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# Stability of Poiseuille Flow of a Weakly Viscoelastic Fluid Through a Horizontal Cylindrical Pipe Subjected to a Longitudinal Surface Current Density

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**Abstract: In this article, we study the stability of the Poiseuille flow of a weakly viscoelastic diamagnetic fluid through a horizontally oriented cylindrical pipe. The pipe in question is traversed by a longitudinal surface current density. The problem obtained is an eigenvalue problem which we will attempt to solve using the Q-Z algorithm. A Fourrier Petrov Galerkin spectral method is described for high accuracy computation of linearized dynamics for flow in a circular pipe. The code used here is based on solenoidal velocity variables and is written in PYTHON. The study will be conducted by manipulating certain flow parameters, such as the dimensionless numbers or the delay parameter . The flow is considered unstable if when the real part of at least on of the eigenvalues is positive, and stable otherwise.**

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## I. INTRODUCTION

Linear flow stability refers to the fluid's ability to maintain its regime in the face of small disturbances. When disturbances dampen over time, the flow is said to be stable. It is said to be unstable when disturbances amplify. The main purpose of studying stability is to determine the precise moment when the flow regime changes.

For a Newtonian fluid, this change in regime is determined by the Reynolds number Re and occurs from Re = 2000 in experimental studies. However, this flow is theoretically and numerically stable regardless of the value of Re. These two facts constitute what is known as the Reynolds paradox.

In our study, where the fluid is weakly viscoelastic and immersed in a magnetic field, other control parameters such as the Weissenberg number and the new dimensionless number will be considered. We derive from elasticity and Ne from the magnetic field. Both regimes (stability or instability) are of technical interest in industry.

Regarding stability, for example, it helps reduce energy losses because laminar flow significantly reduces friction against the walls. It can also help reduce pressure losses, thus lowering the power required by pumps in pipelines. Instability, on the other hand, allows for better and more continuous fluid mixing, and it also promotes thermal efficiency in heat exchangers.

The flows in the pipes are presented in several industrial applications such as heat exchangers, chemical rectors, gas turbine cooling and heating systems and combustion chambers or mixing systems.

Magnetic body forces act on all materials but are generally very weak for paramagnetic and diamagnetic materials. These forces derive from a specific magnetic energy acquired by the material in the presence of an applied magnetic field.

The effects of intense magnetic fields on matter are indeed multiple and arouse the curiosity of researchers. Quantum [1], thermodynamic [2], mechanical [3,4] effects that are imperceptible at low fields are observed in matter under strong magnetic fields. The term magneto-science is sometimes used to group together all the fields application.

One of the fields of magneto-science is the study of fluids under intense magnetic fields. The outstanding

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applications generally concern magnetohydrodynamics and magnetic levitation. In the first case, the fluid is necessarily driver. Currents that flow through fluid placed in magnetic fields intense generate Laplace forces allowing the movement of the fluid. In the second case, the fluid used has no magnetic or electrical properties particular. Volume magnetic forces created by magnetic devices make it possible to locally compensate for gravity and thus to maintain in levitation a fluid over a given volume.

The magnetic volume force will be able to act on the flow of the liquid provided that the term associated with the volume forces is of the same order of magnitude as the other terms of the equation. We will carry out a dimensionless study of this problem to show that it is possible to act on the flow of the fluid which we will consider as being viscoelastic and not conductive.

Existing superconducting devices that act on diamagnetic or paramagnetic non-conductive liquids are system designs for magnetic levitation. They most often result from a problem of inverse synthesis: starting from a desired force field (the uniformity of the force field over the largest volume is often the greatest objective), the magnetic field is determined as well as associated magnetic field sources. We will be interested in the effects that volumetric forces can have on the flow of a fluid and more particularly on the stability of the flow of such a fluid.

## II. EVALUATION OF THE STRESS OF MAGNETIC FIELD

The fluid being neither charged nor conductive, we have the relation $rot\vec{B} = \vec{0}$, knowing that $\vec{B} = rot\vec{A}$, $\vec{A}$ being the vector potential.

$$\overrightarrow{rot}\left(\overrightarrow{rot}\vec{A}\right) = \overrightarrow{grad}\left(div\vec{A}\right) - \vec{\Delta}\vec{A} = \vec{0} \quad (1)$$

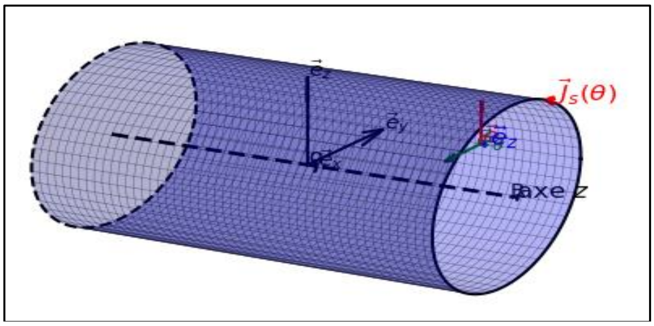

Fig 1 Infinite Cylinder Through which Our Fluid Flows

We are using a stationary current distribution here. The Lorenz's gauge condition entails $div\vec{A} = 0$. We can then determine $\vec{A}$ using the relation $\vec{\Delta}\vec{A} = \vec{0}$. The solenoid is considered very thin, it's radius R and the surface current density is of the form:

$$\vec{j}(\theta)\vec{e}_z = J_0 \, sin(p_1\theta)\vec{e}_z \quad (2)$$

The problem is invariant by translation following $\vec{e}_z$. The magnetic and mechanical quantities therefore only depend on r and $\theta$. any plane containing $\vec{e}_z$ and $\vec{e}_\theta$ is an antisymmetric plane. The potential vector $\vec{A}$ is perpendicular to the antisymmetry planes and will therefore be according to $\vec{e}_z$ only. We will denote it $\vec{A}_z$. The magnetic field belongs to the antisymmetry planes and will consequently have components according to $\vec{e}_r$ and $\vec{e}_\theta$.

In cylindrical coordinate:

$$\vec{\Delta}\vec{A}_z = \vec{0} \Leftrightarrow \frac{\partial^2 A_z}{\partial r^2} + \frac{\partial^2 A_z}{\partial z^2} + \frac{1}{r}\frac{\partial A_z}{\partial r} - \frac{A_z}{r^2} = 0 \quad (3)$$

As

We will denote by " int " the inside of the solenoid and by " ext " the outside.

$$\vec{B} = \frac{1}{r}\frac{\partial A_z}{\partial \theta}\vec{e}_r - \frac{\partial A_z}{\partial r}\vec{e}_\theta$$

The continuity of the normal component of the magnetic field at $r = R$ is written:

$$\left(\frac{\partial A_{zint}}{\partial \theta}\right)_{r=R} = \left(\frac{\partial A_{zext}}{\partial \theta}\right)_{r=R} \quad (4)$$

The discontinuity of the tangential component of the magnetic field at $r = R$ imposes

$$\left(\vec{B}_{int} - \vec{B}_{ext}\right).\vec{e}_\theta = \mu_0 J_0 \, sin(p_1\theta) \quad (5)$$

The vector potential is in the form:

$$\vec{A}_z = \left(\alpha r^{p_1} + \beta r^{-p_1}\right)\left(\gamma cos\left(p_1\theta\right) + \kappa sin\left(p_1\theta\right)\right)$$

α, β, γ and κ are constants that will be determined by boundary conditions and the matching conditions.

On obtient après calculs:

$$\vec{A}_{zint} = \frac{\mu_0 J_0}{2p}\frac{r^{p_1}}{R^{p_1-1}} sin\left(p_1\theta\right)\vec{e}_z$$

$$\vec{A}_{zext} = \frac{\mu_0 J_0}{2p}\frac{R^{p_1+1}}{r^{p_1}} sin\left(p_1\theta\right)\vec{e}_z$$

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Either:

$$\vec{B}_{int} = \begin{pmatrix} \dfrac{\mu_0 J_0}{2}\left(\dfrac{r}{R}\right)^{p_1-1} cos\left(p_1\theta\right) \\ -\dfrac{\mu_0 J_0}{2}\left(\dfrac{r}{R}\right)^{p_1-1} sin\left(p_1\theta\right) \\ 0 \end{pmatrix}$$

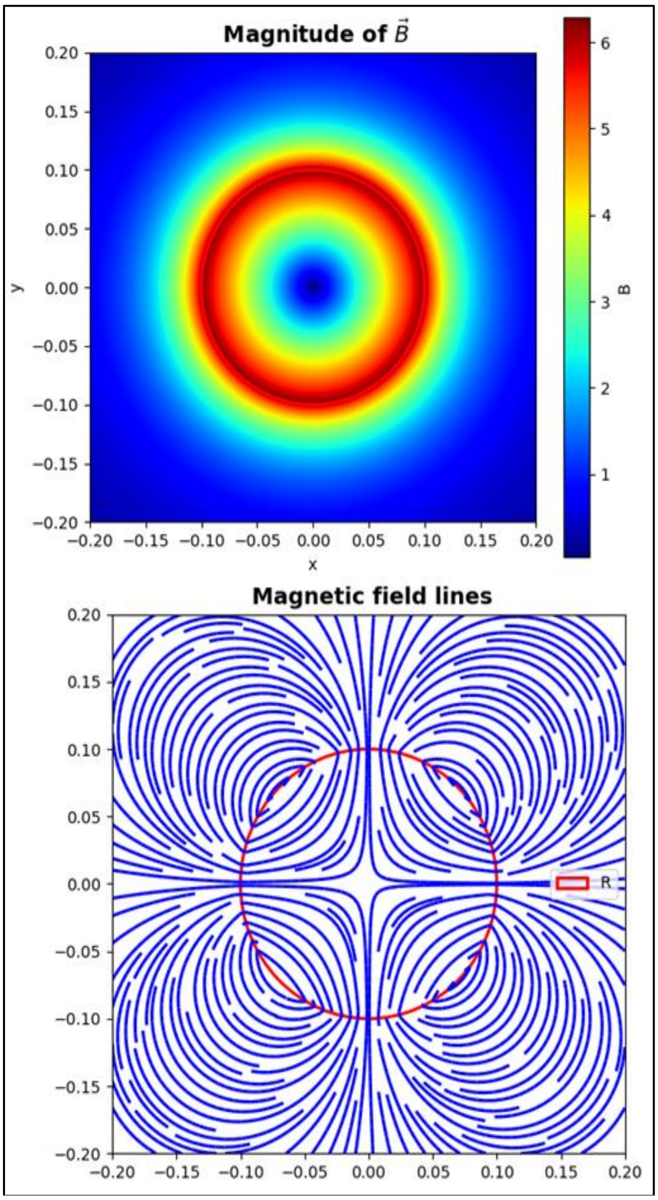


Fig 2 Magnitude of B (up) and Magnetic Field Lines (Down) with $J_0 = 10^5\, A/Cm$, R=10Cm and $p_1 = 2$

The volume magnetic force inside the solenoid is:

$$\vec{f}_{mv}^{\,int} = \frac{\chi\mu_0 j_0^{\,2}\left(p_1-1\right)}{4r}\left(\frac{r}{R}\right)^{2p_1-2}\vec{e}_r$$

On the outside, it is given by:

$$\vec{f}_{mv}^{\,ext} = -\frac{\chi\mu_0 j_0^{\,2}\left(p_1+1\right)}{4r}\left(\frac{R}{r}\right)^{2p_1+2}\vec{e}_r$$

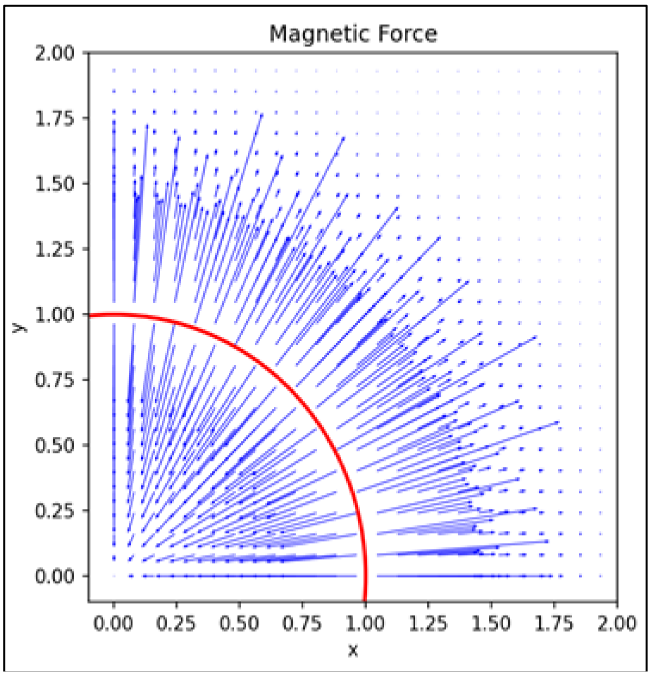


Fig 3 Magnetic Vector Field with $J_0 = 10^5\, A/Cm$, R=10Cm , $\chi = -10^{-6}$ and $p_1 = 2$

The tensor of the magnetic actions in the solenoid is given by:

$$\left\{\overline{\overline{T}}_{int}, \vec{e}_i\right\} = \frac{\chi\mu_0}{8} J_0^{\,2}\left(\frac{r}{R}\right)^{2p_1-2}\begin{pmatrix} 2cos^2\left(p_1\theta\right) & -sin\left(2p_1\theta\right) \\ -sin\left(2p_1\theta\right) & 2sin^2\left(p_1\theta\right) \end{pmatrix}, i = \{r, \theta\}$$

On the Outside

$$\left\{\overline{\overline{T}}_{ext}, \vec{e}_i\right\} = \frac{\chi\mu_0}{8} J_0^{\,2}\left(\frac{r}{R}\right)^{2p_1+2}\begin{pmatrix} 2cos^2\left(p_1\theta\right) & sin\left(2p_1\theta\right) \\ sin\left(2p_1\theta\right) & 2sin^2\left(p_1\theta\right) \end{pmatrix}, i = \{r, \theta\}$$

We can therefore see that it is inside or outside the solenoid, the tensor of the magnetic actions always comprises a tangential component. This tensor is also symmetrical.

$$\overline{\overline{T}}_{int}^{\,r\theta} = -\frac{\chi\mu_0}{8} J_0^{\,2}\left(\frac{r}{R}\right)^{2p_1-2} sin\left(2p_1\theta\right)$$

$$\overline{\overline{T}}_{ext}^{\,r\theta} = \frac{\chi\mu_0}{8} J_0^{\,2}\left(\frac{r}{R}\right)^{2p_1+2} sin\left(2p_1\theta\right)$$

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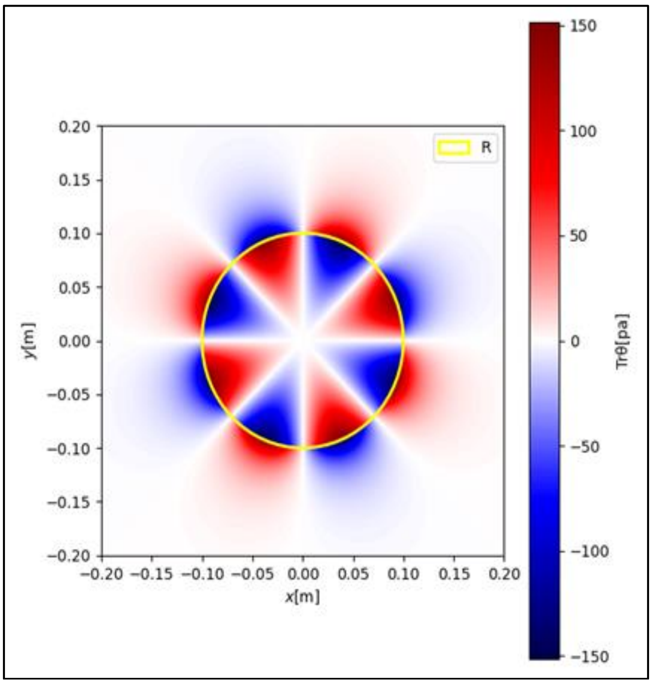


Fig 4 Tensor of Magnetic Actions with p1=2

## III. EQUATIONS GOVERNING THE FLOW AND STUDY STRATEGY

To fully account for the effects generated by the stress of magnetic actions, we will first consider the flow of laminar viscoelastic fluid in absence of magnetic field. The equations governing such flow are given by the system below.

$$\begin{cases} div\vec{u} = 0 \\ \dfrac{D\sigma}{Dt} + \dfrac{\sigma}{\lambda} = \dfrac{2\eta\omega}{\lambda} D(\vec{u}) \\ \rho\left\{\partial_t \vec{u} + (\vec{u}.\vec{\nabla})\vec{u}\right\} = -\vec{\nabla}p + \eta\Delta\vec{u} + div\sigma + \rho\vec{g} \end{cases} \qquad (6)$$

Secondly, we apply a surface and longitudinal current density to the surface of the cylinder $\vec{j} = j(\theta).\vec{e_z}$.

Assuming that the stress of the magnetic actions has a longitudinal component, one can imagine a modification of the velocity $\vec{u}$ and pressure p fields such as $\vec{u}_m = \vec{u} + \vec{u}'$, $p_m = p + p'$ and $\sigma_m = \sigma + \sigma'$The equations governing this new flow are as follows:

$$\begin{cases} div\vec{u_m} = 0 \\ div\vec{B} = 0 \\ \dfrac{D\sigma}{Dt} + \dfrac{\sigma}{\lambda} = \dfrac{2\eta\omega}{\lambda} D(\vec{u_m}) \\ \rho\left\{\partial_t \vec{u}_m + (\vec{u_m}.\vec{\nabla})\vec{u_m}\right\} = -\vec{\nabla}p_m + \eta(1-\omega)\Delta\vec{u_m} + \dfrac{\chi}{\mu_0}\overrightarrow{grad}\left(\dfrac{B^2}{2}\right) \end{cases} \qquad (7)$$

By subtracting from this system, that satisfied by the viscoelastic fluid not subjected to any magnetic field, it comes:

$$\begin{cases} div\vec{u_m} = 0 \\ div\vec{B} = 0 \\ \dfrac{D\sigma}{Dt} + \dfrac{\sigma}{\lambda} = \dfrac{2\eta\omega}{\lambda} D(\vec{u_m}) \\ \rho\left\{\partial_t u' + (\vec{u}'.\vec{\nabla})\vec{u}' + (\vec{u}.\vec{\nabla})\vec{u}' + (\vec{u}'.\vec{\nabla})\vec{u}\right\} = -\vec{\nabla}p' + \eta(1-\omega)\Delta\vec{u}' + \dfrac{\chi}{\mu_0}\overrightarrow{grad}\left(\dfrac{B^2}{2}\right) \end{cases} \qquad (8)$$

In order to generalize the study to several situations, we introduce the dimensionless parameters.

$$\tilde{p} = \frac{p}{\rho u_0^2} \quad ; \tilde{\vec{u}} = \frac{\vec{u}}{u_0} \quad ; \tilde{r} = \frac{r}{R} \quad ; \ \tilde{z} = \frac{z}{R} \quad ; \tilde{t} = \frac{tu_0}{R}; \quad \tilde{\nabla} = R\nabla \ ; \tilde{\mathbf{B}} = \frac{B}{B_0}$$

By replacing each variable by its expression in the previous system, we obtain the following dimensionless equations:

$$\begin{cases} div\vec{u}' = 0 \\ \dfrac{\partial\boldsymbol{\sigma}'}{\partial t} + u'\dfrac{\partial\sigma_b}{\partial r} + \mathbf{W}_b\dfrac{\partial\sigma'}{\partial z} + g_a(\vec{u}', \sigma_b) + g_a(\overrightarrow{W}b, \boldsymbol{\sigma}') + \dfrac{\boldsymbol{\sigma}'}{We} = \dfrac{2\omega}{Re.We} D(\vec{u}') \\ \dfrac{\partial\vec{u}'}{\partial t} + (\vec{u}'.\vec{\nabla})\vec{u}' + (\vec{u}.\vec{\nabla})\vec{u}' + (\vec{u}'.\vec{\nabla})\vec{u} = \vec{\nabla}\left(-p' + Ne\left(\dfrac{B^2}{2}\right)\right) + \dfrac{1-\omega}{R_e}\Delta\vec{u}' + \vec{\nabla}.\sigma' \end{cases} \qquad (9)$$

Taking into account certain simplifying assumptions due to the linearity of the problem, system (9) yields :

$$\begin{cases} div\vec{u}' = 0 \\ \dfrac{\partial\vec{u}'}{\partial t} + (\vec{u}'.\vec{\nabla})\vec{u}' + (\vec{u}.\vec{\nabla})\vec{u}' + (\vec{u}'.\vec{\nabla})\vec{u} = \vec{\nabla}\left(-p' + Ne\left(\dfrac{B^2}{2}\right)\right) + \dfrac{1}{R_e}\Delta\vec{u}' + \vec{\nabla}.\sigma^e \end{cases} \qquad (10)$$

$R_e = \dfrac{\rho u_0 R}{\eta}$ is the Reynolds' number. It is the ratio of inertia effects to viscous effects.

$N_e = \dfrac{\chi B_0^2}{\rho\mu_0 u_0^2}$ is a dimensionless quantity, ratio between the magnetic effects and the effects of inertia.

$We = \dfrac{\lambda u_0}{R}$ is the Weissenberg's number. It is ratio of elastic effect and the effects of inertia.

Inside the cylinder, the projection of the conservation equation on the axis gives:

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$$\begin{cases} \dfrac{\partial u'}{\partial t} = -W_b\dfrac{\partial u'}{\partial z} - u'\dfrac{\partial u'}{\partial r} - \dfrac{v'}{r}\dfrac{\partial u'}{\partial \theta} + \dfrac{v'^2}{r} - w'\dfrac{\partial u'}{\partial z} - \dfrac{\partial p'}{\partial r} + \dfrac{1}{R_e}\left(\Delta u' - \dfrac{u'}{r^2} - \dfrac{2}{r^2}\dfrac{\partial v'}{\partial_\theta}\right) + N_e\dfrac{\chi\mu_0 j_0^2\left(p_1-1\right)}{4r}\left(\dfrac{r}{R}\right)^{2p_1-2} + div\,\sigma^e.\vec{e}_r \\ \dfrac{\partial v'}{\partial t} = -W_b\dfrac{\partial v'}{\partial z} - u'\dfrac{\partial v'}{\partial r} - \dfrac{u'v'}{r} - \dfrac{v'}{r}\dfrac{\partial v'}{\partial \theta} - w'\dfrac{\partial v'}{\partial z} - \dfrac{1}{r}\dfrac{\partial p'}{\partial \theta} + \dfrac{1}{R_e}\left(\Delta v' - \dfrac{v'}{r^2} + \dfrac{2}{r^2}\dfrac{\partial u'}{\partial_\theta}\right) + div\,\sigma^e.\vec{e}_\theta \\ \dfrac{\partial w'}{\partial t} = -W_b\dfrac{\partial w'}{\partial z} - u'\dfrac{\partial W_b}{\partial r} - \dfrac{v'}{r}\dfrac{\partial w'}{\partial \theta} - u'\dfrac{\partial w'}{\partial r} - w'\dfrac{\partial w'}{\partial z} - \dfrac{\partial p'}{\partial z} + \dfrac{1}{R_e}\Delta w' + div\,\sigma^e.\vec{e}_z \\ \dfrac{1}{r}\dfrac{\partial(ru')}{\partial r} + \dfrac{1}{r}\dfrac{\partial v'}{\partial_\theta} + \dfrac{\partial w'}{\partial_z} = 0 \end{cases} \tag{11}$$

## IV. PROCEDURE AND NUMERICAL BASE

We consider that the variation of the perturbation of the fields of velocity, pressure is periodic along the azimuthal and axial directions. These considerations make it possible to approximate u in the following form.

$$u_s\left(r,\theta,z;t\right) = \sum_{l=-L}^{L}\sum_{n=-N}^{N}\sum_{m=0}^{M}\alpha_{mnl}\left(t\right)_{mnl}\left(r\right)e^{i(n\theta+lk_0z)} \tag{12}$$

This approximation was used by Meseguer & Trefethen 2001[5].

The continuity equation leads to a linear dependence between the three components of $u(r,\theta,z)$ leading to a system with two degrees of freedom. Therefore, we note:

$$u_s\left(r,\theta,z;t\right) = \sum_{l=-L}^{L}\sum_{n=-N}^{N}\sum_{m=0}^{M}\left[\alpha_{mnl}^{(j)}\left(t\right)\phi_{mnl}^{(j)}\left(r\right)\right]e^{i(n\theta+lk_0z)} \tag{13}$$

For $j = (1,2)$, the substitution of the expression of $u_s$ in the equation (10) the problem result in a system of ordinary differential equations of coefficients $\alpha_m^{(j)}\left(t\right)$. It should be noted that the expression of $\alpha_m^{(j)}\left(t\right)$ for a at long times is $e^{Ct}$ ., where $c = c_r + ic_i$

This substitution is followed by a projection based on the scalar product:

$$\left(f,g\right) = \int_{-1}^{1} w(r)\,f^*.g\,dr \tag{14}$$

Where $f^*$ designates the conjugated function of $f$.

The basic functions will be chosen so that integrand is even, which will result in:

$$\int_{-1}^{1} G\left(r\right)dr = 2\int_{0}^{1} G\left(r\right)dr \tag{15}$$

The basic functions are also chosen so that the projection of the pressure and magnetic terms are zero. For this purpose functions based on Chebyshev polynomial were considered. This choice imposes two essential criteria. The first consists of choosing weights associated with Chebyshev polynomials, which makes it easier to calculate the scalar product. The second criterion relates to the approximation of the integral by a Gauss- Chebyshev-Lobatto quadrature of the form:

$$\left(f,g\right) = \int_0^1 w(r)\,f^*.g\,dr \approx \sum_{j=0}^{M} w\left(r_j\right)f^*\left(r_j\right)g\left(r_j\right) \tag{16}$$

$$f = \phi_{m,n,l}^{(1,2)} \quad and \quad g = \psi_{m,n,l}^{(1,2)}$$

We define,

$$h_m\left(r\right) = \left(1-r^2\right)T_{2m} \quad ,$$

$$g_m\left(r\right) = \left(1-r^2\right)^2 T_{2m}$$

$T_{2m}$ is the Chebyshev polynomial defined by

$$T_{2m} = \cos(2m\theta)$$

The basic functions and tests were proposed by Leonard & Wray 1982 [6] then used by Meseguer & Trefethen 2001 [5] and are defined as follows:

- *Basic Functions*

1[st] case: $n = 0$

$$\phi_{m,l,n}^{(1)} = \begin{pmatrix} 0 \\ rhm(r) \\ 0 \end{pmatrix} \; ; \quad \phi_{m,l,n}^{(2)} = \begin{pmatrix} ik_0 lrg_m\left(r\right) \\ 0 \\ \begin{cases} D_+\left[rg_m(r)\right] & si \;\; l \neq 0 \\ h_m\left(r\right) & si \;\; l = 0 \end{cases} \end{pmatrix}$$

2[nd] case: $n \neq 0$

$$\phi_{m,l,n}^{(1)} = \begin{pmatrix} -inr^{\delta-1}g_m\left(r\right) \\ D_+\left[r^\delta g_m\left(r\right)\right] \\ 0 \end{pmatrix}, \quad \phi_{m,l,n}^{(2)} = \begin{pmatrix} 0 \\ -ik_0 lr^{\delta+1}h_m\left(r\right) \\ inr^\delta h_m\left(r\right) \end{pmatrix}$$

$$\delta = \begin{cases} 2 \; if \;\; n \, is \, even \\ 1 \; if \;\; n \, is \, odd \end{cases}$$

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➢ *Test Functions*

1st case: $n = 0$

$$\psi^{(1)}_{m,l,n} = \begin{pmatrix} 0 \\ h_m(r) \\ 0 \end{pmatrix} \frac{1}{\sqrt{1-r^2}}$$

$$\psi^{(2)}_{m,l,n} = \begin{pmatrix} -ik_0 1 r^2 g_m(r) \\ 0 \\ \begin{cases} D_+\left[r^2 g_m(r) + r^3 h_m(r)\right] \; if \; l \neq 0 \\ r h_m(r) \end{cases} \end{pmatrix} \frac{1}{\sqrt{1-r^2}}$$

2nd case: $n \neq 0$

$$\psi^{(1)}_{m,l,n} = \begin{pmatrix} -inr^{\beta} g_m(r) \\ 0 \\ D\left[r^{\beta+1} g_m(r)\right] + r^{\beta+2} h_m(r) \end{pmatrix} \frac{1}{\sqrt{1-r^2}}$$

$$\psi^{(2)}_{m,l,n} = \begin{pmatrix} 0 \\ ik_0 l r^{\beta+2} h_m(r) \\ \begin{cases} -inr^{\beta+1} h_m(r) \, si \, l \neq 0 \\ r^{1-\beta} h_m(r) \, si \, l = 0 \end{cases} \end{pmatrix} \frac{1}{\sqrt{1-r^2}}$$

$$\beta = \begin{cases} 0 \; if \; n \, is \, even \\ 1 \; if \; n \, is \, odd \end{cases}$$

Using the Fourier representation, the continuity equation becomes:

$$Du(r) + \frac{in}{r} v(r) + ik_0 w(r) = 0 \qquad (17)$$

$$D_+ = D + \frac{1}{r}$$

## V. NUMERICAL IMPLEMENTATION

Let $$R_u = -\frac{\partial \vec{u}}{\partial t} - \left(\vec{u}.\vec{\nabla}\right)\vec{u} - \vec{\nabla}p + \frac{1}{R_e}\Delta\vec{u} + Ne\nabla\left(\frac{B^2}{2}\right) + div\sigma \qquad (18)$$

Replacing $\vec{u}$ by the basic functions and projecting on the basis of the test functions, it comes:

$$\left(R_u, \psi\right) = 0 \qquad (19)$$

The numerical method chosen is adapted from work in the early 1980 by Leonard and Wray *[6]*. With this Petrov-Galerkin procedure that was then used by *Meseguer & Trefethen 2000* [8] and *Meseguer & Mellibovsky 2007 [7]*, we obtain:

$$\begin{pmatrix} \left(\phi^1_{m,n,l}, \psi^1_{m,n,l}\right) & \left(\phi^2_{m,n,l}, \psi^1_{m,n,l}\right) \\ \left(\phi^1_{m,n,l}, \psi^2_{m,n,l}\right) & \left(\phi^2_{m,n,l}, \psi^2_{m,n,l}\right) \end{pmatrix} \begin{pmatrix} \dot{\alpha}^1_{m,n,l} \\ \dot{\alpha}^2_{m,n,l} \end{pmatrix} + \begin{pmatrix} c^1 \\ c^2 \end{pmatrix} + \begin{pmatrix} d^1 \\ d^2 \end{pmatrix} = \frac{1}{R_e} \begin{pmatrix} \left(\Delta\phi^1_{m,n,l}, \psi^1_{m,n,l}\right) & \left(\Delta\phi^2_{m,n,l}, \psi^1_{m,n,l}\right) \\ \left(\Delta\phi^1_{m,n,l}, \psi^2_{m,n,l}\right) & \left(\Delta\phi^2_{m,n,l}, \psi^2_{m,n,l}\right) \end{pmatrix} \begin{pmatrix} \alpha^1_{m,n,l} \\ \alpha^2_{m,n,l} \end{pmatrix} \qquad (20)$$

Where

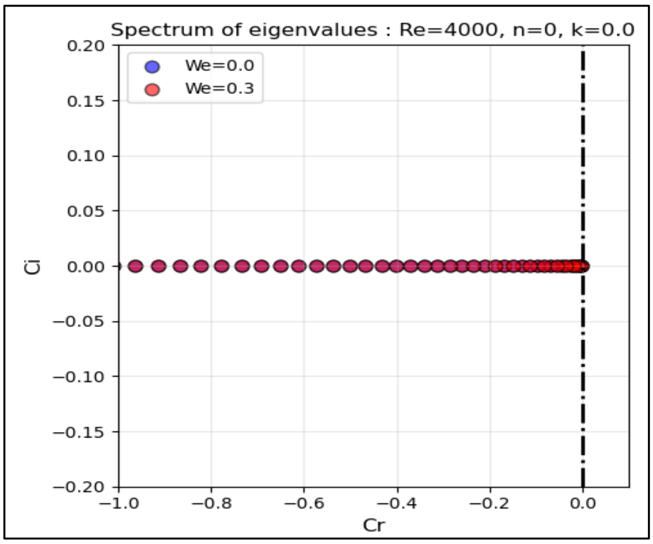
$$c^{1,2} = -\left(\left(\left(\vec{u}.\vec{\nabla}\right)\vec{W}_b + \left(\vec{W}_b.\vec{\nabla}\right)\vec{u}\right), \psi^{1,2}_{m,n,l}\right)$$

The matrix $c^{1,2}$ derives from the projection of the nonlinear terms and can be calculated with a pseudo-spectral method by Fast Fourier Transform (FFT).

Let the linear case where the nonlinear terms can be neglected, the problem obtained after projection is a problem with the generalized eigenvalues ($Av = cBv$) that will be solve numerically with QZ algorithm used by *J. P. Berlioz [8]*. To implement this method, we will use Housholder's unitary reflection matrices and Givens rotation matrices (*A. Quarteroni, R. Sacco and F. Saleri* [9]) and *(L. Amodei and J.P. Dedieu [10] ).*

## VI. RESULTS AND DISCUSSION

➢ *One-Dimensional Disturbance (n=0, k0=0)*



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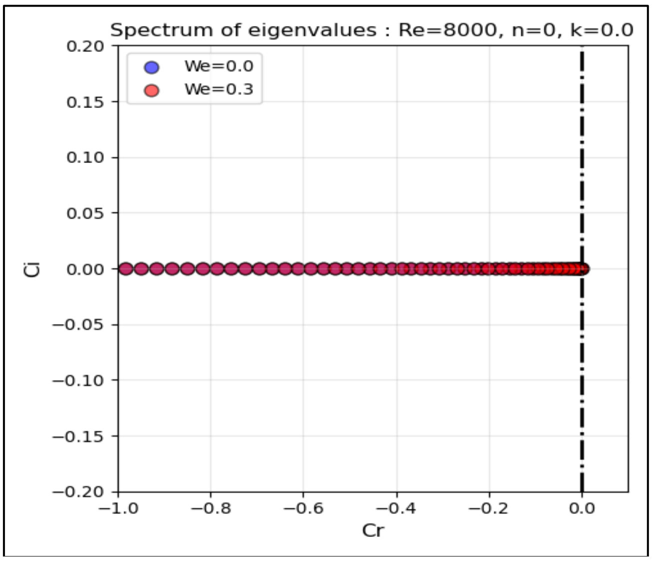
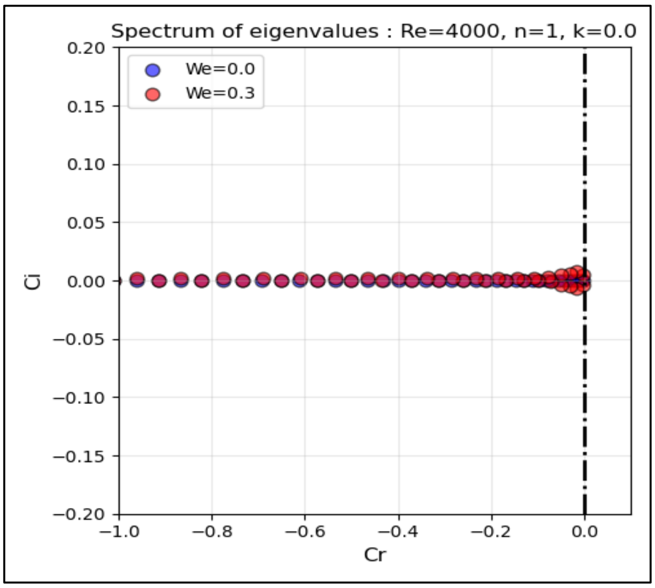



Fig 5 One-Dimensional Disturbance (n=0, k0=0)

➢ *Homogeneous Disturbance (n≠0, k0=0)*



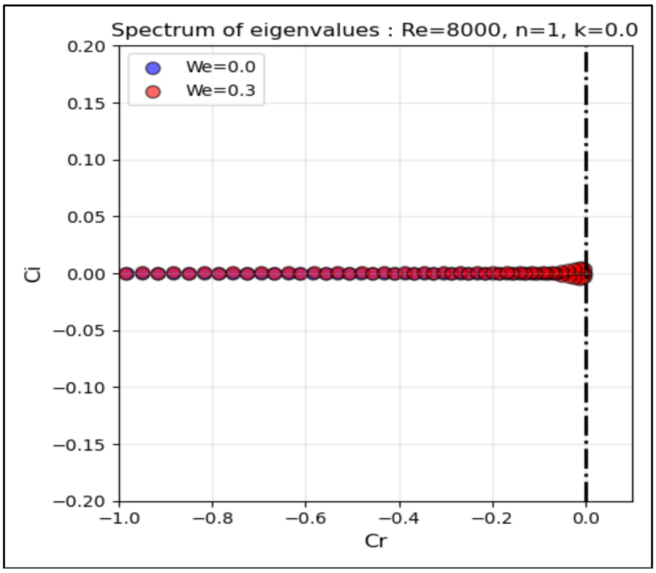
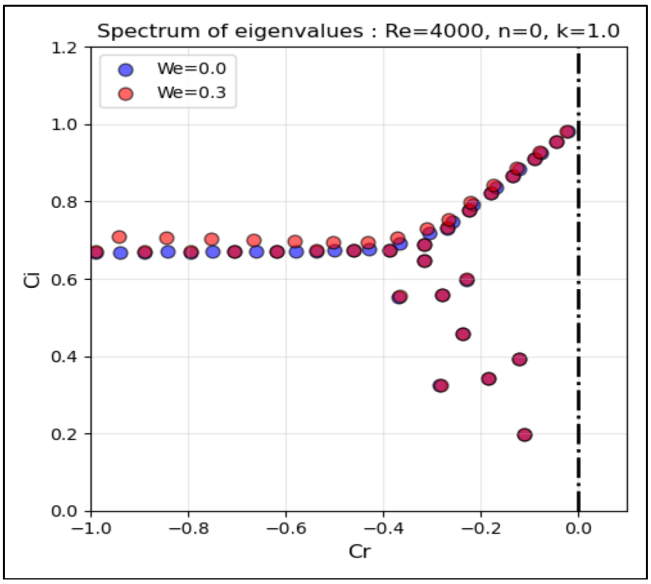
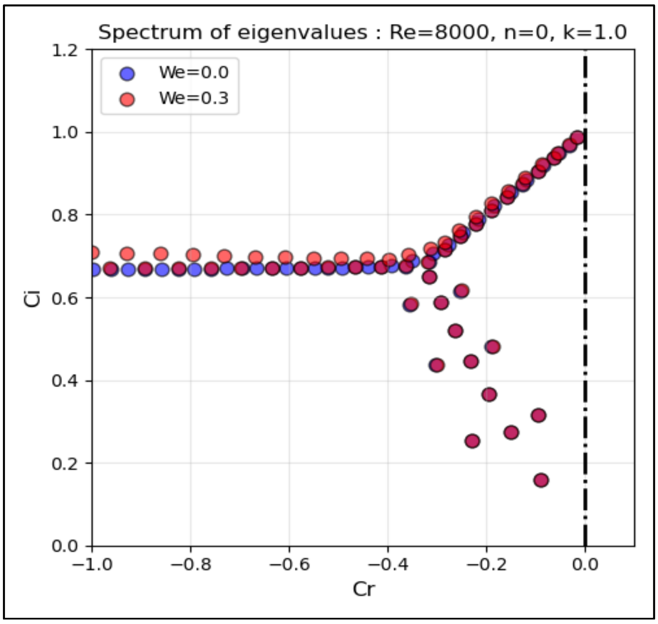
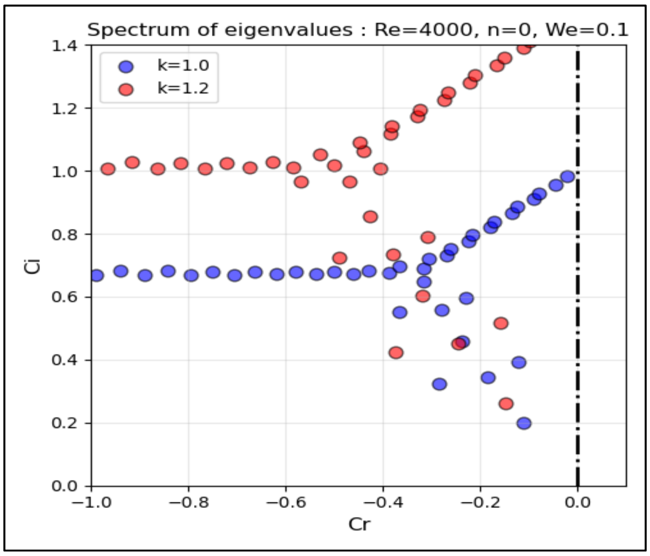




Fig 6 Homogeneous Disturbance (n≠0, k0=0)

➢ *Axisymmetric Disturbance (n=0, k0≠0)*



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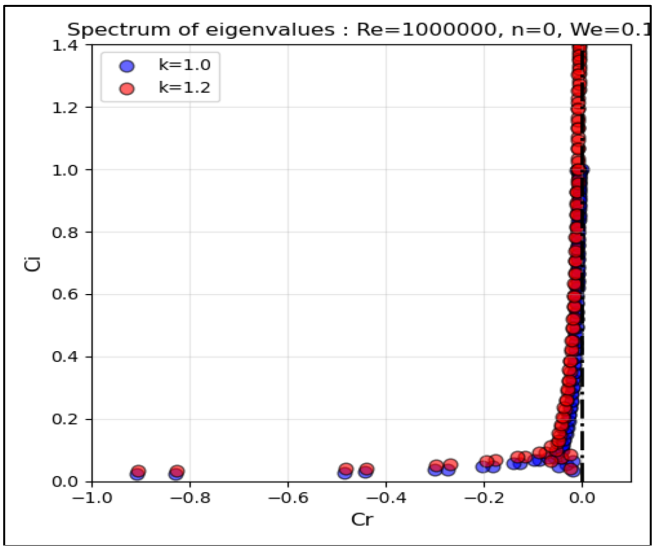


Fig 7 Axisymmetric Disturbance (n=0, k0≠0)

➢ *Three-Dimensional Disturbance (n≠0, k0≠0)*

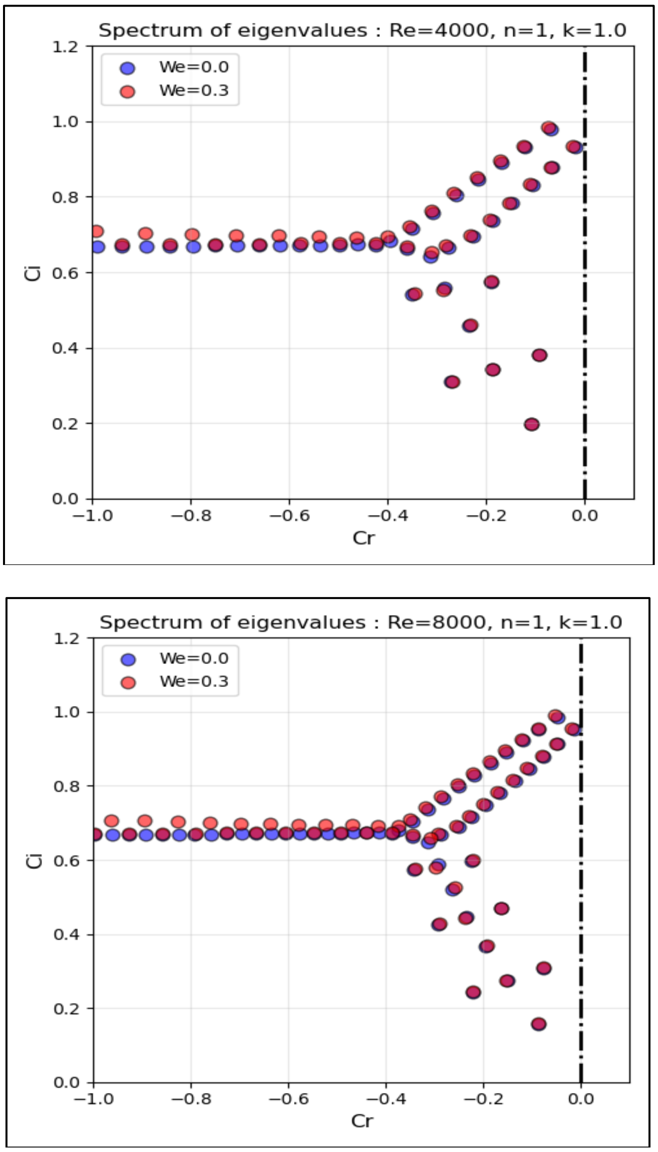


Fig 8 Three-Dimensional Disturbance (n≠0, k0≠0)

Analysis of the various results from our simulations, as shown in the figures, reveals that the elastic effects are closely linked to the variation of instability along the longitudinal direction. Indeed, when k=0, the flow is the same for different values of We, with the other parameters fixed. However, when k≠0, the flow pattern changes with the Weissenberg number and, consequently, with the elasticity. For this flow (k=0), the eigenvalue spectrum shows three zones:

- A zone where the growth rate $C_r$ is less than 0.4. In this zone, the effect of elasticity is negligible. The viscous modes are stable, and the two spectra for we = 0 and We = 0.3 coincide.
- A second zone (-0.4 < $C_r$< -0.2) where the modes of the Newtonian fluid are separated from those of the viscoelastic fluid, and where the points of the viscoelastic fluid spectrum are above those of the Newtonian fluid. We can therefore say that elasticity has a destabilizing effect, meaning that it increases the growth rate.
- In the third zone ($C_r$ > -0.2), a "Y"-shaped bifurcation is observed for both spectra, but the viscoelastic fluid is still less damped.

It is worth noting that these simulations were performed for Re=4000 and Re=8000, and the results are the same. To observe the effect of varying the perturbation along the longitudinal direction, we also studied the spectra of a viscoelastic fluid for two values of the axial wavenumber (k=1 and k=1.2). We clearly observe two separate branches. For k=1, we have a plateau at ci=0.65, and for k=1.2, a plateau at ci=1. However, in both cases, the growth rate is negative, and therefore the flow is stable. We also note that for the same given frequency ci, the growth rate of the viscoelastic flow is higher than, or at least as high as, that of the Newtonian flow. Magnetic force and pressure have no effect on flow stability. Indeed, the gradient functions cancel out when projected onto functions with zero divergence. As for the longitudinal wave number k, its influence is due partly to the inertial term, which causes energy exchanges between the base flow and the disturbances, and partly to the fact that the fluid is more stretched along the axis of the cylinder, which is in fact the principal direction of the base flow.

## VII. CONCLUSION

The flow of a weakly viscoelastic diamagnetic fluid through a horizontally oriented cylindrical pipe is linearly stable at least up to Re=8000. The action of a magnetic field induced by a longitudinal surface current density along this cylinder has no effect on the stability of such a flow; however, elasticity can have a destabilizing effect, with perturbations varying with the longitudinal component z. To expect instabilities, a high Reynolds number and strong or moderate elasticity would be necessary.

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