

A Group-Action Framework for Intelligent Traffic Signal Control and Urban Infrastructure Coordination

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Abstract: Urban traffic congestion is a dynamic systems problem involving interacting traffic streams, signal phases, queues, road capacity and changing demand. This paper develops a mathematically explicit Group-Theoretic framework in which traffic states are acted upon by a set of admissible traffic-control transformations. The key object is therefore not the set of raw traffic states itself, but a transformation group (or, where physical constraints prevent closure or invertibility, an admissible transformation structure) acting on the traffic-state space. Closure, associativity, identity and inverse are interpreted through composition, sequencing, no-change operation and restoration of compatible traffic-control transformations. A four-arm intersection model is formulated using queue-length vectors and signal-control transformations. A performance functional based on queue, delay and stopping cost is then coupled with the group action so that the mathematical structure organizes candidate control operations while an optimization rule selects an operationally feasible action. The framework is compatible with established group-based signal timing, adaptive signal control, queueing and optimization literature. The paper also identifies an important limitation: not every real-world traffic-control operation has an inverse, and therefore a strict group should only be defined over transformations satisfying the required algebraic properties. The proposed framework is intended as a foundation for simulation and field validation rather than as an experimentally proven traffic-improvement method.

Keywords: Group Theory; Group Action; Intelligent Transportation Systems; Traffic Signal Control; Urban Infrastructure; Adaptive Signal Control; Queue Length; Mathematical Modelling; Traffic Optimization.

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I. INTRODUCTION

Urban traffic networks contain multiple interacting components whose operating conditions change continuously. Signalized intersections are particularly important because vehicles arriving from different approaches compete for limited right-of-way. Modern traffic-control research therefore combines signal timing, queue estimation, optimization and real-time information. Group-based signal timing is already an established concept in traffic engineering: prior studies formulate signal settings through group-based control variables and optimize cycle time, start time and green duration. [1,2]

The present paper addresses a different mathematical question: whether the algebraic structure of Group Theory can be used to formalize the transformations by which a

traffic-control system changes from one admissible configuration to another. The proposed answer is based on a group action. Let S be a set of traffic states and G a set of admissible transformations. The action $G \times S \rightarrow S$ describes how a control operation changes a traffic state.

This distinction is essential. A vector of queue lengths such as (20, 30, 40, 50) does not automatically form a group under a physically meaningful traffic operation. Instead, Group Theory is applied to transformations acting on traffic states. This provides a mathematically defensible bridge between abstract algebra and traffic-control engineering.

II. LITERATURE REVIEW

Existing research demonstrates several mature mathematical approaches to traffic signal control. Wong

developed group-based optimization of signal timings using the TRANSYT traffic model, treating common cycle time, start time and green duration as group-based control variables. [1] A later time-dependent extension used group-based variables and a performance index based on estimated delay and stops, with simulation-based evaluation. [2]

Lee et al. developed a hierarchical adaptive group-based traffic-signal-control formulation combining proactive global optimization with local reactive adjustment based on real-time traffic information. [3] Their work demonstrates that group-based signal control can be integrated with adaptive methods rather than being restricted to fixed-time operation.

These studies establish the engineering relevance of the term 'group-based' in traffic-signal control. The present paper uses Group Theory in the algebraic sense—groups, transformations and group actions—to propose a complementary mathematical layer. The contribution is therefore not the claim that group-based traffic control is new, but the formulation of traffic-control transformations through algebraic group actions.

➤ Research Objectives

- To formulate a mathematically consistent representation of traffic-control transformations using Group Theory.
- To interpret closure, associativity, identity and inverse in the context of admissible traffic-control operations.
- To define a group action on a traffic-state space for a signalized intersection.
- To couple the algebraic model with an objective function involving queue and delay.
- To provide a framework suitable for simulation and later field validation.

➤ Mathematical Preliminaries

Let S denote the traffic-state space. For a four-arm intersection, a basic state may be represented as:

$$s = (q_N, q_S, q_E, q_W)$$

Where q_N , q_S , q_E and q_W are queue-length measures on the four approaches. Additional variables such as arrival rates, turning ratios, occupancy and signal phase may be appended when data are available.

Let G be a set of admissible transformations $g: S \rightarrow S$. A transformation may represent a change in signal priority, green allocation, phase sequence, lane priority or another control parameter. The action is written as:

$$G \times S \rightarrow S, (g, s) \mapsto g \cdot s.$$

If G is a group, it satisfies the four standard axioms under composition.

III. INTERPRETATION OF THE FOUR GROUP PROPERTIES

➤ Closure

$$g_1, g_2 \in G \Rightarrow g_1 \circ g_2 \in G.$$

If two admissible control transformations can be composed and the resulting operation remains admissible, closure holds. For example, a permissible green-time adjustment followed by a permissible priority adjustment must produce another permitted controller configuration.

➤ Associativity

$$(g_1 \circ g_2) \circ g_3 = g_1 \circ (g_2 \circ g_3).$$

Associativity allows a sequence of compatible transformations to be grouped without changing the resulting transformation. This is useful when control actions are coordinated across several stages or intersections.

➤ Identity

$$e \circ g = g \circ e = g.$$

The identity e represents the no-change operation. When the current traffic-control configuration is acceptable, the controller may retain the existing configuration.

➤ Inverse

$$g \circ g^{-1} = g^{-1} \circ g = e.$$

An inverse represents restoration of a compatible control configuration. For example, a temporary priority transformation introduced for an emergency movement may be followed by a restoration transformation. In real traffic systems, however, a strict inverse exists only when the control operation is reversible and all relevant state information is preserved.

➤ Proposed Group-Action Traffic Model

Consider a four-arm intersection $I = \{N, S, E, W\}$. Define the state vector

$$s_t = (q_N(t), q_S(t), q_E(t), q_W(t)).$$

Let $G = \{e, g_1, g_2, \dots, g_r\}$ be a set of admissible control transformations. Examples include a transformation increasing East-West green allocation within safety constraints, a transformation increasing North-South priority, a phase-sequence transformation, and a temporary emergency-priority transformation.

For each candidate transformation g , the controller predicts

$$s_{t+1}(g) = g \cdot s_t.$$

The proposed model then evaluates each candidate state using a performance functional.

➤ Performance Functional

$$J(s) = w_Q Q(s) + w_D D(s) + w_S S_p(s) + w_C C(s)$$

Where Q is total queue, D is aggregate delay, S_p is a stopping-related measure and C is a congestion/conflict penalty; the weights w_Q, w_D, w_S, w_C are selected according to the study design. The candidate transformation is chosen by an optimization rule such as:

$$g^* = \arg \min_{\{g \in G_{feasible}\}} J(g \cdot s_t).$$

The subscript feasible is important: traffic-safety, minimum green, inter-green, pedestrian and controller constraints must be satisfied. Group Theory supplies the

transformation structure; the performance criterion supplies the decision rule.

➤ Illustrative Numerical Example

Consider an illustrative intersection state

$$s = (20, 25, 90, 85).$$

The total queue is 220 vehicle-equivalent units under the chosen measurement definition. Suppose three candidate transformations are available: g_1 (increase North-South priority), g_2 (increase East-West priority) and e (retain the current plan). For illustration only, suppose a simulation model returns the following objective values:

Table 1 Illustrative Numerical Example

Candidate	Illustrative J value
g_1	245
g_2	180
e	220

Under the stated objective, g_2 would be selected in this illustrative example. These numbers are hypothetical and are not presented as experimental evidence. In a journal study they must be replaced by outputs from a calibrated traffic simulator or field dataset.

➤ Network-Level Extension

For a road network $N=(V,E)$, let $V=\{I_1, \dots, I_n\}$ denote intersections and E the connecting road links. Each intersection may have a local transformation structure G_i . Coordinated control can then be represented by tuples of compatible local transformations. The framework can incorporate constraints between adjacent intersections so that a change at one junction is consistent with downstream capacity and signal coordination.

$$s_{t+1} = (g_1, \dots, g_n) \cdot s_t.$$

This extension permits the framework to address corridor coordination, queue spillback and network-level control, but the exact algebraic structure must be established from the admissible transformation set rather than assumed.

➤ Urban Infrastructure Applications

- Lane reallocation: transformations can represent permitted changes in lane-use configuration.
- Public-transport priority: a transformation can temporarily allocate signal priority to buses or other designated movements.
- Emergency corridors: temporary priority transformations can be represented and followed by restoration operations when reversibility conditions hold.
- Intersection coordination: compatible transformations can be composed across neighbouring junctions.
- Road-network symmetry: structurally equivalent intersections can be mapped through symmetry

transformations, subject to capacity and safety constraints.

➤ Proposed Algorithm

- Step 1. Acquire real-time traffic measurements.
- Step 2. Construct the current traffic state s_t .
- Step 3. Generate the set of safety-feasible transformations $G_{feasible}$.
- Step 4. Predict the state resulting from each transformation.
- Step 5. Evaluate $J(g \cdot s_t)$ for every candidate.
- Step 6. Select the feasible transformation satisfying the chosen optimization objective.
- Step 7. Apply the signal/control action.
- Step 8. Measure the resulting state and update the model.

➤ Research Hypotheses

- H1: A group-action representation can provide a mathematically consistent framework for composing compatible traffic-control transformations.
- H2: Coupling the transformation framework with a traffic-performance objective can support systematic selection among feasible control actions.
- H3: The proposed framework can be implemented in a simulation environment and compared with a fixed-time or existing adaptive controller.

These are research hypotheses. They should not be reported as confirmed until simulation or field experiments are completed.

➤ Validation Plan

- Select one or more real signalized intersections.
- Collect traffic volume, queue length, signal timing, turning movement and speed data.

- Calibrate a microscopic or macroscopic traffic model.
- Implement the proposed transformation set.
- Run baseline and proposed-control scenarios under identical demand.
- Compare average delay, maximum queue, throughput, stops and travel time.
- Perform sensitivity analysis under low, medium and high demand.
- Report statistical uncertainty and reproducibility details.

IV. LIMITATIONS

- Not every real-world traffic operation is reversible; therefore, a strict group may need to be restricted to a reversible subset of transformations.
- Traffic dynamics are continuous and stochastic, whereas a simple group model may be discrete.
- Safety constraints can prevent algebraic closure of a naive transformation set.
- The illustrative numerical example is hypothetical and must not be presented as empirical validation.
- Field deployment requires traffic-engineering standards, controller compatibility, pedestrian safety provisions and regulatory approval.

V. DISCUSSION

The principal mathematical contribution is the separation of traffic states from transformations acting on those states. This avoids treating queue-length vectors as a group without a justified operation. The group-action formulation also provides a common language for local signal changes, coordinated control and temporary interventions.

The framework is complementary to established group-based traffic-signal optimization rather than a replacement for it. Prior literature already demonstrates group-based signal timing and adaptive group-based control. [1–4] The proposed research adds an abstract-algebraic interpretation that can be tested computationally.

VI. CONCLUSION

This paper developed a Group-Action Framework for Intelligent Traffic Signal Control and Urban Infrastructure Coordination. The framework represents traffic conditions by a state space S and admissible control operations by transformations G acting on S . Closure, associativity, identity and inverse provide precise mathematical interpretations for composition, sequencing, no-change operation and restoration of compatible control actions.

The model is intentionally presented as a research framework rather than as a claim of proven traffic improvement. Its next scientific step is simulation and empirical validation. A successful implementation would provide quantitative evidence regarding whether the proposed transformation-based control layer can reduce

delay, queue accumulation or unnecessary stops under defined traffic conditions.

➤ Author Note

The mathematical framework in this manuscript is proposed for further computational and empirical investigation. Any future journal submission should include the target journal's required formatting, declarations, data-availability statement, ethics statement where applicable, and experimentally generated results.

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