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Transforming Mathematics Student Teachers' Wrong Answers into Knowledge and Skill Development Catalysts

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Abstract: This qualitative research focussed on the use of written wrong answers to enhance mathematics student teachers' knowledge and skills development for learning and teaching. Descriptive research design, grounded in the interpretivist paradigm, was adopted. All the sixty-seven (67) third year mathematics student teachers in 2024/2025 academic year were purposively selected to participate in the study. They wrote a quiz on Matrices as part their Mathematics methods course content coverage. Fifty-one (51) scripts displayed correct working and answer while sixteen (16) did not. The 16 scripts were purposively selected for analysis. The main findings, that emerged from the reflective journals and in-depth group discussions show emerged that the analysis of wrong answers contributed to the student teachers' ability to (a) not only notice, but also interpret their peers' Mathematical thinking; (b)deepen their reflection not only about the wrong answers, but also about their own knowledge of the topic (c) focus on their ability to communicate and address wrong answers and; (d)coherently and logically present mathematical ideas orally and in writing. These competencies are critical for their future effectiveness in teaching within the competence-based curriculum. The study concludes that incorrect responses can be productively reframed as opportunities for knowledge and skills development in mathematics teacher education. It is therefore recommended that teacher preparation programmes systematically integrate error-analysis tasks into Mathematics Methods courses. Such analysis of incorrect responses and their underlying reasoning should be recognised as a core pedagogical strategy for fostering professional knowledge and skills development.

Keywords: *Mathematics Student Teachers, Wrong Answers, Knowledge and Skills Development.*

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I. INTRODUCTION

Mathematical calculations come up frequently in teaching and learning Mathematics. In classroom practice, both wrong and right written responses/answers/solutions are inevitable, but there is generally a focus on right answers which seem to promote more the culture of 'rightness.' Often times in this environment, correct answers are rewarded and celebrated, while wrong answers are typically viewed as failures. The wrong answers, generally conceptualised negatively, signify a knowledge gap to be corrected and moved past. This approach, however, overlooks a profound pedagogical opportunity. This is because wrong answers and the incorrect steps leading to the wrong answers can lead to knowledge construction (Vosniadou & Verschaffel, 2004) built through experience and reflection. As Borasi (1996) argues, errors are not endpoints but critical opportunities for deep learning. This study looks at the written wrong answers of pre-service mathematics student teachers (PSMTs), the working leading to it and their strategic use in fostering knowledge and skill development among pre-service mathematics student teachers.

II. LITERATURE REVIEW

➢ *The Potential Value of Wrong Answers*

The potential value of wrong answers, and the working leading to it, in learning is not new, but its application in Mathematics teacher education preparation especially among preservice secondary mathematics teachers (PSMT) in the Zambian context remains underexplored. The pioneering work by Borasi (1996) framed 'errors as springboards for inquiry', suggesting that analysing wrong

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answers and error resulting in the wrong answers can lead to a deeper understanding of mathematical concepts than simply practicing correct procedures. It offers learning opportunities a teacher. Herholdt and Sapire (2014, p.60) argue that while it requires mathematics content and pedagogical content knowledge (PCK) on the part of teachers, it also serves to 'broaden teachers' knowledge of mathematical cognition and concept development'. Degirmenci (2022) and Peng & Luo (2009) also acknowledge the importance of studying the wrong answers. When a learner produces a wrong answer, it reveals a misapplication of a rule, an underlying misconception, or a flawed problem-solving strategy (Swan, 2001). Engaging with these pathways provides a unique window into the learner's mind, offering important diagnostic information for the teacher and a concrete starting point for conceptual repair for the learner. Even though there is value in analysing wrong answers and the working leading to the wrong answers, researcher as Peng & Luo (2009), Kilic (2010), Herholdt and Sapire (2014) also accept that such an analysis can be challenging for teachers of mathematics. This may be argued to qualify the point that teachers of mathematics may need professional development activity with a focus on analysing wrong answers. It also implies that PSMT are prepared to engage in such a task during teacher preparation.

- *The Dual role for Student Teachers*

Mathematics student teachers are simultaneously learners of advanced mathematics and emerging teachers of the subject. An error in their own work can therefore present what may be considered as a two-fold opportunity. First, addressing it solidifies their own subject matter knowledge (SMK), ensuring their conceptual foundations are sound. Secondly, it builds their pedagogical content knowledge (PCK) as presented in Shulman(1986)'s seminal work on teacher knowledge and expounded upon by other researchers including Van Driel, Verloop & De Vos (1998). Being aware of errors can shape both the pedagogical and mathematical content knowledge (Durkaya, Aksu, Öçal, Şenel, Konyalıoğlu, Hızarcı, & Kaplan, 2011). Researchers such as Kontrová, Biba & Šusteková (2022) further argue that explaining and justifying correct and incorrect solutions to problems is more beneficial for achieving better results in mathematics education than justifying the right solutions. Learning is not only directly related to giving correct answer, but also determining the mistakes and incorrect solutions'(Durkaya et al., 2011, p. 2570)-something that student teachers need learn for themselves and for the purpose of modelling it to their future learners. 'Teachers come across errors not only in tests but also in their mathematics classrooms virtually every day'(Sapire, Shalem, Wilson-Thompson & Paulsen, 2016, p.1). This expresses the need to prepare student teachers for dealing with wrong answers, and the steps leading to it and doing so with less challenge (Paulovics & Csapodi, 2025). This study mostly considered the written wrong answers, the working procedure leading to it. The purpose was to consider their strategic use as catalysts for knowledge and skill development among pre-service mathematics student teachers

III. METHODOLOGY

The study used a qualitative descriptive research design grounded in the interpretivist paradigm. Descriptive research design provides a clear, accurate and systematic picture of a phenomenon as it exists. It helps in identifying trends, patterns and relationships providing detailed information about existing conditions (Hassan, 2024; Singh, 2023). As for the interpretivist paradigm, Pervin and Mokhtar (2022), said, that reality is socially and subjectively constructed through social interactions. This paradigm unlike the positivist for quantitative approach and pragmatist for mixed methods approach, in this context, is appropriate because it gives human feelings, beliefs and understanding why things are what they are. In this context researchers mostly considered the written wrong answers, the working leading to it and their strategic use as catalysts for knowledge and skill development among third year (3^{rd}) pre-service mathematics student teachers. All the sixty-seven (67) 3^{rd} year pre-service secondary mathematics student teachers pursuing Bachelor's degree programme, with Mathematics as one of the chosen subjects of specialisation, in the 2024/2025 academic year took part in the study. Their participation started with taking the quiz that was presented and thereafter in analysing and discussing the anonymised scripts with incorrect working and/or answers.

In this context, all students with Mathematics as one of the chosen subject of specialisation take a Mathematics Methods course where they have opportunity to discuss selected secondary school mathematics topics and concepts therein with an extension to how best to teach them. Matrices was one of the topics they covered and the PSMTs wrote a quiz on aspects of Matrices they had covered up to this point. One problem involving multiplication of a matrix by a matrix, specifically multiplication of 1x3 matrix by a 3x2 matrix as shown below in Fig 1 was purposively selected as it was the one that majority of the students did not solve correctly and/or got an incorrect answer.

Express $\begin{pmatrix}-1 & 2 & -3\end{pmatrix}\begin{pmatrix}5 & -2\\ 4 & 2\\ 3 & 1\end{pmatrix}$ as a single matrix.

Fig 1 Quiz Item PSMTs had to Solve

Fifty-one (51) scripts out of the 67 displayed correct working and answer, while sixteen (16) did not. The 16 scripts with wrong answer were purposively selected for analysis after being anonymised. Below is the representative depiction of 6 of the16.

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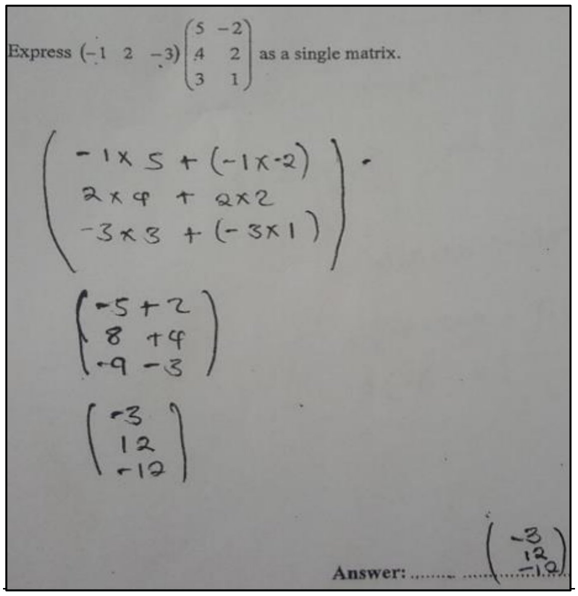

Fig 2 PSMT Working and Solution A

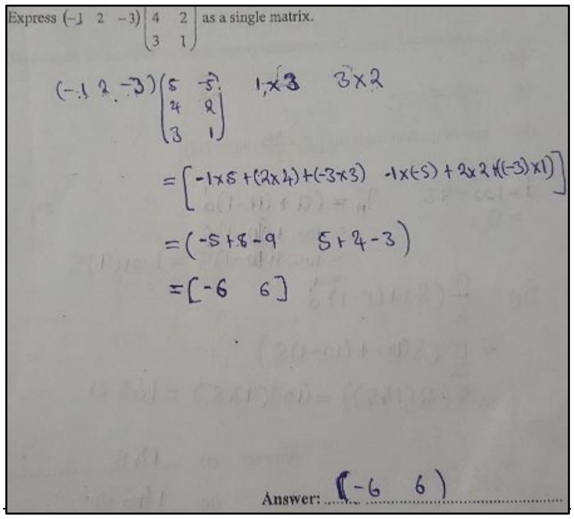

Fig 3 PSMT Working and Solution B

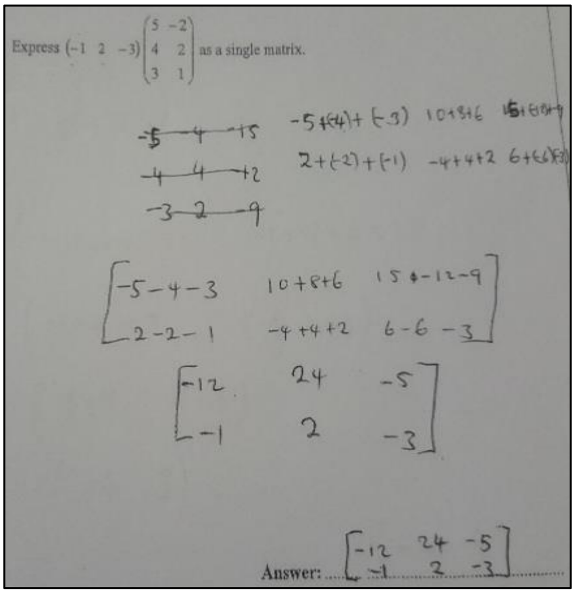

Fig 4 PSMT Working and Solution C

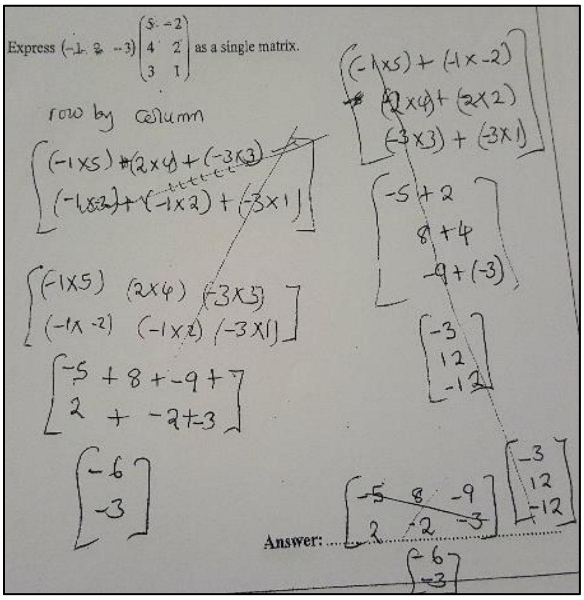

Fig 5 PSMT working and Solution D

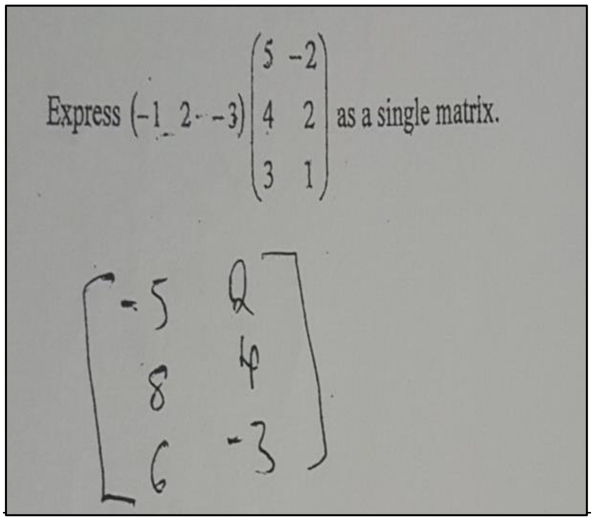

Fig 6 PSMT working and Solution E

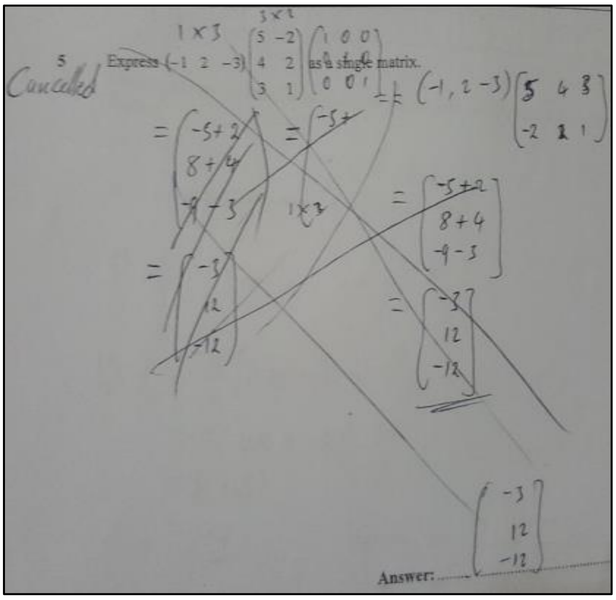

Fig 7 PSMT working and Solution F

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The PSMTs' views about the incorrect answers of the quiz item were collected through the individual reflective notes and in-depth group discussions. They were asked to explain the reason(s) for the incorrect answers, and or the working/steps leading to it. The PSMTs wrote their reflective notes first and thereafter engaged in the discussions which were centred on the specific wrong answers and working identified in their peers' quiz scripts. The data collected were analysed using thematic analysis following the six-phases by Braun and Clarke (2006). Through member checking, audit trail, as well as triangulation trustworthiness was implemented.

For ethical considerations, consent was gotten from all the students. From the students whose scripts where analysed with caution that their privacy was upheld, and as such, their request was upheld. Researchers committed themselves to confidentiality, the reason why there is no mention, by name, of any affected student. Participation in the study for all the PSMTs was entirely voluntary and all PSMTs were fully informed about the scope of the study, the processes involved and their right to withdraw at any stage (Mpolomoka, 2024).

IV. FINDINGS, INTERPRETATION AND DISCUSSION

The purpose was to provide PSMTs an opportunity to carry out an in-depth analysis of peers' wrong responses. The following coding scheme was used: The PSMTs either analysed the wrong answers and possible reasons for the wrong answers (such as for Fig 2, 3, 4 and 5) or could not/had difficulties stating the possible reason(s) for the wrong answers (such as with respect to Fig 6). Then, they either suggested how the problem at hand should have been solved or explained the rules and procedures to be followed to solve the given problem correctly, or suggested a reasonable way different and aside the above-mentioned.

- *Analysed Incorrect Answers and Possible Reasons for Incorrect Answers*

The PSMTs identified different kinds of errors made by their peers and categorised them as such. These were narrowed down to: conceptual mistakes/errors; computational mistakes/errors and including: Careless (calculation-related) mistakes/errors. PSMT3's comment during the discussion with reference to the errors made emphasised that: *'it is important to have the correct understanding of the concepts one needs, like the order of matrices, to use to solve the matrix problem…conceptual error cost everything'*. PSMT8 stated that: *'one can have the correct understanding of concepts, but still get it wrong may be by rushing through the work and making silly or careless mistakes….'*

PSMT's also analysed the wrong answers with respect to the level or depth of the mathematics content knowledge they or their peers had. They emphasised the need to correctly state the order of the matrices and confirm possibility of multiplication before proceeding with the actual multiplication. With reference to Fig 2 and 4 one PSMT1 stated: *'the chances of you getting it wrong when you don't state the order of the matrices and confirm compatibility for multiplication are very high are very high…'*

Other PSMT made reference to their actual experience of analysing wrong answers and the steps that led to them. For instance, PSMT12 wrote: *'I like the thinking going into this kind of task…'*. PSMT6 also indicated that *'it is an eye opener…I never thought I could learn Maths like this…we need more of such as it is making us think deeply about Mathematics and even about ourselves and how we can communicate our understanding to other'*. Overall the PSMTs seemed to have had a positive learning experience.

- *Could not or had Difficulties Stating the Possible Reason(s) for the Wrong Answers*

Some PSMTs indicated that they found it difficult to state reason(s) for the wrong answers. PSMT14, making reference to working similar to what is shown in Fig 6, remarked: *'I am not sure what made my colleague to write down this answer. This is difficult to work with as it requires a lot of thinking and reflection…it would have been easier if the working was shown'*.

Other PSMTs attributed their challenge to state the possible reasons for the wrong answers to their ability to explain. PSMT9 recounted: *'explaining the error to or for your self is easier and doesn't involve much as explaining to someone what the error is'*. As if amplifying PSMT9's comment PSMT6 stated that: *'I know where and what the problem or mistake is, but writing or explaining in words, and in English, is a challenge…it requires deep thinking'*

This opportunity and experience however led some of them to explore how best they needed to communicate. This is evidenced by what one PSMT wrote: *'... I had to find a way of communicating the error, not just the technical words…so constructing my sentences in way that the point is clear enough and the maths words are correctly used too.'* It challenged them to use their knowledge of mathematics, mathematics as a language and use of English as a language of instruction to relay the message or express their thoughts.

Some PSMTs did not show the working and steps leading to it. Based on this some PSMT stated the missed opportunity for their peer to follow instructions that were given to show all the working. PSMT2's comment during the group discussion was: *'we have to follow instructions ourselves before we even talk about helping learners to remember to follow instructions.'* PSMT8, making reference to Fig 6, above stated *'looking at the working helps to know their thinking about the work, misconceptions… and which we could use to support their learning but we have no working to look at here and this makes it difficult for us.'*

Some PSMT extended the discussion to neat and orderly presentation of one's working. For instance, PSMT1 stated that: *'we can make mistakes when calculating, but our work should still be neatly presented'*. PSMT5 added that *'we can cancel work done, but the way we cancel matters a lot…and one should not be left in doubt concerning what has been cancelled and what hasn't'*. As if summing up the discussion PSMT12 stated that: *'writing is a way of communicating and should be done logically and clearly…'* Some of these comments were made with reference to working such as presented under Fig 5 and Fig 7.

This study considered the written wrong answers, and the working procedure leading to the wrong answers. The findings endorse this kind of analysis as it provides learning opportunities for further exploring and advancing both mathematical content knowledge and pedagogical content knowledge (Durkaya et al., 2011). While it is acknowledged its benefits may depend on factors such as how errors are explained or highlighted or on students' prior knowledge and cognitive capabilities Dieterich, Rumann, & Rodemer (2025). It remains to be argued that error analysis not only leads to understanding the ideas driving the errors in the work given, but also encourages students to test the quality of their own understanding of ideas and concepts (Safadi & Hawa, 2024). In addition, the findings are in line with the works of Lin, Fu-Yu Yang, Wu, Yeh, Liao and Chan (2025) who state that knowing the steps leading to wrong answers may provide opportunities for sharpening one's skills including communication skills, critical analysis skills among others. By analysing their own errors, student teachers can develop skills including confidence, decision-making and the metacognitive skills to anticipate and understand the potential errors of their future learner(s). This process can transform a personal learning moment into a lesson not only in content knowledge and instructional strategy, but also in hard and soft skills development. It can thus be concluded that analysis of error in students' own work can present what may be considered as a three-fold learning opportunity aligned with knowledge, and both hard and soft skills. Thus confirming that the strategic use of wrong answers, and steps leading to the wrong answers can be catalysts for knowledge and skill development among pre-service mathematics student teachers. Wrong answer and error-based activities may need to be used more in Mathematics teacher preparation in order help '…prevent the formation of misconceptions, incomplete and/or incorrect information in students or prospective teachers, and to correct existing misconceptions' (Degirmenci, 2022, p.87). This contributes to solidifying their knowledge and teaching skills. The process of analysing the wrong answers, and the step leading to them, can also contribute to the development of relevant competences such as presented in in the competence -based curriculum in Zambia (MOE, 2023) and 21st century skills (Binkley, Erstad, Herman, Raizen, Ripley & Rumble, 2012) including effective communication, resilience, problem solving among others.

V. CONCLUSION

In this study the wrong answers from a quiz on matrices were leveraged as central objects of inquiry. Through reflective notes and in-depth group discussions, the pre service Mathematics student teachers were guided to deconstruct their errors and articulate the resulting details. This paper argues that such a process did more than rectify misunderstandings, but contributed to enhancing knowledge related both to Mathematics content and how to teach the content, simultaneously developing essential soft skills such as resilience, communication, and critical thinking. It emerged that the analysis of wrong answers contributed to the students teachers' ability to: (a) not only notice, but also interpret their peers' Mathematical thinking; (b)deepen their reflection not only about the wrong answers, but also about their own knowledge of the topic (c) focus on their ability to communicate and address wrong answers and; (d) coherently and logically present mathematical ideas orally and in writing, all of which are essential for Mathematics student teachers' future success in teaching and implementing the now new competence-based-curriculum. It is concluded that wrong answers can be reframed as knowledge and skills development opportunities for mathematics student teachers. The preservice teachers may need to be given more opportunities to engage with wrong answers in the methods courses for the valuable learning opportunities it offers.

RECOMMENDATIONS

One of the recommendations is that Mathematics teacher preparation programmes should institutionalise error analysis. They should formally embed error-analysis tasks into their methods modules/courses. Analysing wrong answers and their underlying reasoning should be treated as a core pedagogical strategy for consolidating pre-service teachers' subject matter and instructional content knowledge. Secondly, Mathematics teacher preparation programmes should aim at strengthening mathematical communication competencies. We recommend that curriculum designers and teacher educators should stress the importance of developing student teachers' ability to clearly explain, justify and communicate mathematical errors using appropriate mathematical language, both orally and in writing. The use of promote the use of reflective journals and integrative discussions focused on incorrect solutions should also be promoted, as these practices can foster critical thinking, metacognition, resilience and readiness to address learner misconceptions in school classrooms.

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