution. It is found that the pressure gradient inspires the flowing fluid velocity profile all over the channel. Thus, the pressure gradient acts as a driving force propelling the fluid flow along the flow axis. Enhancing the pressure gradient magnitude strengthens the mechanical energy applied to the system, ensuring the fluid flow dominance over the viscous resistance induced by the variable viscosity. Therefore, the velocity rises near both surfaces and in the channel domain, leading to an encouraged velocity distribution. Figure 6 illustrates a rise in the temperature distribution for increasing Brinkmann number (Br). An increase in the Brinkmann number causes an enhancement in the reactive fluid temperature along the flow channel. The term (Br) depicts the viscous dissipation ratio to thermal conductive propagation in the system. A rise in the term implies that the fluid friction viscous heating is well-pronounced in relation to the fluid thermal conduction removal. As such, more mechanical energy is transformed into internal energy, leading to an enhanced temperature level in the fluid. Therefore, the noticed rise in the temperature with increasing Brinkmann number highlights the important role of viscous dissipation in controlling thermal behaviour in an electro-osmotic and hydromagnetic system where high shear and electromagnetic forces co-occur. Figure 7 shows the impact of variation in the Frank Kamenetskii term on the reactive fluid heat distribution. As obtained, boosting the Frank Kamenetskii term prompts the temperature field along the flow regime. The Frank Kamenetskii term defines the exothermic species reactions' strength relative to thermal diffusion. The system is susceptible to critical behaviour and thermal runaway with an increase in the thermal distribution. Hence, excessive heat production must be prevented to avoid an explosion. Figure 8 displays the effect of Prandtl number on the heat transfer. Enhancing the Prandtl number results in a pronounced increase in the temperature profile and a thermal boundary layer thickness. The Prandtl number (Pr) presents the momentum diffusivity ratio to heat diffusivity. A larger Prandtl number corresponds to lower heat diffusivity, leading to slow conduction relative to momentum transfer. As such, there is a high accumulation of thermal energy at the device surface, causing rising temperature levels and a stronger thermal gradient close to the channel surface.

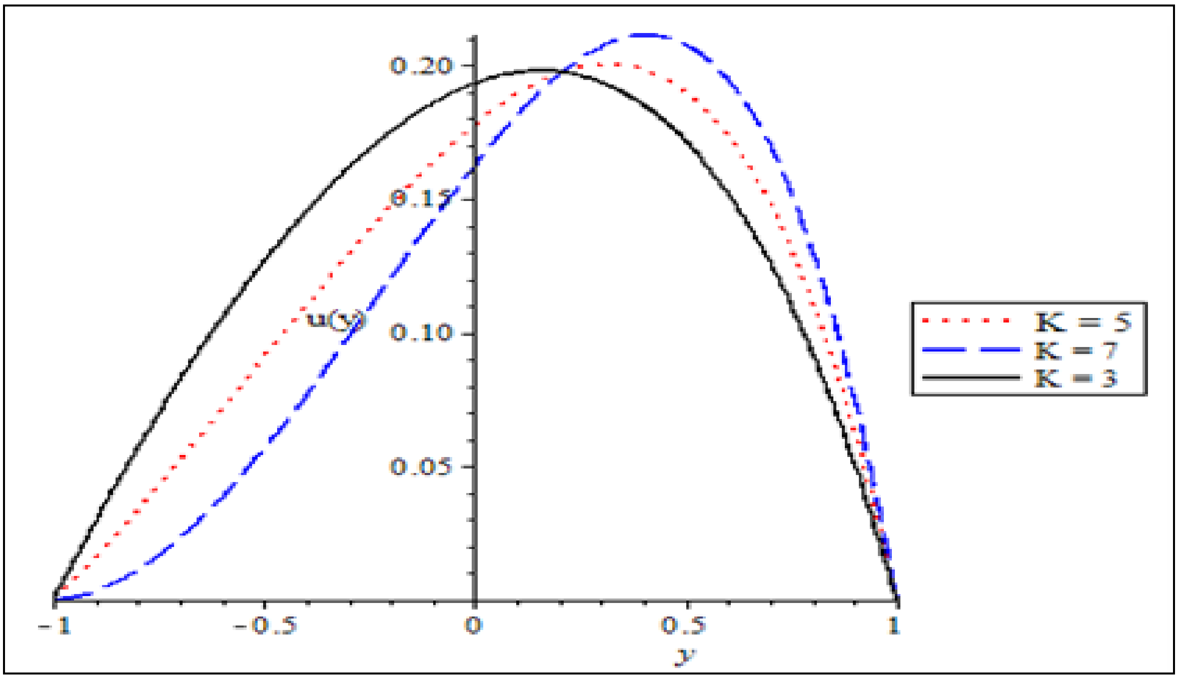

Fig 2 Effect of Debye-Hückel Parameter on the Flow Velocity

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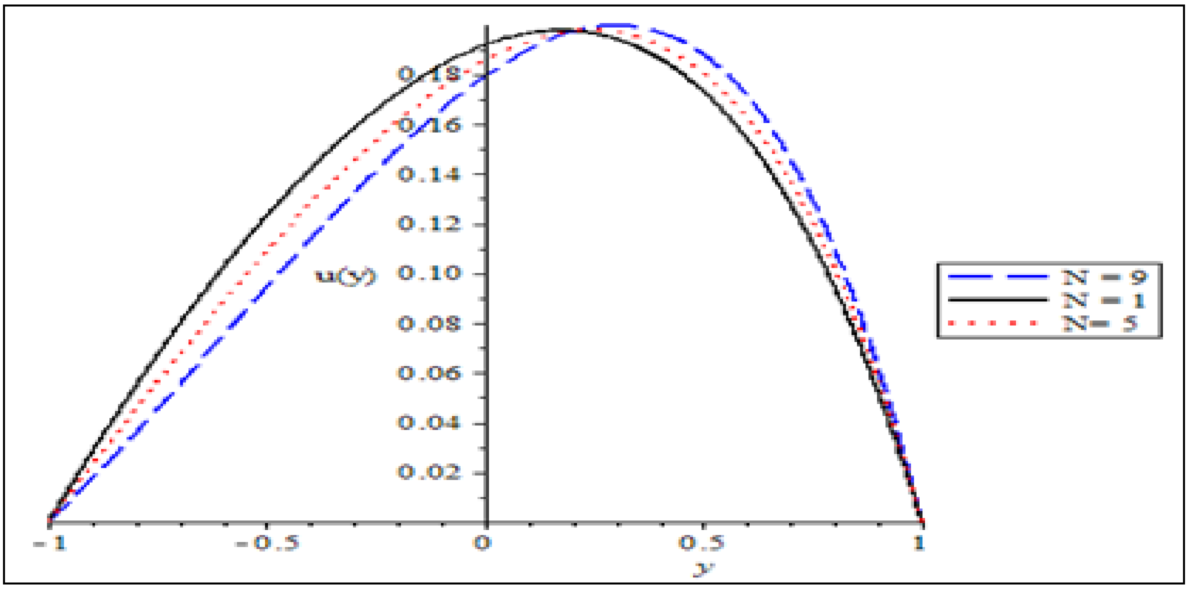

Fig 3 Electro-Osmosis Term Effect on the Velocity Field

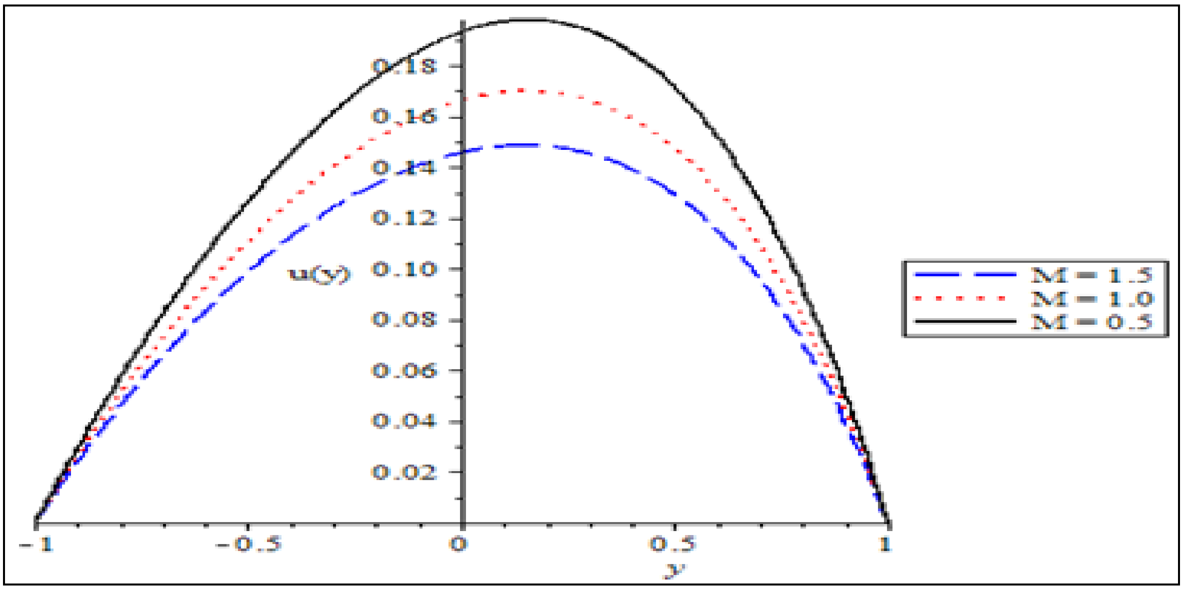

Fig 4 Velocity Profile for Variation in Magnetic Field Term

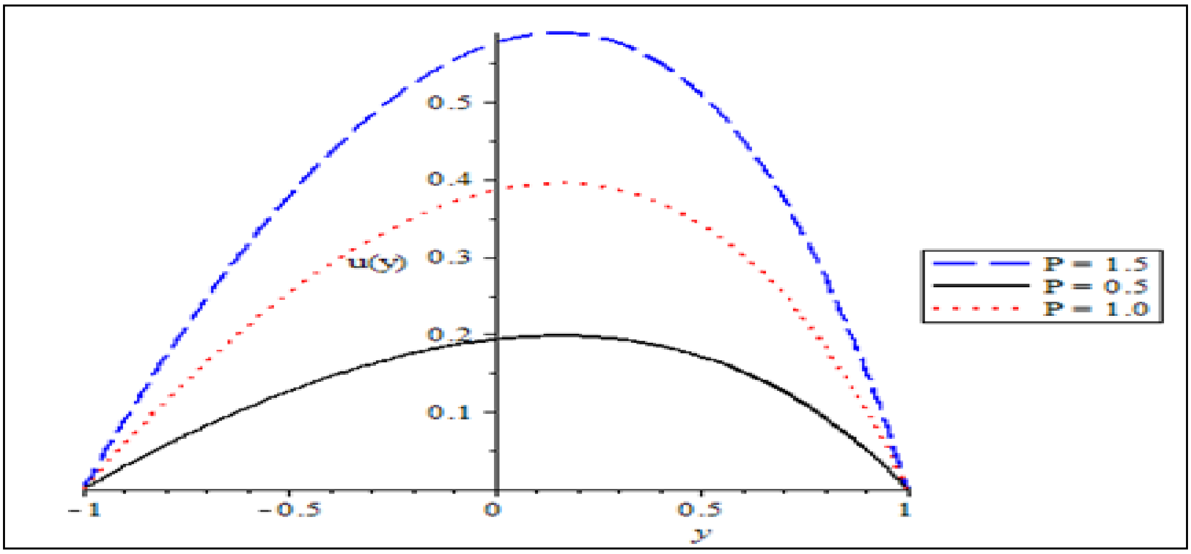

Fig 5 Pressure Gradient Impact on the Velocity Distribution

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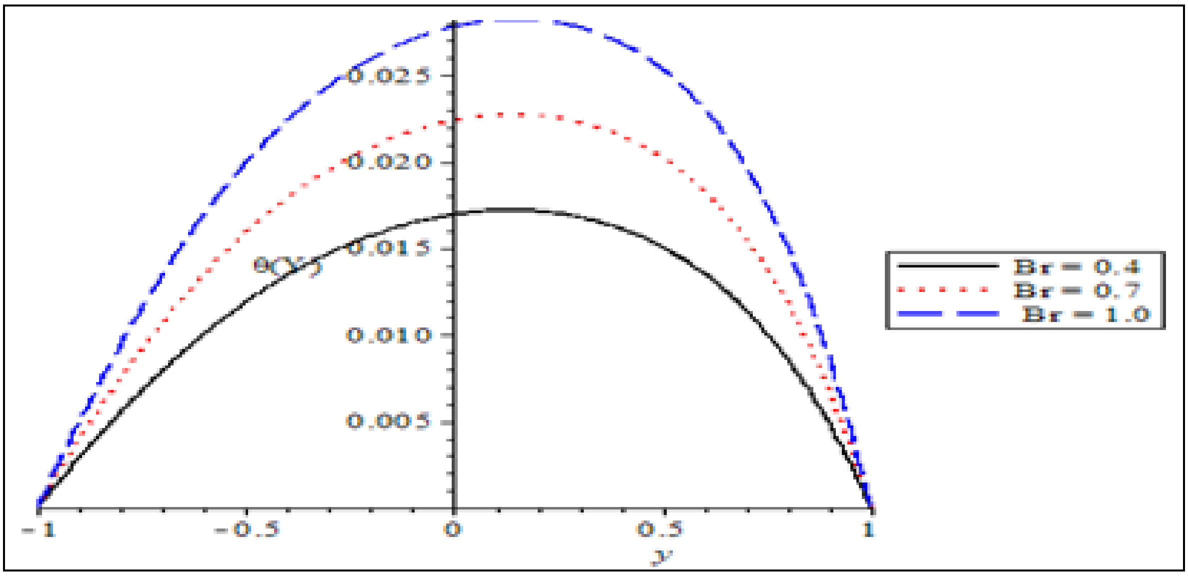

Fig 6 Temperature Field for Increasing Brinkmann Number

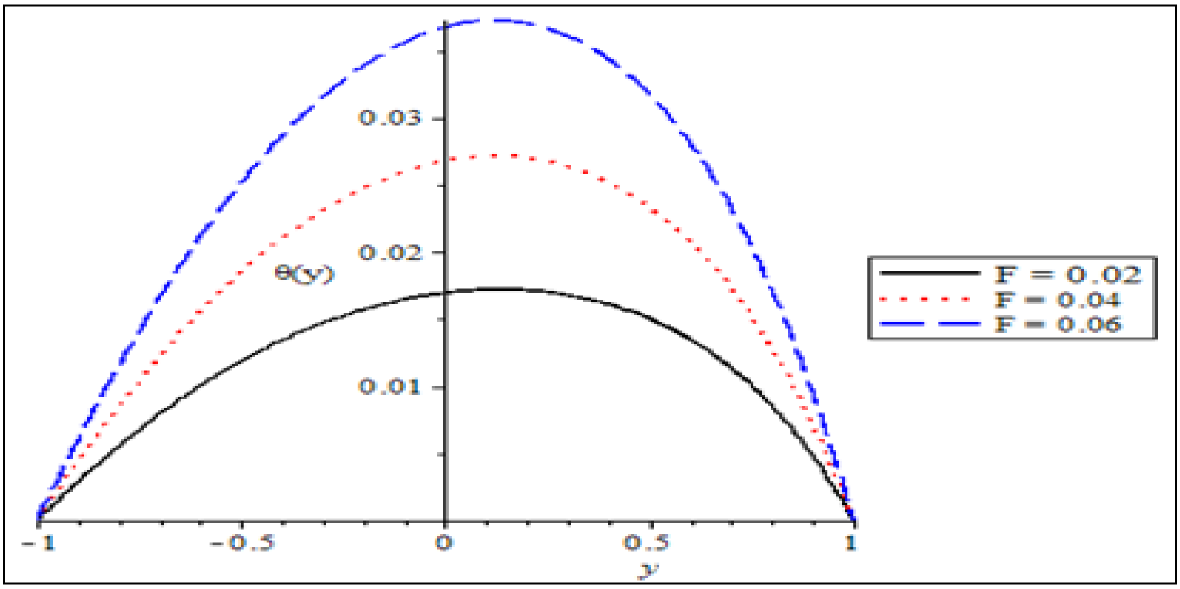

Fig 7 Impact of Frank Kamenetskii on Heat Distribution

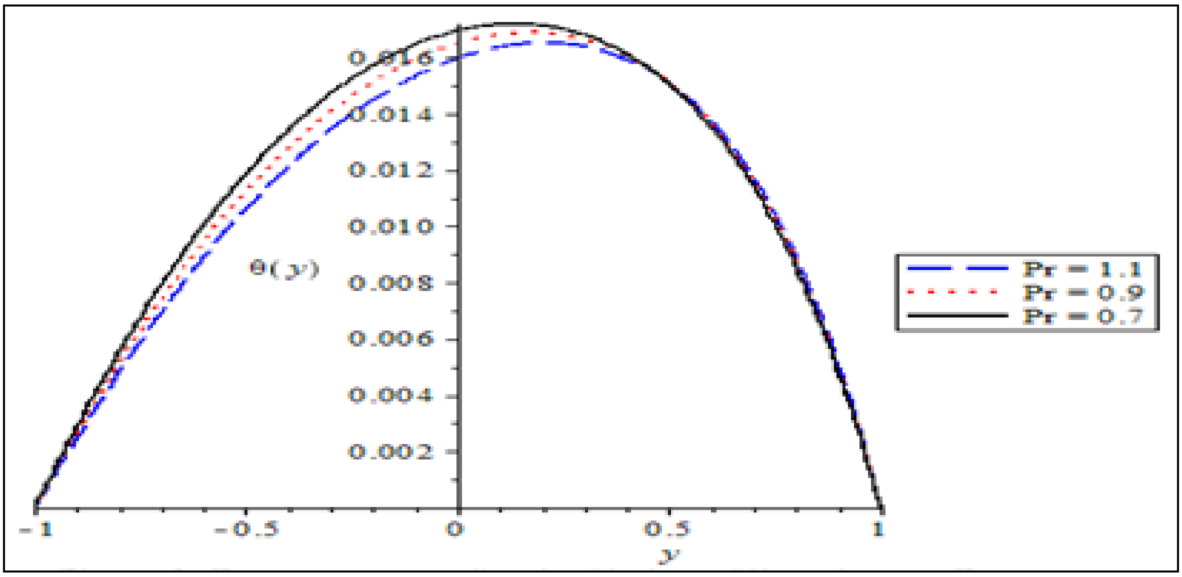

Fig 8 Increasing Effect of Prandtl Number on the Thermal Profile

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Thermal criticality occurs when reacting species advance into runaway heating, pushing it away or toward this critical threshold. The effect of the Brinkman number on the thermal criticality is presented in Figure 9. The Brinkman number (Br) accounts for the species' self-heating due to viscous dissipation. A boost in Br decreases the reacting fluid species' thermal criticality, owing to the dominance of the fluid friction. The thermal boundary layer reduces as the system damps the turning point quickly, as the internal generation and conduction balance slopes away from runaway. Therefore, inspiring (Br) decelerates thermal ignition tendencies. The impact of variable viscosity on the bifurcation chain reaction is presented in Figure 10. As observed, enhancing heat convection can cool the system as thermal advection dominates or disperse heating more proficiently and strengthen reaction rates. In exothermic flow reactive systems, reducing viscosity decreases the thermal critical term, indicating that runaway needs smaller perturbations. As such, variable viscosity shows to destabilize the system, because heated regions get more mobile, shuttling heat reactive into new regimes. Figure 11 shows the impact of variable thermal conductivity on the thermal branched-chain. As noticed, thermal conductivity induces fluid temperature and micro-mixing. The heated fluid gets more conductive; thus, higher temperature produces better heat conductivity, leading to heat diffusion. Meanwhile, higher conductivity variations sharpen gradients to prompt internal heat, which correspondingly increases the thermal runaway. Hence, variable thermal conductivity leads to an unstable and unsteady system by encouraging the temperature distribution, increasing the thermal criticality threshold. Figure 12 displays the influence of activation energy on the fluid reacting thermal criticality. The activation energy controls the response of reaction rates to temperature rise. High activation energy leads to a high reaction sensitivity rate to a change in temperature, to ignite excessive heat release. The Frank-Kamenetskii term rises under the same conditions; thus, the system gets prone to thermal runaway. Increasing activation energy stimulates the critical threshold and prompts the explosion-like behaviour.

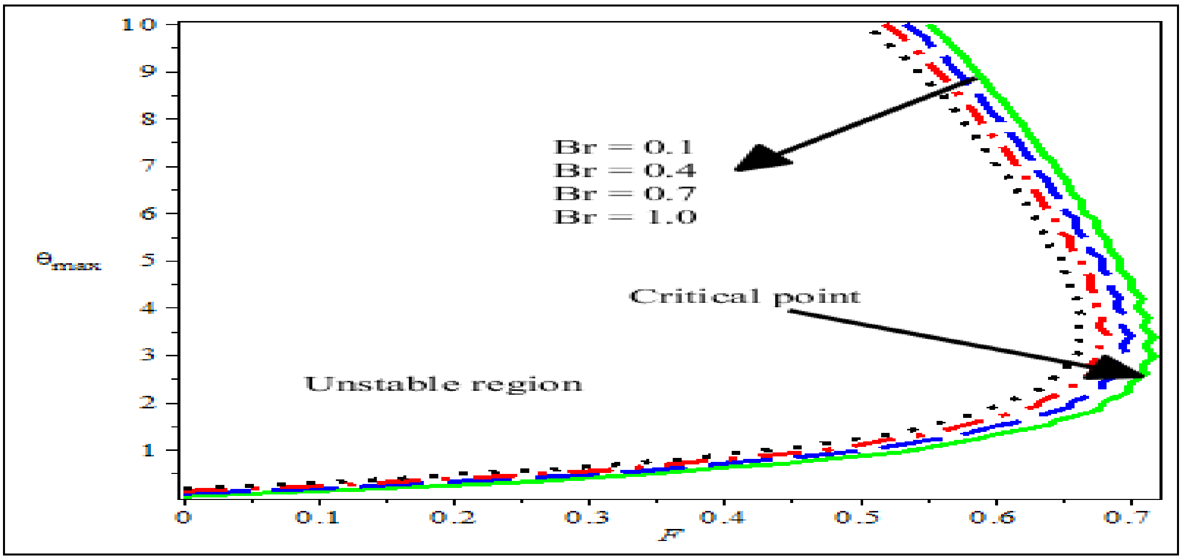

Fig 9 Thermal Criticality for Diverse (Br)

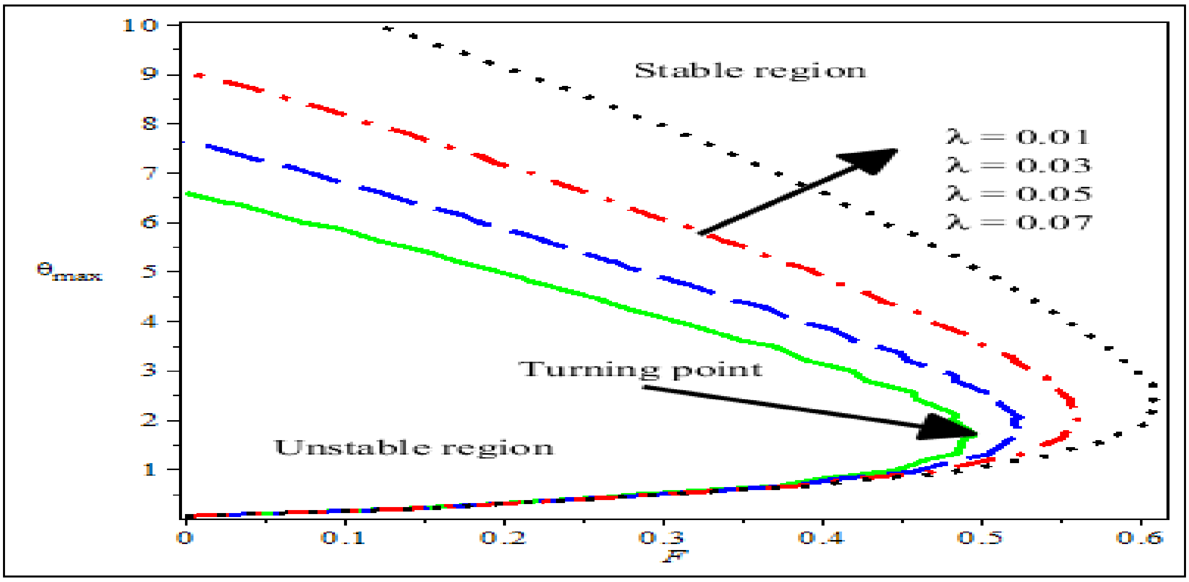

Fig 10 Bifurcation Chain for Different Variable Viscosity

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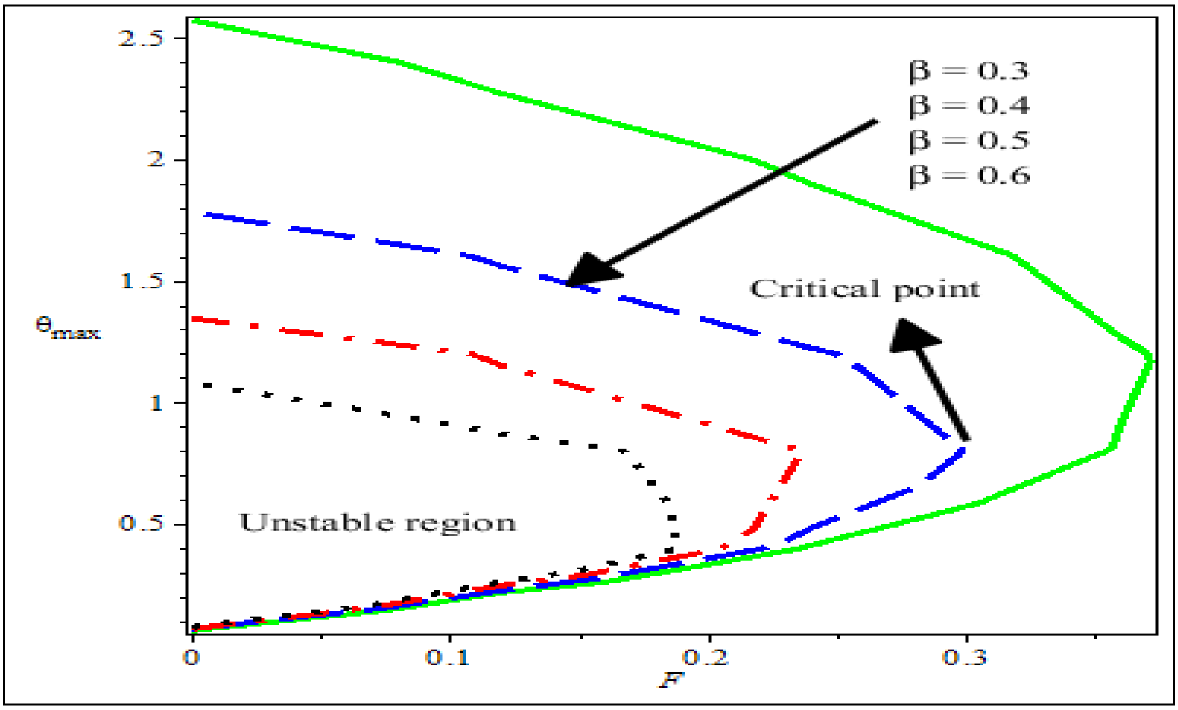

Fig 11 Thermal Branched-Chain for Various (β)

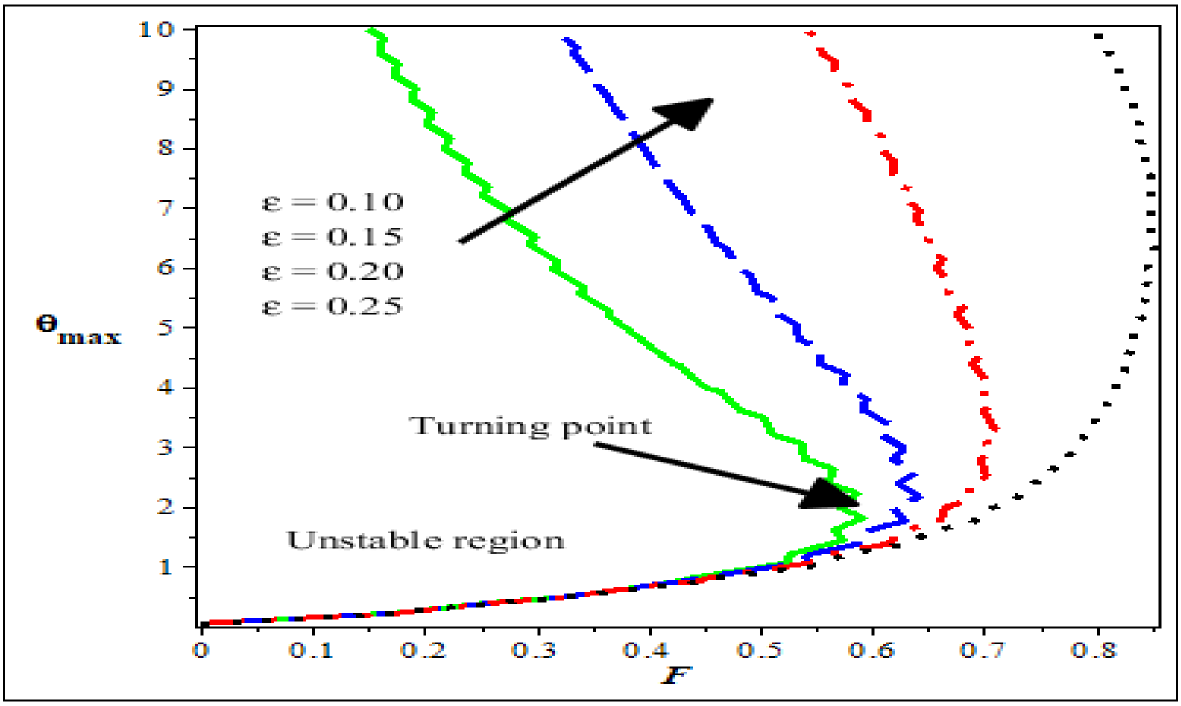

Fig 12 Criticality Bifurcation for Different (ε)

V. CONCLUSION

This study examined the influences of electro-osmotic flow and magnetohydrodynamics on a reactive fluid with thermal-dependent viscosity and conductivity flowing within parallel walls. The nonlinear governing momentum and energy balances were developed using appropriate non-dimensional terms and numerically solved through the Weighted Residual Collocation Method, permitting an exhaustive analysis of the flow and thermal characteristics with the effect of key physical terms. The outcomes revealed that the:

- Electro-kinetic terms, the electro-osmosis and Debye-Hückel terms, pointedly augment the flow velocity by enhancing the electrical body force within the electric double layer.
- Applied magnetic field suppresses fluid motion due to the opposing Lorentz force, signifying its efficiency as a flow control mechanism.

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- Pressure gradient distinctly augmented the fluid, countering magnetic damping and viscous resistance.
- Brinkmann numbers strengthened viscous dissipation, leading to rising temperature levels.
- Frank-Kamenetskii values augmented internal heat generation due to exothermic reactions, pushing the system toward thermal criticality.
- Prandtl number reduces thermal diffusion, ensuing in larger heat retention within the fluid.

Overall, the results give valuable insight into the interaction between temperature-dependent fluid properties, chemical reactions, magnetic fields, and electro-osmotic forces. The study findings are pertinent to the design and optimization of thermal management systems, chemical reactors, and microfluidic devices, where flow dynamics and thermal stability is control essential.

REFERENCES

[1]. Akgul, M. and Pakdemirli, B. M. (2008). *Analytic and Numerical Solution of Electro-Osmotically driven flow of a third grade fluid between micro-parallel plates*: International Journal of non-linear Mechanics. 43, 985-992.

[2]. Andrew J. P. and Todd M.S. (2009). *Induced Charge Electroosmosis over Controllably Contaminated Electrodes Department of Chemical Engineering*: University of California, Santa Barbara, California, USA.

[3]. Billingham (2003). *Steady-Static Solutions for strongly exothermic ignition in symmetric geometrics*: IMAJ. APPL. Math 2003; 65 283-313.

[4]. Ehsan, R., Kharazmi, S and Yaghoub F. (2009). *Application of the Homotopy method for analytical solution of non-Newtonian channels flows:* Phys. Scr. 79, 065009.

[5]. Escandon, J. P., Santiago, F. and Bautista O. (2013). *Temperature distributions in a parallel flat plate microchannel with electro-osmotic and magneto-hdyrodynamic micropumps*, proceedings of the International Conference on Mechanics, Fluids, Heat, Elasticity and Electomagnetic Fields.

[6]. Iseries, A. (1996). *A first course in the Numerical Analysis of Differential Equations:* Cambridge University Press, ISBN 978-0-521.

[7]. Kamenetskii, D. A. (1969). Diffusion and Heat Transfer in Chemical Kinetics, second ed. Plenum press, New York.

[8]. Makinde, O. B. (2009). On thermal stability of a reactive third grade fluid in a channel with convertible cooling there walls, Appl Math. Comp. 213, 170-176.

[9]. Odutayo, R. R., Ademola M. R., Ismail B. A., Kazeem O. S. and Sarafa O. A. (2011*). Temperature dependent Poiseuille fluid flow between parallel plates*, Canadian, Journal on Sciences and Engineering Mathematics: 2(3), 146-152.

[10]. Suvankar, G., Sandip, S., Tapan, K. H., Manoranjan M. (2015). *Thermally developing combined electro-osmotic and pressure driven flow of nanofluids in a microchannel under the effect of magnetic field*: Chemical Engineering science. 26, 10-12.

[11]. White, M. F. (1999), Viscous fluid flow 2nd Edition (New York: McGraw Hill).

[12]. Zhery, L. Alejandro G. and Aldor, J. B. (2002*). Comparison of Kinetic Theory hydrodynamics for poiseuille flow*: J. of Stat. Phys. 109. (314), 495-505.

[13]. Animasaun, I. L., Prakash, J., Vijayaragavan, R. and Sandeep, N. (2017). *Stagnation Flow of Nanofluid Embedded with Dust Particles Over an Inclined Stretching Sheet with Induced Magnetic Field and Suction.* Journal of nanofluids. 6 (10), 28-37.

[14]. Animasaun, I.. L.. Raju, C. S. K and Sandeep, N. (2016). *Unequal diffusivities case of homogeneous–heterogeneous reactions within viscoelastic fluid flow in the presence of induced magnetic-field and nonlinear thermal radiation.* Alexandria Engineering Journal. 55, 1595-1606.

[15]. Sandeep, N., Sulochana, C, snd Animasaun, I. L. (2016). *Stagnation point flow of a Jeffrey nanofluid over a stretching surface with induced magnetic field and chemical reaction*. International Journal of Engineering Research in Africa, 20, 93-111.