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Fractional Viscoelastic Models for Non-Newtonian Fluid Dynamics: A Comprehensive Review of Rheology, Constitutive Modeling and Numerical Approaches

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Abstract: The stress–strain response of non-Newtonian fluids is highly nonlinear and shows long-term memory effects, anomalous relaxation behavior, and frequency-dependent responses when stress is applied. Some classical viscoelastic models such as the Maxwell, Kelvin–Voigt, and Burgers models [1, 4], are not well suited to the hereditary behavior of polymers, biological tissues, gels, suspensions, or industrial materials. Fractional calculus is an effective mathematical tool for modeling these types of systems because fractional derivatives inherently exhibit nonlocal and memory-dependent effects [3, 5]. This review addresses viscoelastic models for non-Newtonian fluid dynamics using a fractional approach. The Caputo derivative, Riemann-Liouville derivative Caputo-Fabrizio derivative and Atangana-Baleanu derivative are some known fractional operators. They have been discussed in terms of interpretation, constitutive behavior and rheology [5, 8 13]. When comparing models like Maxwell, Kelvin-Voigt, Zener and generalized models we look at key features. These include creep and relaxation behavior, power-law rheology and computational efficiency [1, 4 5]. Recent advances in equations have been summarized. This includes developments, in Navier-Stokes formulations, multi-term and variable-order operators and advanced numerical methods [6, 7 10]. Existing problems in parameter estimation, computational complexity, numerical stability, and experimental validation are also discussed in this paper. Finally, future research directions on the use of machine learning for parameter identification, hybrid CFD–fractional models, adaptive fractional models, and biomedical modeling of rheological properties are indicated in this paper. This review provides a general understanding of the role of fractional viscoelasticity in complex fluid systems.

Keywords: *Fractional Calculus, Non-Newtonian Fluids, Viscoelasticity, Memory Effects, Fractional Constitutive Equations, Power-Law Rheology, Fractional Derivatives.*

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I. INTRODUCTION

Non-Newtonian materials exhibit significant interest as they have many applications in engineering, polymer processing, biomechanics, biomedical systems, lubrication technology, and the transport of fluids in industry [2, 3]. These fluids tend to follow a constitutive law that is typically nonlinear, where the stress level is dependent upon both the existing rate of strain of the material and the history of the previously established strains [4, 5]. Such hereditary behavior produces long-term memory effects that cannot be accurately described using only conventional integer-order differential equations.

The Maxwell, Kelvin–Voigt, and Burgers models are traditional viscoelastic models that are important for describing the viscoelastic responses of complex materials. However, these classical models typically predict exponential relaxation and finite creep, which do not agree with the experimentally observed power-law rheological behavior of many biological and polymeric systems [1, 4]. Experimental investigations have demonstrated that several viscoelastic materials exhibit distributed relaxation spectra and anomalous transport behavior over multiple timescales [3, 5].

Fractional calculus extends ordinary differentiation and integration to non-integer orders and naturally incorporates memory and hereditary characteristics into constitutive equations [5, 13]. Fractional derivatives possess nonlocal kernels, allowing the present material responses to depend continuously on the previous states. Thus, fractional viscoelastic models are able to give a better description of

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creep, stress relaxation and frequency-dependent behaviors than the traditional integer order models [1, 3]. In recent years, fractional constitutive equations have been successfully used in the fields of biological tissues, generalized Oldroyd-B fluids, viscoelastic nanofluids, porous media transport and magnetohydrodynamic flows [7, 9]. Moreover, the modern nonsingular operators, such as Caputo–Fabrizio and Atangana–Baleanu derivatives, have been employed to improve the numerical stability and physical realism of the considered operators using exponential and Mittag-Leffler kernels, respectively [8,11]. Despite the fact that many studies have been devoted to specific fractional models in a stand-alone manner, a comprehensive study unifying the three concepts of rheological interpretation, constitutive modeling, fractional operators, and computational methods under a single framework is not available in the literature.This review addresses this gap by systematically analyzing fractional viscoelastic models for non-Newtonian fluid dynamics.

II. RESEARCH GAP AND NOVELTY

Despite substantial developments in fractional rheology, several limitations remain.

- Existing reviews often discuss fractional derivatives and rheological applications separately, without integrating constitutive modeling, physical interpretation, and computational analysis within a single framework [4, 5].
- Comparative analyses between classical fractional operators and modern nonsingular operators remain insufficient, particularly regarding numerical stability and physical interpretation [8, 11].
- Multi-term and variable-order fractional models have not been systematically reviewed for complex non-Newtonian fluid systems [7, 10].
- Many studies emphasize mathematical formulations while providing limited discussions on experimental validation and practical engineering relevance [2, 3].
- The integration of fractional viscoelasticity with heat transfer, nanofluid dynamics, and magnetohydrodynamic transport remains fragmented in the literature [7, 9].

Unlike previous reviews that primarily focused on isolated rheological models or fractional operators, this review integrates fractional constitutive equations, power-law rheology, advanced operators, computational methods, and experimental interpretations within a unified framework of non-Newtonian fluid dynamics. A special focus was dedicated to comparing the Caputo, Caputo–Fabrizio, and Atangana–Baleanu operators to model memory-dependent transport phenomena and viscoelastic responses.

III. FUNDAMENTALS OF FRACTIONAL CALCULUS IN VISCOELASTICITY

- *Fractional Derivatives*

Fractional derivatives generalize classical differentiation to non-integer orders and provide efficient mathematical tools for modeling hereditary systems [5].

- *Caputo Fractional Derivative*

The Caputo fractional derivative is widely used because it preserves the classical initial conditions and possesses a strong physical interpretation [5, 13]

$$ {}^{C}D_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha-n+1}}\, d\tau \quad (1) $$

Where

- $0 < \alpha < 1$ = order of the fractional derivative
- $\Gamma(\cdot)$ = Gamma function
- $f^{(n)}(\tau)$ = integer-order derivative of f

The parameter α controls the intensity of the memory effects. Smaller values of α corresponded to stronger hereditary behavior.

- *Riemann–Liouville Fractional Derivative*

The Riemann–Liouville derivative is expressed as follows [5]:

$$ {}^{RL}D_t^{\alpha} f(t) = \frac{1}{\Gamma(1-\alpha)} \frac{d^n}{dt^n} \int_0^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}}\, d\tau \quad (2) $$

Unlike the Caputo derivative, this formulation requires fractional initial conditions, thereby limiting its applicability to various physical systems.

- *Caputo–Fabrizio Derivative*

The Caputo–Fabrizio derivative uses a nonsingular exponential kernel [8]:

$$ {}^{CF}D_t^{\alpha} f(t) = \frac{1}{1-\alpha} \int_0^t f'(\tau)\, Exp\left(-\frac{\alpha(t-\tau)}{1-\alpha}\right) d\tau \quad (3) $$

Where

- $0 < \alpha < 1$ = order of the fractional derivative
- $Exp\left(-\frac{\alpha(t-\tau)}{1-\alpha}\right)$ = exponential memory kernel (is a nonsingular exponential kernel)
- $f'(\tau)$ = past rate of change.

Unlike classical fractional derivatives, it does not contain a singular, power-law kernel. This operator represents systems with finite memory and improved numerical stability [8].

- *Atangana–Baleanu Derivative (Nonlocal Kernel)*

The Atangana–Baleanu derivative employs a Mittag–Leffler kernel as follows [11]:

$$ {}^{AB}D_t^{\alpha} f(t) = \frac{B(\alpha)}{1-\alpha} \int_0^t f'(\tau)\, E_{\alpha}\left(-\frac{\alpha\,(t-\tau)^{\alpha}}{1-\alpha}\right) d\tau \quad (4) $$

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Where

- $0 < \alpha < 1$ = order of the fractional derivative
- $Exp\left(-\frac{\alpha(t-\tau)}{1-\alpha}\right)$ = exponential memory kernel (is a nonsingular exponential kernel)
- $f'(\tau)$ = past rate of change
- $B(\alpha)$ = normalization constant
- E_α is the Mittag–Leffler function

IV. PHYSICAL INTERPRETATIONOF FRACTIONAL OPERATORS

Fractional derivatives represent hereditary behavior because the current state depends on the entire deformation history [4, 5].

➢ *Power-Law Relaxation*

Many viscoelastic materials exhibit power-law stress relaxation [4].

$$G(t) \propto t^{-\alpha}, \quad 0 < \alpha < 1 \tag{5}$$

Where

- $G(t)$ = relaxation modulus
- α = controls memory intensity

This behavior is commonly observed in polymers and biological tissues [1, 4].

➢ *Long-Term Creep*

Fractional models accurately describe the creep compliance as follows [5]:

Creep compliance $J(t)$ often follows a power-law form:

$$J(t) \propto t^{\alpha} \tag{6}$$

Unlike classical models, deformation progresses over time.

➢ *Frequency-Dependent Viscoelasticity*

The complex modulus is expressed as [3]:

$$G^*(\omega) = G'(\omega) + iG''(\omega) \tag{7}$$

Where

- $G'(\omega)$ = storage modulus that represents elastic behavior
- $G''(\omega)$ = The loss modulus represents the viscous behavior.

Fractional models accurately reproduce the experimentally observed frequency-dependent rheological responses.

V. CLASSICAL AND FRACTIONAL VISCOELASTIC MODELS

Classical and fractional viscoelastic models are also discussed. Viscoelastic materials respond to applied loads with a combination of elastic and viscous properties. This dual nature is often represented by mechanical models comprising springs (elastic) and dashpots (viscous). Classical rheological models are a good approximation for describing these materials but do not always account for the time-dependent phenomena observed in several real systems.

- *Classical Maxwell Model*

The classical Maxwell constitutive equation is as follows:

$$\frac{d\varepsilon}{dt} = \frac{1}{E}\frac{d\sigma}{dt} + \frac{\sigma}{\eta} \tag{8}$$

Where

- $\sigma(t)$ = stress
- $\varepsilon(t)$ = strain
- E = elastic modulus
- η = viscosity.

This model captures the distributed memory effects and power-law rheology of materials [4].

- *Fractional Maxwell Model*

The fractional Maxwell model generalizes the classical formulation using fractional derivatives as follows [1]:

$$\sigma(t) + \lambda^{\alpha} D_t^{\alpha}\sigma(t) = E\lambda^{\alpha}\, {}^{C}D_t^{\alpha}\varepsilon(t) \tag{9}$$

The fractional order parameter introduces a long-term memory into the constitutive response.

- *Fractional Kelvin–Voigt Model*

The fractional Kelvin–Voigt model is expressed as follows [5]:

$$\sigma(t) = E\varepsilon(t) + \eta\, {}^{C}D_t^{\alpha}\varepsilon(t) \tag{10}$$

This model accurately describes the creep and oscillatory viscoelastic behaviors.

- *Fractional Zener Model*

The generalized fractional Zener model is as follows:

$$\sigma + \tau^{\alpha} D_t^{\alpha}\sigma(t) = E_1\varepsilon(t) + E_2\tau^{\alpha} D_t^{\alpha}\varepsilon(t) \tag{11}$$

This formulation simultaneously captures the creep and stress relaxation responses.

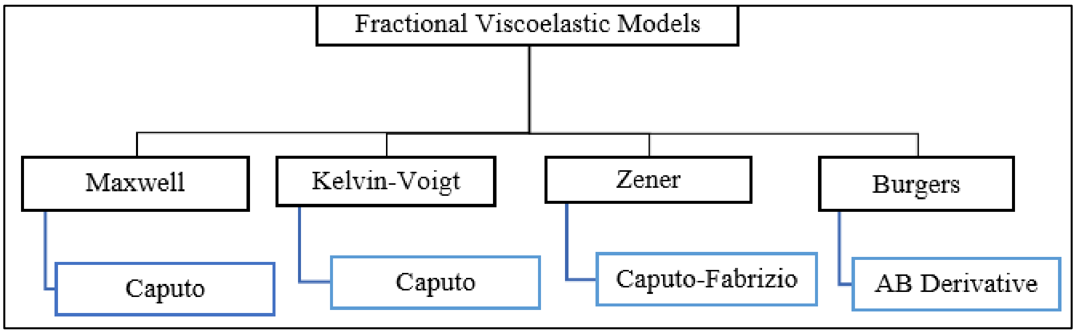

Fig 1 Classification of Fractional Viscoelastic Models

Table 1 Comparative Analysis of Fractional Operators

Operator	Kernel Type	Memory Behavior	Advantages	Limitations
Caputo Fractional Derivative	Singular power-law	Long memory	Classical initial conditions	Singular kernel
Riemann -Liouville Fractional Derivative	Singular power-law	Long memory	Strong mathematical formulation	Fractional initial conditions
Caputo-Fabrizio Fractional Derivative	Exponential	Finite memory	Numerical stability	Limited long-memory representation
Atangana-Baleanu Fractional Derivative	Mittag–Leffler	Multi-scale memory	Physical realism	Computational complexity

VI. FRACTIONAL CONSTITUTIVE EQUATIONS FOR NON-NEWTONIAN FLUIDS

➢ *Fractional Stress-Strain Relation*

The generalized constitutive equation is expressed as follows [6]:

$$\tau = \mu D_t^{\alpha} \gamma \tag{12}$$

Where:

- $\tau =$ shear stress
- $\mu =$ dynamic viscosity
- $\gamma =$ strain rate.

This equation introduces nonlocal rheological effects that are absent in classical constitutive laws.

➢ *Fractional Navier–Stokes Equation*

The fractional Navier–Stokes equation is represented as follows [7]:

$$\rho \frac{\partial^{\alpha} u}{\partial t^{\alpha}} = -\nabla p + \mu \nabla^2 u + F \tag{13}$$

Where:

- $\rho =$ fluid density
- $p =$ pressure
- $F =$ external forces.

This equation is primarily used in magnetohydrodynamic flows, porous media transport, viscoelastic nanofluids, and thermal convection systems.

VII. ADVANCED FRACTIONAL MODELS

➢ *Multi-term Fractional Models*

Multi-term models simultaneously represent multiple memory mechanisms [7].

$$\sum_{i=1}^{n} a_i D_t^{\alpha_i} u(t) = f(t) \tag{14}$$

These formulations improve the transient rheological simulations.

➢ *Variable-Order Fractional Models*

Variable-order derivatives allow the fractional order to vary dynamically.

$$D_t^{\alpha(t)} f(t) \tag{15}$$

These models are suitable for materials whose rheological properties evolve with temperature, deformation or time [10].

VIII. NUMERICAL AND ANALYTICAL METHODS

➢ *Laplace Transform Method*

The Laplace transform of the Caputo derivative is given by [5]:

$$\mathcal{L}\{\, {}^{C}D_t^{\alpha} f(t)\} = s^{\alpha} F(s) - \sum_{k=0}^{n-1} s^{\alpha-k-1} f^{(k)}(0) \tag{16}$$

Where:

- $\mathcal{L}\{\cdot\} =$ Laplace transform operator
- $F(s) = \mathcal{L}\{f(t)\}$

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This technique provides exact solutions for linear -fractional systems.

➢ Grünwald–Letnikov Approximation

The finite difference approximation is given by [7]:

$$D_t^\alpha f(t) \approx \frac{1}{h^\alpha} \sum_{k=0}^{n} (-1)^k \binom{\alpha}{k} f(t-kh) \quad (17)$$

Where

- $h =$ time-step size

➢ *Numerical Stability Considerations*

Fractional differential equations require substantial computational resources because fractional derivatives depend on all the previous states. Major numerical challenges include large memory storage requirements, slow convergence rates, increased computational complexity, and numerical instability when solving nonlinear systems [7, 10].

IX. CHALLENGES AND FUTURE RESEARCH DIRECTIONS

➢ *Despite their Advantages, Fractional Viscoelastic Models Face Several Challenges.*

- Difficult experimental parameter estimation
- High computational cost
- Limited physical interpretation of fractional order
- However, there is a lack of standardized numerical frameworks.

➢ *Future Research Should Focus on the Following Aspects:*

- Adaptive variable-order operator
- Hybrid CFD–fractional frameworks
- Machine-learning-assisted parameter identification
- Biomedical viscoelastic modeling
- Multi-scale computational rheology
- Efficient numerical algorithms for large-scale simulations are required to resolve this issue.

The integration of artificial intelligence with fractional rheology may significantly improve the predictive capabilities and computational efficiency of complex fluid systems.

X. EMERGING TRENDS IN FRACTIONAL RHEOLOGY

Recent developments in fractional rheology have paved the way for modelling complex viscoelastic and non-Newtonian fluid systems. However, fractional differential equations have nonlocal and memory-dependent characteristics, which may pose difficulties for traditional parameter estimation methods. Consequently, machine learning (ML) and artificial intelligence (AI) techniques have emerged as promising tools for identifying model parameters and improving predictive accuracy [7, 10].

Physics-Informed Neural Networks (PINNs) and Fractional Rheology-Informed Neural Networks (FRINNs) combine experimental data with governing fractional differential equations to provide efficient solutions for inverse and forward rheological problems. These approaches reduce computational costs while maintaining high accuracy in the representation of complex material behaviors [10].

In addition, hybrid Computational Fluid Dynamics (CFD) and fractional constitutive frameworks have gained attention for simulating industrial processes involving polymers, suspensions, nanofluids, and biological fluids [6, 7, 9]. Variable-order fractional operators and adaptive memory kernels have also been investigated to model materials whose rheological properties evolve with time, temperature, and deformation [10, 12].

The integration of data-driven methods with fractional constitutive equations is expected to significantly enhance the modeling of multiscale transport phenomena, biomedical systems, and advanced engineering materials in future studies [2, 7, 10].

XI. CONCLUSION

Fractional calculus has emerged as an effective mathematical framework for describing the complex rheological behaviors of viscoelastic non-Newtonian fluids. Unlike classical integer-order models, fractional constitutive equations successfully incorporate memory effects, hereditary characteristics, and power-law responses, which are frequently observed in polymers, biological tissues, gels, suspensions, and other complex materials [1, 3, 5].

This review presents a comprehensive analysis of fractional viscoelastic models, with an emphasis on the mathematical foundations of fractional calculus, constitutive modeling, physical interpretation, and numerical approaches. The Caputo, Riemann–Liouville, Caputo–Fabrizio, and Atangana–Baleanu fractional operators have been discussed and compared in terms of their kernel properties, memory behavior, physical significance, and computational characteristics [5, 8, 11, 12]. Furthermore, classical and fractional viscoelastic models, including the Maxwell, Kelvin–Voigt, and Zener models, were examined to illustrate the advantages of fractional-order formulations in representing creep behavior, stress relaxation, and frequency-dependent rheological responses [1, 4, 5].

This review also highlighted recent developments in fractional constitutive equations, fractional Navier–Stokes formulations, multi-term models, variable-order operators, and numerical solution techniques for non-Newtonian fluid dynamics [6, 7, 10]. Comparative analysis has demonstrated that fractional models provide improved agreement with experimental observations compared to traditional rheological formulations, particularly for materials exhibiting long-term memory and anomalous transport behavior [3, 4].

Despite significant progress, challenges associated with parameter estimation, computational complexity, numerical

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stability, and experimental validation continue to limit the widespread application of fractional viscoelastic models [2, 7, 10]. Addressing these challenges remains an important area for future research.

Overall, the literature reviewed in this study confirms that fractional viscoelastic modeling provides a robust and physically meaningful approach for analyzing complex, non-Newtonian fluid systems. The integration of advanced fractional operators, constitutive equations, and numerical methodologies is expected to further enhance the understanding and predictive capabilities of rheological models for a wide range of scientific and engineering applications.

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