

Mixing-Water Temperature Governs the Precision, Not the Validity, of Abrams' Law in Early-Age Cement Mortar

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Abstract: Abrams' law holds that, at a given age, the compressive strength of a cementitious composite falls monotonically as the water-to-cement (w/c) ratio increases. Whether it continues to hold when the mixing water is delivered hot, a routine occurrence in tropical construction and in accelerated precasting, has not been established for mortar at the early ages that govern formwork striking and handling. We cast seventy-two 50 mm cubes of 1:3 ordinary Portland cement mortar from 24 independent batches, spanning eight w/c ratios (0.50–0.64 in 0.02 steps) and three mixing-water temperatures (25, 50 and 75 °C), and tested in compression at 24, 48 and 72 h. Abrams' law was not invalidated at any temperature: the fitted exponent was negative in all nine temperature–age series, and a Monte-Carlo test showed that the 31 local strength reversals observed across 63 adjacent w/c steps are indistinguishable from the 28.1 ± 2.6 expected of a strictly monotone law corrupted by single-specimen scatter ($p = 0.17$). Once the pseudo-replication created by the shared-batch structure was removed, no reproducible departure from monotonicity survived ($F(6,14) = 1.10$, $p = 0.41$). What mixing-water temperature changes is precision. The coefficient of variation about the fitted Abrams line was 6.6 % at 25 °C and 4.5 % at 50 °C but reached 22.0 % at 75 °C, a 24-fold inflation of residual variance relative to 50 °C (Levene $W = 30.4$, $p < 0.0001$), while the mean coefficient of determination of the Abrams fit fell from 0.53 to 0.20. The w/c $\times$ temperature interaction was not significant ($p = 0.27$): temperature shifts the strength level and disperses the response without altering the sensitivity of strength to w/c. Mixing water at 50 °C maximised early strength, exceeding 25 °C by 0.94 MPa and 75 °C by 3.29 MPa in paired comparison ($p = 0.0004$ and $p < 0.0001$). Gradient-boosted and random-forest surrogates, validated with whole batches held out, did not outperform the four-parameter Abrams-form regression, confirming that the specimen-level noise floor rather than model form limits predictability at this level of replication. In practice, hot mixing water above roughly 50 °C should be treated as a variability hazard in quality control, not as a strength-reduction factor.

Keywords: Abrams' Law; Cement Mortar; Mixing-Water Temperature; Early-Age Compressive Strength; Heteroscedasticity; Batch Pseudo-Replication.

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I. INTRODUCTION

$$f_c = A / B^{w/c} \quad (1)$$

Cementitious materials are the most widely produced construction materials in the world. They are conventionally divided into concrete, in which coarse and fine aggregate are bound together by hydrated cement, and mortar, which contains only fine aggregate [1]. In both, water plays two opposing roles: it is the reagent that drives hydration and, in excess, the agent that leaves behind the capillary porosity that limits strength. Abrams [2] captured the engineering consequence of this duality. His extensive testing at the Lewis Institute produced the relationship.

Where f_c is the compressive strength, w/c is the water-to-cement ratio by mass, and A and B are empirical constants that absorb the influence of materials, age and test conditions [3]. The underlying physical idea is older than the exponential form: six years before Abrams, Férét [4] had already identified the compactness of hydraulic mortars as the controlling variable. More than a century later, the w/c ratio is still regarded as the single largest determinant of concrete strength [5].

The law has since been generalised and qualified many times. Nagaraj and Banu [6] treated the solidification of cement

paste as a constant-volume process, in which high-density cement particles and low-density water yield a hydration product of intermediate density with interspersed residual porosity, and derived a normalised linear form in the reciprocal of w/c . Rao [3] extended the treatment explicitly to cement mortars, and Bhanja and Sengupta [5] modified the law for silica-fume systems. Others have pushed back. De Brito et al. [7] showed that strength cannot be predicted reliably when aggregate characteristics are ignored, and Kargari et al. [8] demonstrated that both cement strength class and the sand-to-cement ratio enter the generalisation. Within mortar specifically, Schulze [9] found that the water–cement ratio dominated while cement content was secondary in polymer-modified systems; Haach et al. [10] reported the expected trade-off between workability gain and mechanical loss as w/c rose in lime-blended mortars; and Kim et al. [11] quantified a steep strength penalty for increased water content at constant cement mass. Hashmi et al. [12] reached the same conclusion for fly-ash concrete. Taken together, this body of work supports a monotonically decreasing strength– w/c relationship.

Almost all of that work, however, was carried out under ambient or unreported mixing-water temperature. The omission matters, because temperature is a first-order control on hydration kinetics. Eskandarsefat [13] reported that the seven-day cylinder strength of high-early-strength concrete rose parabolically with mixing-water temperature to an optimum near 50 °C before declining, and Naser et al. [14] found peak strength at 40 °C in silica-fume concrete, attributing the gain to accelerated hydration. The mechanism is well understood. Elevated temperature accelerates the dissolution of C_3S and the nucleation of C-S-H, so more product forms early; but that product is deposited as a dense, poorly distributed shell around the residual clinker rather than being spread uniformly through the capillary space [15]. Early strength therefore rises even as the microstructure becomes coarser and less homogeneous. Mixing-water quality has drawn parallel attention: Reddy Babu et al. [16] catalogued roughly twenty water-quality parameters that influence strength, and standard practice restricts mixing water accordingly [17]. Temperature, by contrast, is largely absent from that literature.

These two strands have not been brought together. If mixing-water temperature alters hydration kinetics, and if Abrams' constants absorb test conditions, then it is fair to ask

whether the law itself remains descriptive when the water is hot, and, if so, with what fidelity. The question is not merely academic. In tropical and subtropical construction, stored mixing water routinely reaches 40–50 °C by the afternoon, and hot water is used deliberately in accelerated precasting and in cold-weather concreting. If the strength– w/c relationship degrades under those conditions, mix-design charts calibrated at ambient temperature will misstate not necessarily the mean strength, but the confidence with which it can be predicted.

The present study therefore targets a specific gap: the fidelity, as distinct from the direction, of Abrams' law in ordinary Portland cement mortar at the very early ages (24–72 h) that govern formwork striking, unit handling and early load transfer, as a function of mixing-water temperature. The contribution is threefold. First, mixing-water temperature is treated as an experimental factor in an Abrams-law assessment for mortar, which to our knowledge has not been done before. Second, conformity is assessed statistically rather than by visual inspection of the strength curves, using a Monte-Carlo reference distribution that asks whether the observed non-monotonicity exceeds what measurement scatter alone would generate. Third, the analysis quantifies the effect of temperature on the dispersion of the relationship and identifies that dispersion, not a change in the exponent, as the principal consequence of hot mixing water.

II. MATERIALS

➤ Cement

A single consignment of ordinary Portland cement (Holcim, Bangladesh), supplied as CEM I to BDS EN 197-1:2003 [18] and broadly equivalent to ASTM C150 Type I [19], was used throughout and stored in sealed, impermeable bags. The physical properties measured on the consignment are given in Table 1. A representative chemical composition for Bangladeshi OPC of this type is reproduced from Ali et al. [20] in Table 2; the oxide composition of the specific consignment was not determined, so Table 2 should be read as indicative rather than as a measurement on the test material. We also note that the measured 28-day mortar strength of 43.4 MPa corresponds to strength class 42.5 N rather than to the 52.5 N class nominally declared for the product, and the cement is accordingly treated as 42.5 N throughout this paper.

Table 1 Physical Characteristics of the Ordinary Portland Cement (Measured; ASTM C187 [21], C188 [22], C191 [23] and C109 [24]).

Property	Unit	Value
Specific gravity	–	3.14
Normal consistency	% by mass	28.5
Initial setting time	min	110
Final setting time	min	230
Compressive strength, 3 d	MPa	13.70
Compressive strength, 7 d	MPa	19.33
Compressive strength, 28 d	MPa	43.38

Table 2 Indicative Chemical Composition of Bangladeshi OPC, after Ali et al. [20]. These Values were not Measured on the Consignment used in this Study. The Reported SO₃ Content Exceeds the Tabulated British Standard Figure but Remains Within the 4.0 % Limit of BS EN 197-1 [25] for CEM I.

Oxide / parameter	Typical BS range (% by mass)	Reported value (% by mass)
SiO ₂	21.00	21.45
Al ₂ O ₃	6.00	4.30
Fe ₂ O ₃	3.50	3.28
CaO	65.00	64.32
MgO	0.70	1.18
Free lime	2.00	1.27
SO ₃	1.50	3.56
Insoluble residue	1.50	0.35
Loss on ignition	4.00	2.06

➤ Fine Aggregate

Locally available riverbed sand from the Mahananda River in Panchagarh District, Bangladesh, sold commercially as Panchagarh sand, was used as the fine aggregate. Its grading, determined to ASTM C136 [26], is given in Table 3 and Fig. 1. A fineness modulus of 2.32 places the sand at the fine end of, and marginally within, the ASTM C33 [27] grading envelope. Physical properties determined to ASTM C128 [28] were: bulk specific gravity 2.57 (oven-dry) and 2.61 (saturated surface-

dry), apparent specific gravity 2.65, and absorption capacity 1.21 %. We emphasise that ASTM C109 [24] nominally requires graded standard sand to ASTM C778 [29]; a locally sourced natural sand was used here by design, because the object of the study is the behaviour of a mortar representative of regional practice. Absolute strengths are therefore not directly comparable with nominal C109 values, although all comparisons within this study remain internally consistent because a single sand consignment was used throughout.

Table 3 Sieve Analysis of the Panchagarh Sand (ASTM C136 [26]).

Sieve no.	Opening (mm)	Mass retained (g)	Retained (%)	Cumulative retained (%)	Finer (%)
4	4.75	1.1	0.22	0.22	99.78
8	2.36	7.2	1.46	1.68	98.32
16	1.18	41.2	8.34	10.02	89.98
30	0.60	169.5	34.29	44.31	55.69
50	0.30	160.2	32.41	76.72	23.28
100	0.15	110.8	22.41	99.13	0.87
Pan	—	4.3	0.87	100.00	0.00
Total		494.3	100.00	FM = 2.32	

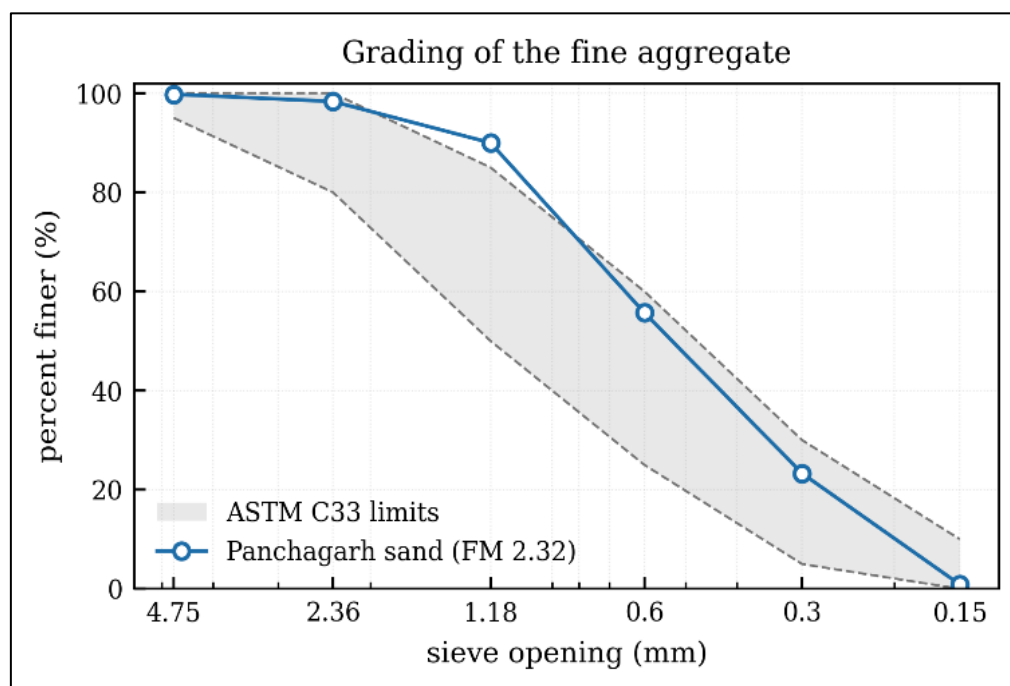


Fig 1 Grading Curve of the Panchagarh Sand Against the ASTM C33 Envelope for Fine Aggregate. The Grading Lies at the Fine Boundary of the Envelope Across the 0.60–0.15 mm Range.

➤ Water

Deionised water from the laboratory demineralisation plant was used for mixing, so that any effect of dissolved species, whose solubility and reactivity are themselves temperature-dependent, was excluded and the temperature effect isolated. Verification testing confirmed pH 7.3 and non-detectable turbidity, colour, carbon dioxide, iron, manganese, chloride, total dissolved solids and coliforms.

Potable water from the university treatment plant was used for curing (Table 4). This water carries 695 mg/L chloride

and 1410 mg/L total dissolved solids. The chloride concentration exceeds the 500 mg/L limit that ASTM C1602 [17] sets for prestressed concrete and for reinforced concrete bridge decks, although it remains below the 1000 mg/L limit that the same specification applies to other reinforced concrete. The specimens tested here are unreinforced and the water was used only for curing, but the point is recorded because it is a systematic condition shared by every specimen in the programme.

Table 4 Characteristics of the Curing Water. The Mixing water was Deionised and Returned Non-Detectable Values for Every Parameter Except pH (7.3).

Parameter	Unit	Curing water
pH	–	7.8
Colour	Pt-Co unit	12
Turbidity	NTU	3
Carbon dioxide	mg/L	5
Total iron	mg/L	0.20
Manganese	mg/L	0.07
Chloride	mg/L	695
Total dissolved solids	mg/L	1410
Total coliform	no./100 mL	0
Faecal coliform	no./100 mL	0

III. EXPERIMENTAL PROGRAMME

➤ Design of Experiments

The programme was a full factorial in two controlled factors, mixing-water temperature at three levels (25, 50 and 75 °C) and w/c ratio at eight levels (0.50 to 0.64 in steps of 0.02), with compressive strength measured at three ages (24, 48 and 72 h). The cement-to-sand ratio was fixed at 1:3 by mass throughout, because cement content is itself a strength determinant [30] and varying it would have confounded the factors of interest. The w/c window was chosen to fall within the 0.3–1.2 range over which Abrams' law has been reported valid for structural concrete [31], while remaining workable for a 1:3 mortar without admixture.

One batch of mortar was mixed for each of the 24 temperature × w/c combinations, and three cubes were cast from each batch and tested at 24, 48 and 72 h. The design therefore comprises 72 specimens drawn from 24 independent batches, with one specimen per batch–age cell. We state this structure explicitly because it fixes the correct unit of statistical replication, a point developed in Section IV.C and one that is frequently mishandled in the mortar literature.

➤ Specimen Preparation

The sand was screened for visible contaminants, oven-dried at 110 ± 5 °C for 24 h to remove inherent moisture that would otherwise raise the effective w/c ratio, and cooled to laboratory temperature under forced air so that it would not perturb the mixing-water temperature. Cement was passed through a No. 50 sieve to break up any lumps and weighed to 0.01 g. Mixing water was heated on a magnetic-stirrer hotplate, and the target temperature was confirmed with a mercury-in-glass thermometer immediately before addition. Mixing

followed the rotation and revolution schedule of ASTM C109 [24] in a digital planetary mortar mixer. Two-inch (50 mm) cube moulds were cleaned and lightly oiled; the fresh mortar was placed and compacted, the surface struck off level, and the specimens demoulded at 24 h, marked, and transferred to the curing tank. All operations were carried out in an air-conditioned laboratory nominally held at 25 °C.

Two departures from ASTM C109 [24] are recorded for transparency. First, natural sand replaced C778 standard sand, as noted in Section II.B. Second, the specimens were surface-dried in air before testing rather than tested directly from moist storage; because surface drying tends to raise the measured cube strength, the absolute values reported here are expected to be modestly higher than strictly conformant testing would give. Both departures were applied identically to every specimen and therefore do not bias comparisons between temperature or w/c levels.

➤ Testing

Cubes were loaded to failure in a calibrated compression machine at the stated ages. Compressive strength was computed as the failure load divided by the 2500 mm² nominal bearing area of the 50 mm cube. The complete set of 72 failure loads and derived strengths is provided as supplementary data.

➤ Statistical Framework

Assessing a monotone law from single specimens per condition demands more care than visual inspection of a strength–w/c plot allows, because any measurement scatter will generate apparent local reversals in a genuinely monotone underlying relationship. Four analyses were therefore specified in advance.

➤ *Fitting Abrams' Law*

Equation (1) linearises as $\ln f_c = \ln A - (w/c) \ln B$, so ordinary least squares of $\ln f_c$ on w/c recovers both constants. The fit was performed independently for each of the nine temperature $\times$ age series, and the sign, magnitude and significance of the slope, together with the coefficient of determination and the residual standard deviation, were recorded. Spearman's rank correlation was computed in parallel as a distribution-free check on monotonicity.

➤ *Reversal Test Against a Monotone Null*

A violation of Abrams' law, as the term is normally used, is an increase in strength between two adjacent w/c levels. Across nine series of eight points there are 63 such adjacent steps. To establish how many reversals a strictly monotone law would itself produce, 200,000 synthetic datasets were simulated from the nine fitted monotone Abrams lines with Gaussian noise of the pooled residual standard deviation estimated from the data, and the reversal count was tallied in each. The observed count was then referred to this null distribution. The procedure answers the question that visual analysis cannot: is the observed non-monotonicity greater than measurement scatter alone would produce?

➤ *Batch Structure and Pseudo-Replication*

Because the three specimens tested at 24, 48 and 72 h from a given temperature $\times$ w/c cell come from one batch, any batch-level error, a mis-weighed water addition, locally deficient compaction, or an unrepresentative air content, displaces all three in the same direction. Treating the 72 specimens as independent would then overstate the evidence for any pattern. This was tested by comparing the correlation of Abrams-fit residuals between ages within a temperature block (same batches) against the correlation between temperature blocks at equal age (different batches), using

Fisher's z-transformation. A nested F-test for reproducible departure from linearity was then run both naively, at specimen level, and correctly, at batch level.

➤ *Global Model and Surrogate Learners*

A global regression of $\ln f_c$ on w/c , mixing-water temperature and its square, $\ln(\text{age})$, and the $w/c \times$ temperature interaction was fitted to characterise the main effects and to test whether temperature modifies the sensitivity of strength to w/c . Random-forest and gradient-boosting regressors were then fitted on the same three predictors as flexible surrogates. Cross-validation held out whole batches (leave-one-batch-out); a conventional random K-fold split was also run and is reported only to show the magnitude of the optimism that batch leakage introduces. Homogeneity of residual variance across temperature was tested with Levene's and Bartlett's statistics and with the variance-ratio F test. All analyses were performed in Python 3 with NumPy, SciPy and scikit-learn.

IV. RESULTS

➤ *Abrams' Law Is Directionally Upheld at Every Temperature*

Table 5 reports the fitted constants for all nine series, and Fig. 2 shows the fits with 95 % prediction bands. The exponent constant B exceeds unity and the fitted slope is negative in all nine series without exception; that is, strength decreases with increasing w/c at every mixing-water temperature and every age examined. The Spearman correlations are negative in all nine series as well. Formal significance at the 5 % level is reached in only two series, but this reflects the low power available from eight single specimens spanning a narrow w/c window, not any inconsistency of direction: the probability of obtaining nine negative slopes out of nine by chance, under a null of no relationship, is $2^{-9} = 0.002$.

Table 5 Abrams' Law Fitted Independently to Each Mixing-Water Temperature $\times$ age Series. CoV is the Residual Coefficient of Variation About the Fitted line. All Nine Slopes are Negative, i.e. in the Direction Abrams' law Predicts.

T (°C)	Age (h)	A (MPa)	B	Slope $\pm$ SE	p	R ²	CoV (%)
25	24	19.79	2.66	-0.980 ± 0.380	0.042	0.526	4.9
25	48	29.27	3.47	-1.244 ± 0.531	0.058	0.478	6.9
25	72	26.75	2.34	-0.848 ± 0.590	0.201	0.256	7.7
50	24	30.63	4.57	-1.519 ± 0.352	0.005	0.756	4.6
50	48	19.75	1.72	-0.544 ± 0.294	0.114	0.363	3.8
50	72	29.25	2.41	-0.882 ± 0.378	0.059	0.475	4.9
75	24	42.13	12.97	-2.563 ± 1.547	0.149	0.314	20.1
75	48	36.57	6.96	-1.940 ± 1.484	0.239	0.222	19.2
75	72	25.85	3.51	-1.255 ± 2.002	0.554	0.062	25.9

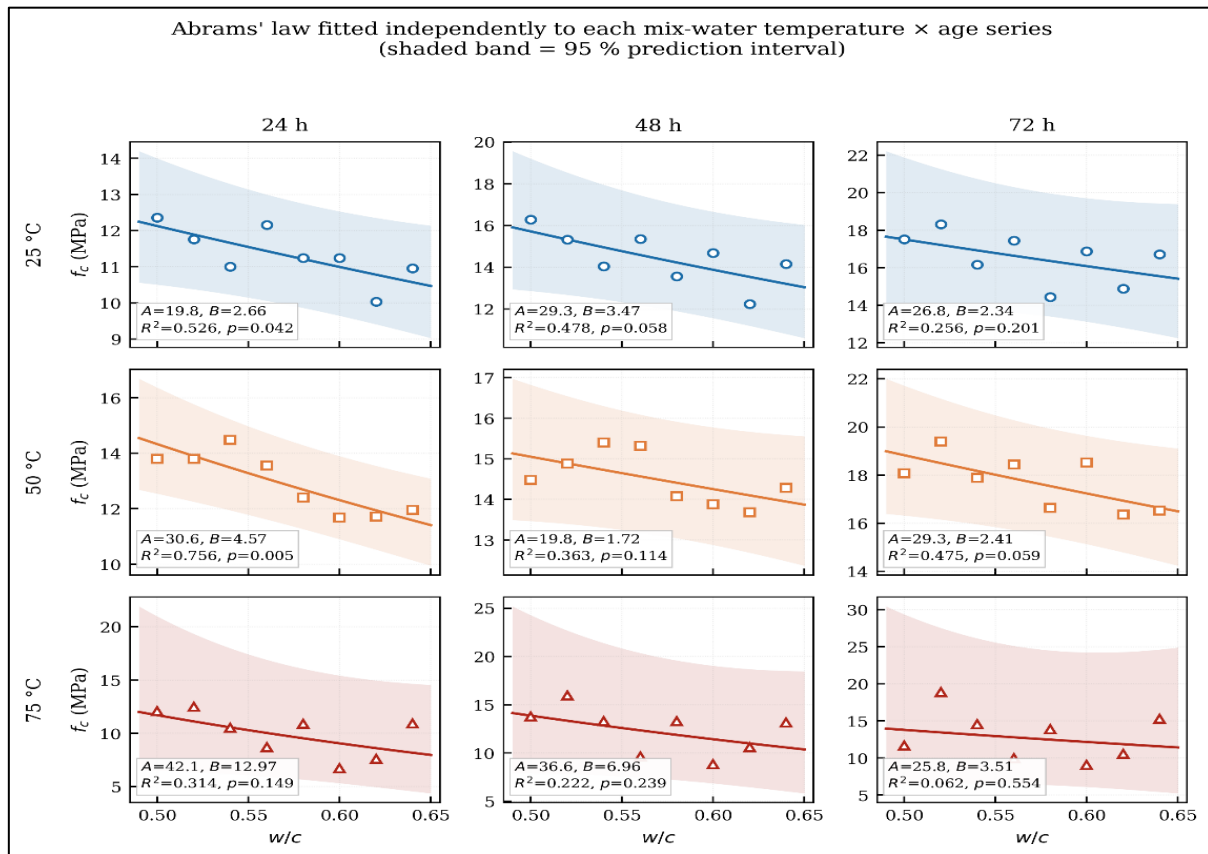


Fig 2 Compressive Strength Against w/c Ratio, with Abrams' law Fitted Separately to Each of the Nine Temperature $\times$ Age Series. Shaded Bands are 95 % Prediction Intervals for a Single new Specimen. Almost Every Apparent Violation lies Inside the Prediction band of a Monotone fit, and the Bands widen Sharply in the Bottom row (75 °C).

➤ *Apparent Violations Are Indistinguishable from Specimen Scatter*

Thirty-one strength reversals occur across the 63 adjacent w/c steps in the dataset. Simulation from strictly monotone Abrams lines, corrupted only by the measured single-specimen scatter, yields 28.1 ± 2.6 reversals, with a 95 % interval of 23 to 33 (Fig. 3). The observed count sits comfortably inside that

interval ($p = 0.17$ for the one-sided test that the data contain more reversals than a monotone law would produce). There is therefore no statistical basis for concluding that Abrams' law is violated at any of the three temperatures. The reversals that a visual reading of the strength curves so readily picks out are the expected consequence of testing one cube per condition.

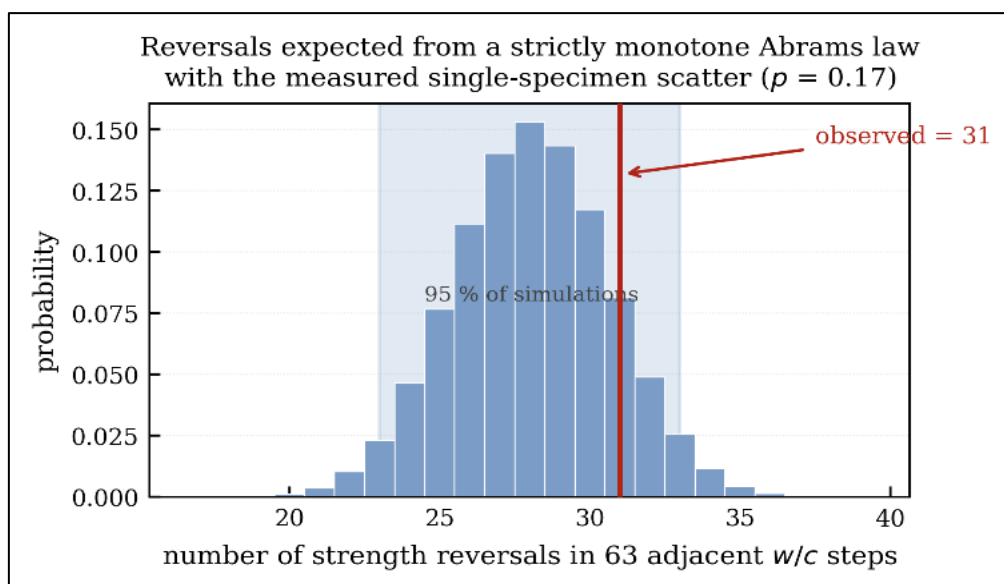


Fig 3 Distribution of the number of strength reversals generated by 200,000 simulations of a strictly monotone Abrams law subject to the measured specimen scatter. The observed value of 31 lies within the central 95 % of the null distribution.

➤ The Apparent Reproducibility of the Pattern Is Batch Pseudo-Replication

A nested F-test applied naively at specimen level appears to detect reproducible structure in w/c beyond a straight line ($F(6,56) = 3.66$, $p = 0.004$), which would be a substantive finding if the 72 specimens were independent. They are not. Residuals about the Abrams fits correlate strongly between ages within a mixing-water temperature block, where

specimens share a batch (mean $r = +0.67$), and negligibly between temperature blocks at equal age, where they do not (mean $r = +0.10$); the Fisher-z contrast gives $p = 0.002$ (Fig. 4). Repeating the shape test at batch level, the correct unit of replication, gives $F(6,14) = 1.10$, $p = 0.41$. No reproducible departure from a monotone Abrams relationship survives. The quadratic term in w/c is likewise not significant ($F(1,61) = 3.49$, $p = 0.067$).

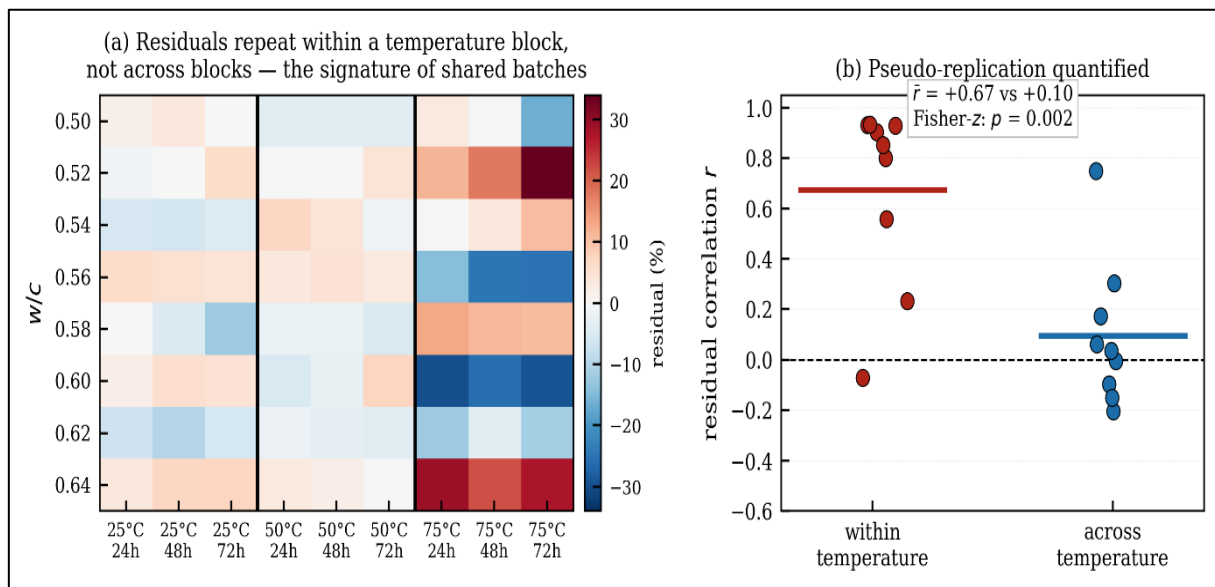


Fig 4 (a) Residual map: vertical stripes of matching colour within each temperature block, and their absence between blocks, are the signature of batch-level rather than specimen-level error. (b) Residual correlations for specimen pairs that share a batch versus pairs that do not; horizontal bars are group means.

➤ Temperature Inflates Dispersion without Changing the Exponent

The principal result of this study is shown in Fig. 5. The residual coefficient of variation about the fitted Abrams line is 6.6 % at 25 °C and 4.5 % at 50 °C, but rises to 22.0 % at 75 °C. For context, the precision statement of ASTM C109 [24] gives a single-operator (within-laboratory) coefficient of variation of about 3.7 % and a multilaboratory coefficient of about 6.6 % for Portland-cement mortar cubes; the dispersion observed at 25 and 50 °C is therefore of the same order as routine testing variability, whereas the value at 75 °C is several times any figure the standard anticipates. Levene's test rejects equality of residual variance decisively ($W = 30.39$, $p = 3.4 \times 10^{-10}$), as does Bartlett's ($\chi^2 = 60.33$, $p = 8.0 \times 10^{-14}$); the variance ratio between 75 °C and 50 °C is 24.3 ($p = 6.8 \times 10^{-11}$). The mean

coefficient of determination of the Abrams fit falls correspondingly, from 0.53 at 50 °C and 0.42 at 25 °C to 0.20 at 75 °C; at 75 °C and 72 h the law explains only 6 % of the variance in strength.

This degradation is a loss of precision, not a change of form. In the global model (Table 6) the w/c $\times$ temperature interaction is not significant ($p = 0.27$), which means the sensitivity of strength to w/c, the slope of the Abrams relationship, is statistically indistinguishable across the three temperatures. Temperature enters instead through the intercept, with significant negative linear and quadratic terms, and through the variance. Age enters as expected through $\ln(\text{age})$. The model explains 68.8 % of the variance in $\ln f_c$, with a residual standard error of 12.8 %.

Table 6 Global regression of $\ln f_c$ on the mix variables ($n = 72$; $R^2 = 0.688$, adjusted $R^2 = 0.664$, residual SE = 0.128 in $\ln$ units, approximately 12.8 % in strength). The non-significant interaction indicates that mixing-water temperature does not alter the w/c sensitivity of strength.

Term	Coefficient	SE	t	p
Intercept	2.7009	0.0262	103.00	< 0.0001
w/c (centred)	-1.3083	0.3304	-3.96	0.0002
T (centred)	-0.0039	0.0007	-5.28	< 0.0001
T ²	-0.0003	0.0001	-5.18	< 0.0001
$\ln(\text{age})$	+0.2869	0.0334	+8.60	< 0.0001
w/c $\times$ T	-0.0179	0.0162	-1.11	0.273

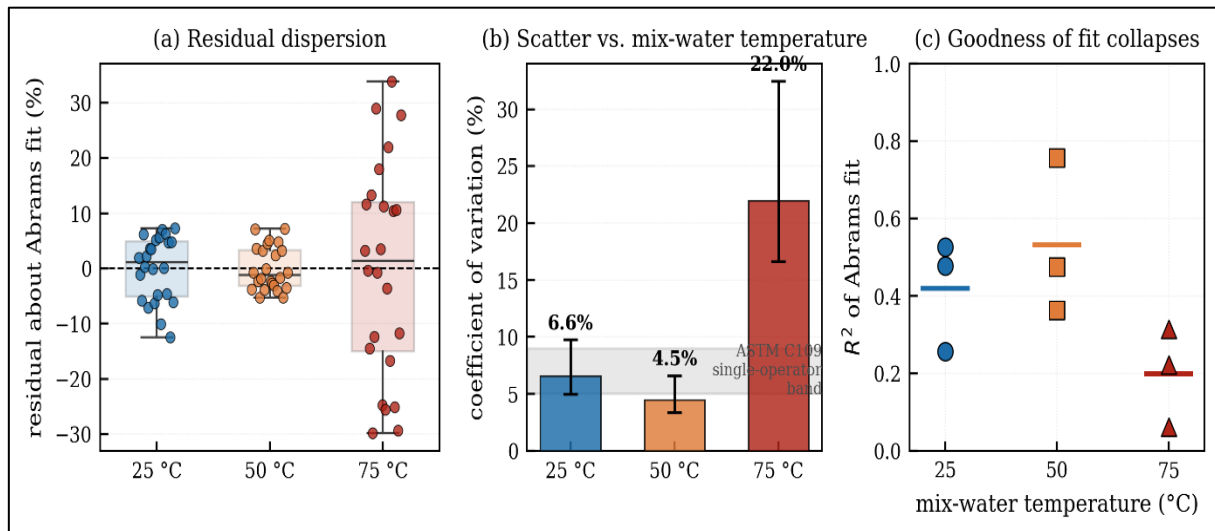


Fig 5 (a) Residuals about the fitted Abrams lines by mixing-water temperature. (b) Residual coefficient of variation with 95 % chi-square confidence intervals; the shaded band spans the single-operator and multilaboratory coefficients of variation given in the ASTM C109 precision statement. (c) Coefficient of determination of the Abrams fit for each of the nine series, with group means.

➤ *A Mixing-Water Temperature of 50 °C Maximises Early Strength*

Mean strength at 50 °C exceeds that at both other temperatures at every age (Fig. 6a), and one-way analysis of variance rejects equality of means at 24 h ($F = 8.45$, $p = 0.002$), 48 h ($F = 4.86$, $p = 0.019$) and 72 h ($F = 10.64$, $p = 0.0006$). Pairing observations by w/c ratio and age, which removes between-mix variation, the 50 °C mixture exceeds the 25 °C

mixture by a mean of 0.94 MPa ($t = 4.15$, $p = 0.0004$; Wilcoxon $p = 0.001$; higher in 19 of 24 pairs) and the 75 °C mixture by 3.29 MPa ($t = 6.10$, $p < 0.0001$; Wilcoxon $p < 0.0001$; higher in 23 of 24 pairs). This corroborates the optimum near 50 °C that Eskandarsefat [13] identified for high-early-strength concrete and extends it to unmodified 1:3 mortar.

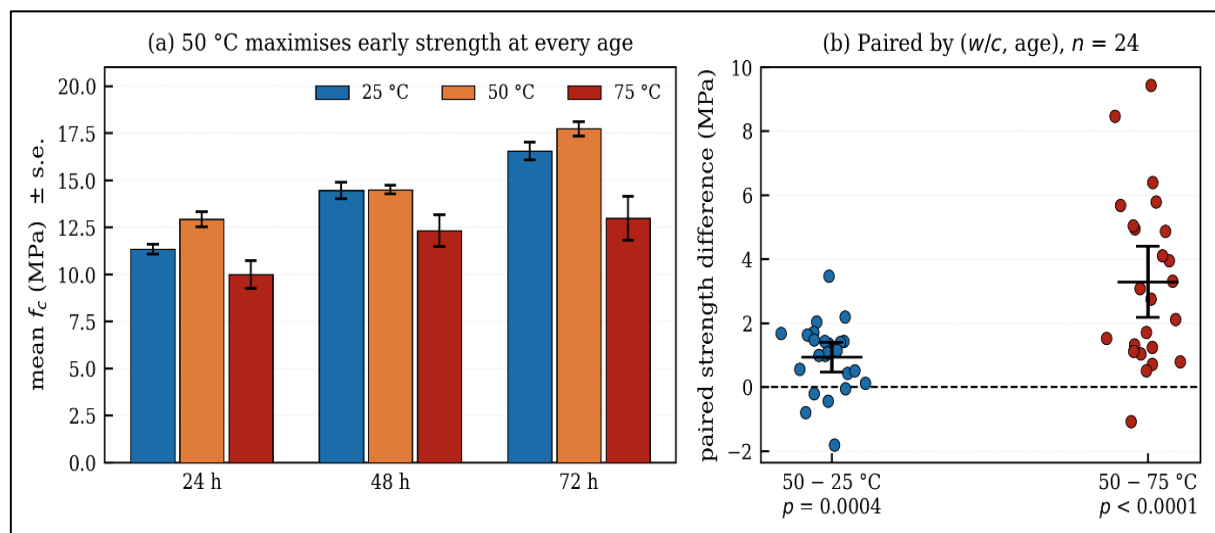


Fig 6 (a) Mean compressive strength by mixing-water temperature and age, with standard errors over the eight w/c ratios. (b) Paired differences relative to 50 °C, blocked by w/c ratio and age; heavy bars are means with 95 % confidence intervals.

➤ *Flexible Learners Do Not Outperform the Physics-Based Form*

Under leave-one-batch-out cross-validation, the four-parameter Abrams-form regression achieved a root-mean-square error of 0.145 in $\ln$ units (1.78 MPa), against 0.159 (1.94 MPa) for a random forest and 0.175 (2.21 MPa) for gradient boosting (Fig. 7). Neither learner improved on the parametric model. The reason is visible in the noise floor: the pooled within-series residual standard deviation is 0.135 in $\ln$

units, so the best cross-validated model is already operating at the limit set by single-specimen measurement scatter, and any additional model flexibility can only fit noise. Permutation importance ranks curing age (0.73) above mixing-water temperature (0.69) and w/c ratio (0.46), consistent with the global regression.

The comparison also carries a methodological warning. Repeating the same cross-validation with conventional random

ten-fold splits reduces the apparent error of gradient boosting to 0.121 and raises its apparent R^2 from 0.37 to 0.70, an improvement that is entirely artefactual, because sibling cubes from the same batch are then distributed across the training and

test folds. Machine-learning studies of cementitious materials that draw several specimens from single batches and validate with random splits will overstate predictive accuracy for exactly this reason.

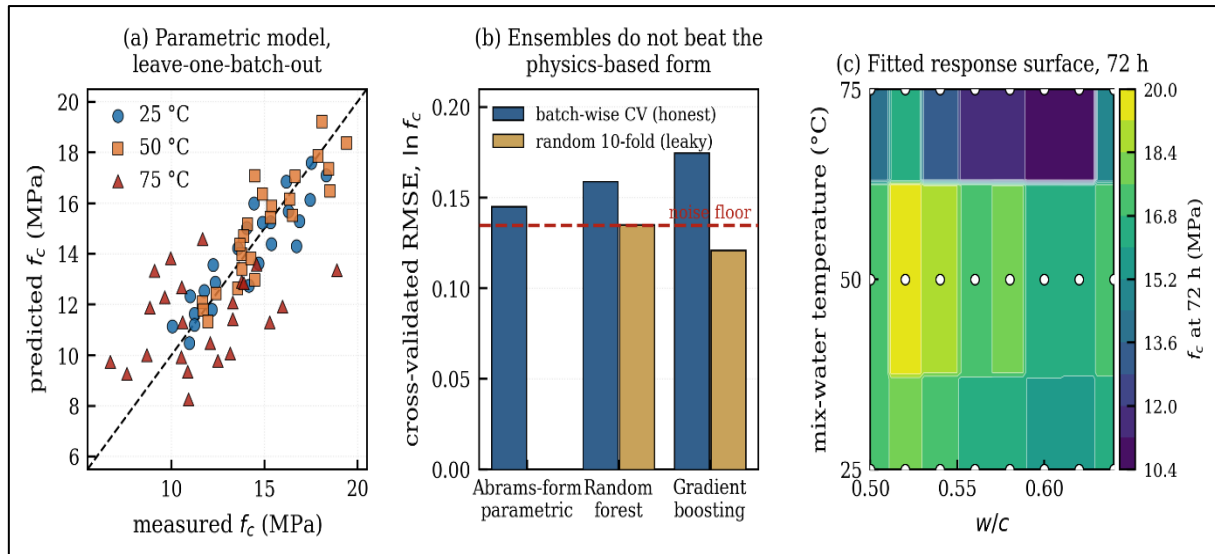


Fig 7 (a) Cross-validated predictions of the Abrams-form model with whole batches held out. (b) Cross-validated error of the three models under honest batch-wise and leaky random-split validation; the dashed line is the estimated specimen noise floor. (c) Response surface at 72 h fitted by gradient boosting, with the experimental grid superimposed.

V. DISCUSSIONS

➤ Why the Visual Reading Fails

The strength– w/c curves in this dataset are visibly ragged, and it is tempting to read each upward step as a counter-example to Abrams' law. That reading conflates two distinct claims: that the underlying relationship is non-monotone, and that a particular realisation of it contains upward steps. With one specimen per condition and a coefficient of variation of even 5 %, the second follows without the first being true. The reversal simulation quantifies precisely this: a perfectly monotone law tested in this design is expected to yield about 28 upward steps in 63, because adjacent w/c levels differ by only 0.02 and the true strength difference between them is smaller than the measurement error. Any conclusion about the validity of a monotone law therefore requires either replication sufficient to resolve steps of that size or a test referenced to a monotone null. Reporting the present dataset as a refutation of Abrams' law would have been a Type I error of exactly this kind.

➤ Mechanism of the Dispersion Increase

That mixing at 75 °C produces erratic, rather than merely lower, strength is consistent with several interacting mechanisms. Elevated temperature accelerates C_3A dissolution relative to the rate at which sulfate can be supplied from the gypsum, which promotes false or flash setting and makes the working time between mixing and compaction both short and variable. Compaction quality then becomes the dominant uncontrolled variable, and small differences in placing time translate into large differences in entrapped air. Rapid early hydration also drives the classic mechanism described by Verbeck and Helmuth [15]: hydration products form too quickly to diffuse away from the clinker surface and deposit as

a dense, non-uniform shell, leaving coarser residual porosity distributed heterogeneously. Heterogeneous porosity produces heterogeneous strength, the same principle that, run in reverse, underlies the very high strengths achievable once porosity and processing defects are minimised in chemically bonded cements [32]. Evaporative loss from hot mortar during mixing and placing, which shifts the effective w/c ratio away from its nominal value by an amount that depends on exposure time, would compound all of these effects. That the mean strength at 75 °C is depressed only moderately while its variance rises more than twenty-fold points specifically to a defect-controlled, rather than a matrix-controlled, failure population.

➤ Practical Implications

Three implications follow for practice. First, hot mixing water should be managed as a variability hazard rather than as a strength penalty: a specification that controls only mean strength will not detect the condition identified here, whereas one that monitors within-batch standard deviation will. Second, where hot water is used deliberately to accelerate early strength gain, in precasting or in cold-weather work, the results support a target near 50 °C and argue against exceeding it, because the marginal strength available above 50 °C is negative while the variability cost is severe. Third, because the w/c sensitivity itself does not change with temperature, existing Abrams-type mix-design charts remain usable for central estimates provided the intercept is recalibrated; what must be revised is the safety margin, not the slope.

➤ Relation to Previous Work

The finding that strength peaks near 50 °C reproduces Eskandarsefat [13] in a different material system and at earlier ages, and is consistent in direction with the 40 °C optimum that Naser et al. [14] reported for silica-fume concrete. The

persistence of Abrams' direction under thermal perturbation is consistent with the generalisations of Nagaraj and Banu [6] and Rao [3], both of which rest on porosity arguments that temperature affects quantitatively but not qualitatively. The present contribution is orthogonal to the criticisms of De Brito et al. [7] and Kargari et al. [8], who showed that aggregate characteristics and cement class enter the constants: this study shows that mixing-water temperature enters the constants and, more consequentially, the residual variance.

VI. LIMITATIONS

The findings are bounded by several features of the experimental design, which we state in full because they condition the strength of every inference above.

The design provides one specimen per batch–age cell. Within-condition variability therefore cannot be estimated directly and has been inferred from residuals about the fitted Abrams lines, which conflates specimen scatter with any genuine lack of fit. Three replicate cubes per cell, as ASTM C109 [24] anticipates, would separate the two and would sharpen every test reported here. The 24 batches also mean that the effective sample size for statements about the w/c profile is 24, not 72.

Only the mixing-water temperature was controlled. The temperature of the fresh mortar at casting, the physically operative quantity, was neither measured nor held constant, and depends additionally on the temperature and thermal mass of the dry constituents. The curing-tank temperature was not recorded, and at ages of 24–72 h the curing history may exert an influence comparable to, or larger than, that of the mixing water; the temperature effect reported here should therefore be read as the effect of the mixing-water temperature as delivered, within an uncontrolled but nominally constant laboratory environment. Laboratory air temperature was nominally 25 °C but subject to interruptions in the air-conditioning supply, and ambient relative humidity was neither controlled nor recorded.

No fresh-state measurements were made. Flow to ASTM C1437 [33], air content and setting time would each have tested the mechanisms proposed in Section VI.B directly, and their absence means that the interpretation of the variance inflation, though consistent with the data, remains inferential. The oxide composition of the cement was taken from the literature rather than measured. Testing was confined to 24, 48 and 72 h, so these data say nothing about whether the 50 °C advantage persists, vanishes or reverses at 28 days, and the crossover behaviour documented by Verbeck and Helmuth [15] gives specific reason to expect that it may reverse. Finally, a single cement source, a single sand and a single 1:3 proportion were used, so generalisation to other binders, gradings and mix proportions is not warranted without further testing.

VII. CONCLUSIONS

Seventy-two mortar cubes from 24 batches, spanning eight w/c ratios and three mixing-water temperatures, were tested at 24, 48 and 72 h and analysed against a monotone null. The following conclusions are supported.

- Abrams' law was not invalidated at any mixing-water temperature examined. The fitted exponent was negative in all nine temperature × age series, and the 31 local strength reversals observed across 63 adjacent w/c steps fall within the 23–33 interval expected of a strictly monotone law subject to single-specimen scatter ($p = 0.17$).
- The apparent reproducibility of the non-monotonic pattern is an artefact of batch pseudo-replication. Residuals correlate at $r = +0.67$ between ages that share a batch and at $r = +0.10$ between independent batches; at the correct level of replication, no departure from monotonicity remains ($p = 0.41$).
- Mixing-water temperature acts principally on the precision of the strength–w/c relationship. The residual coefficient of variation rises from 4.5 % at 50 °C and 6.6 % at 25 °C to 22.0 % at 75 °C, a 24-fold inflation of variance ($p = 6.8 \times 10^{-11}$), while the w/c × temperature interaction is not significant ($p = 0.27$). Hot mixing water disperses the response without changing its slope.
- A mixing-water temperature of 50 °C maximises early compressive strength, exceeding 25 °C by 0.94 MPa and 75 °C by 3.29 MPa in paired comparison ($p = 0.0004$ and $p < 0.0001$).
- Random-forest and gradient-boosting surrogates do not outperform the four-parameter Abrams-form regression under batch-wise cross-validation, because the specimen noise floor rather than model form limits predictability. Random-split validation of the same learners overstates accuracy substantially, a trap that batch-structured cementitious datasets make easy to fall into.

For practice, the results argue that hot mixing water should be controlled as a source of variability rather than of mean strength loss, and that 50 °C represents a practical upper bound where hot water is used deliberately to accelerate early strength gain.

AUTHOR CONTRIBUTIONS

Md. Zihad Hossain: conceptualization, methodology, investigation, formal analysis, writing – original draft. Mehrabuzzaman: investigation, data curation, visualization. Tanveer Tausif: methodology, software, formal analysis, writing – review and editing. Tanvir Alam: supervision, validation, resources, project administration, writing – review and editing. All authors have read and agreed to the submitted version of the manuscript.

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➤ Data Availability

The complete dataset of 72 failure loads and derived compressive strengths, together with the analysis scripts that reproduce every figure and statistic reported in this paper, are provided as supplementary material and will be made available in a public repository upon acceptance.

➤ *Declaration of Competing Interest*

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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