

Reassessing Pólya's Problem-Solving Model in Contemporary Mathematics Education: A Critical Synthesis and a Proposed Recursive Framework

Dr. Tamashwar Budhoo¹; Dannyram Seawpersaud²

¹Deputy Dean, Faculty of Education, University of Guyana, Turkeyen, Guyana

²Mathematics Educator, Ministry of Education, Guyana

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Abstract: George Pólya's four-phase model of problem solving, which comprises understanding the problem, devising a plan, carrying out the plan, and looking back, remains one of the most widely cited heuristics in mathematics education. This conceptual paper examines the extent to which the model, as it is commonly enacted in secondary school curricula and classrooms, accounts for problem solving as contemporary research describes it. The paper distinguishes Pólya's original heuristic programme from the reductive checklist into which it has frequently been transformed, and it directs its critique principally at the latter. Adopting a theory synthesis design, the paper examines six dimensions: the recursive rather than linear character of problem solving, the continuous role of metacognitive regulation, the diversity of learners' conceptual, linguistic, and affective resources, the mediating influence of digital technology, the socially mediated nature of classroom problem solving, and the place of creativity and insight. The paper also engages counterarguments from cognitive load theory, which support explicit and structured guidance for novices. The synthesis concludes that Pólya's model is not a historical artefact; rather, it offers a sound foundation that requires reconceptualisation. The paper therefore proposes a recursive, regulated, and mediated framework in which Pólya's phases form an iterative core, metacognitive regulation operates across all phases, and social, digital, linguistic, and affective conditions shape the process. The paper closes with implications for classroom practice, teacher education, curriculum and assessment design, and future empirical research.

Keywords: Problem Solving; Pólya; Heuristics; Metacognition; Socio-Constructivism; Digital Technology; Mathematical Creativity; Secondary Mathematics Education.

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I. INTRODUCTION

Problem solving occupies a central place in mathematics education. Curriculum frameworks across many jurisdictions position it both as a goal of instruction and as a vehicle through which students develop reasoning, conceptual understanding, and mathematical dispositions [1], [2]. Among the many attempts to describe and teach problem solving, George Pólya's *How to Solve It* [3] holds a foundational position. The book revived the study of heuristics, offered a repertoire of guiding questions, and organised the solver's activity into four phases: understanding the problem, devising a plan, carrying out the plan, and looking back. Pólya's phases remain widely known among mathematics educators, teachers, and students, and his work continues to shape process models of problem solving [4], [5]. The persistence of the model is not difficult to explain. The four phases offer teachers and students an accessible vocabulary and a clear structure, which is particularly valuable for learners who

encounter non-routine problems for the first time. A novice who does not know where to begin can use the phases to orient attention, and a teacher can use them to organise questioning and feedback.

A careful reassessment must nevertheless separate two objects that are often conflated. The first object is Pólya's own heuristic programme, which he presented through teacher–student dialogue, an extensive list of questions, and later studies of plausible reasoning, induction, and analogy [3], [6]. The second object is the version of the model that circulates in many classrooms and textbooks, in which the four phases appear as a fixed sequence of steps to be completed in order. Schoenfeld [7] observed that the label *problem solving* came to be attached to a wide range of curricular practices, many of which reduced heuristic strategies to routine exercises. Schoenfeld's [8] empirical work also showed that general heuristic descriptions, when taught without domain-specific tactics and managerial control,

were too weak to improve students' performance on their own. This paper directs its critique principally at the second object, which may be termed *Pólya-as-practised*. The argument is not that Pólya misunderstood problem solving; the argument is that the checklist enactment of his model misrepresents problem solving in ways that contemporary research now makes visible. Where limitations inhere in the model itself, such as its focus on the individual solver, the paper identifies them explicitly. The analysis concentrates on secondary school mathematics, although it draws on evidence from primary and tertiary settings where that evidence illuminates the argument.

Three bodies of research motivate the reassessment. Studies of problem-solving processes show that solvers move recursively among phases rather than progressing through them once [9], [10]. Research on metacognition and self-regulation shows that monitoring and control operate throughout problem solving rather than only at its conclusion [7], [11]. Socio-constructivist and technology-focused research shows that classroom problem solving is frequently collaborative and mediated by digital tools [12], [13]. The paper addresses two research questions. The first question asks to what extent the checklist enactment of Pólya's model accounts for problem solving as contemporary research describes it. The second question asks what reconceptualisation of the model would preserve its strengths while addressing its limitations. Section II describes the methodology. Section III distinguishes Pólya's original proposal from its classroom enactment. Sections IV to IX examine six dimensions in turn: linearity, metacognition, learner diversity, technology, social mediation, and creativity. Section X engages counterarguments that support structured heuristic instruction. Section XI synthesises the analysis and answers the question of whether the model has become a historical artefact. Section XII proposes a revised framework, Section XIII sets out implications for practice and research, and Section XIV concludes the paper.

II. METHODOLOGY

➤ Research Design

The paper adopts a conceptual research design. Jaakkola [14] argues that conceptual articles require an explicit research design in which the focal phenomenon, the theories brought to bear on it, and the intended contribution are stated and justified. Among the templates Jaakkola describes, the

present paper follows a *theory synthesis* approach, which integrates insights from several theoretical perspectives to offer a new or refined understanding of a focal concept. The focal concept is Pólya's four-phase model as enacted in secondary mathematics education. The theoretical lenses are cognitive and metacognitive theories of problem solving, socio-constructivist theories of learning, research on digital technology in mathematics education, and research on mathematical creativity. Cognitive load theory serves as a counter-lens that tests the critique against the strongest case for structured guidance.

➤ Identification and Selection of Sources

The literature was identified through purposive, theory-driven searching rather than an exhaustive systematic review. Searches were conducted in Scopus, ERIC, and SpringerLink using combinations of the terms “mathematical problem solving”, “Pólya”, “heuristics”, “metacognition”, “self-regulation”, “collaborative problem solving”, “digital technology”, “dynamic geometry”, “computational thinking”, “mathematical creativity”, and “problem posing”. Backward citation tracking from key review articles [1], [2], [15] supplemented the database searches.

Three inclusion criteria guided selection. First, sources had to be peer-reviewed journal articles or peer-reviewed conference papers. Second, sources had to address at least one of the six analytic dimensions directly. Third, empirical sources had to report data on problem-solving processes, metacognition, learner characteristics, technology use, classroom interaction, or creativity in mathematics or a closely related domain. A limited number of landmark books and handbook chapters were retained as exceptions because they define the constructs under examination; these are Pólya [3], [6], Schoenfeld [7], [8], and Vygotsky [16].

➤ Analytic Procedure

The analysis proceeded in four steps for each dimension. The first step identified Pólya's own position on the dimension, drawing on his primary texts. The second step characterised how the dimension is typically represented in the checklist enactment of the model. The third step examined what contemporary research reports about the dimension. The fourth step identified the implications of any gap for a revised framework. Table 1 summarises the six dimensions, the guiding question for each, and the principal sources consulted.

Table 1 Analytic Dimensions, Guiding Questions, and Principal Sources

Dimension	Guiding question	Principal sources
Linearity	Do solvers progress through the phases once and in order?	Schoenfeld [8]; Carlson & Bloom [9]; Rott et al. [10]; Borromeo Ferri [17]; Verschaffel et al. [15]
Metacognition	Where and how does regulation operate during problem solving?	Flavell [18]; Garofalo & Lester [11]; Schoenfeld [7]; Veenman et al. [19]; Zimmerman [20]; Mevarech & Kramarski [21]; Kramarski et al. [22]
Learner diversity	Why, rather than where, do students' attempts break down?	Tambychik & Meerah [23]; Phonapichat et al. [24]; Abedi & Lord [25]; Ramirez et al. [26]
Technology	How do digital tools reshape the phase structure?	Drijvers [27]; Kuzle [28]; Barrera-Mora & Reyes-Rodríguez [29]; Hegedus & Otálora [30]; Wing [31]; Weintrop et al. [32]; Engelbrecht & Borba [12]
Social mediation	How does interaction distribute problem-solving activity?	Vygotsky [16]; Wood et al. [33]; Cobb & Yackel [34]; Yackel et al. [35]; Webb [36]; Goos et al. [13]; Walshaw & Anthony [37]
Creativity	How does the model accommodate	Pólya [6]; Silver [38]; Mann [39]; Leikin & Pitta-Pantazi [40]; Liljedahl &

Dimension	Guiding question	Principal sources
	generation, insight, and posing?	Cai [1]
Counter-lens	What supports structured heuristic guidance?	Kirschner et al. [41]; Lester [42], [43]; Schoenfeld [8]; Kapur [44]

➤ Limitations of the Design

The design carries limitations that readers should weigh. A theory synthesis is interpretive by nature, and a different selection of sources could foreground different emphases. The corpus is not exhaustive, and the synthesis does not estimate effect sizes or aggregate findings statistically. The authors' positionality as mathematics educators who have taught with the four-phase model also shapes the analysis. These limitations are mitigated, though not eliminated, by the explicit inclusion of counterarguments in Section X and by the formulation of testable propositions in Section XII.

➤ What Pólya Proposed and what Classrooms Adopted

Pólya's *How to Solve It* is not primarily a list of four steps. The book presents the phases as a way of organising a much larger set of heuristic questions and suggestions, such as asking whether the solver knows a related problem, whether the problem can be restated, or whether all the data have been used. The book models these questions through dialogue in which a teacher helps a student discover a solution rather than receive one. Pólya's guidance for carrying out the plan asks the solver to check each step and to consider whether each step is correct, which constitutes a form of ongoing monitoring rather than a purely terminal check. His guidance for looking back asks the solver to verify the result, to derive it differently, and to consider whether the method might serve other problems [3].

Pólya's later work extended this programme. *Mathematics and Plausible Reasoning* [6] treated guessing, induction, analogy, and generalisation as legitimate and teachable components of mathematical activity. Schoenfeld [5], [45] characterises Pólya's contribution as the revival of heuristics as a serious object of study and emphasises that Pólya regarded *How to Solve It* as a first step in a larger programme rather than a finished method. Rott [4] similarly traces Pólya's influence on subsequent process models, noting that researchers who build non-linear models of problem solving frequently start from Pólya's phases.

The classroom enactment of the model often looks different. When the four phases are presented as a numbered sequence, students may learn to label their work with phase names without engaging in the heuristic reasoning that the phases were intended to organise. Schoenfeld's [8] studies with undergraduates showed that students could name heuristic strategies yet fail to use them productively, because they lacked the domain-specific knowledge to implement the strategies and the control to decide when to use or abandon them. Lester [42] reached a similar conclusion in his review of two decades of problem-solving research, noting that instruction focused narrowly on heuristics yielded limited improvement in students' problem-solving performance.

The distinction between Pólya's programme and its checklist enactment frames the remainder of the paper.

Several limitations examined below arise from reification of the phases rather than from Pólya's thinking. Other limitations, particularly the model's silence on peer interaction and on digital tools, reflect the historical context in which Pólya wrote and therefore belong to the model itself.

III. FROM LINEAR SEQUENCE TO RECURSIVE PROCESS

The checklist enactment implies that a solver completes understanding before planning, planning before execution, and execution before review. Research on problem-solving processes does not support this sequence as a description of what solvers actually do. Schoenfeld's [8] analyses of problem-solving protocols divided solution attempts into episodes such as reading, analysis, exploration, planning, implementation, and verification. The analyses showed that productive solvers moved among these episodes repeatedly and made deliberate decisions at transition points, whereas unproductive solvers often committed early to a single approach and pursued it without reconsideration.

Carlson and Bloom [9] observed twelve mathematicians working on four tasks and developed the Multidimensional Problem-Solving Framework from their observations. The framework has four phases, namely orientation, planning, executing, and checking, which closely resemble Pólya's phases. The framework departs from a linear reading, however, in two important respects. After orienting themselves to a problem, the mathematicians repeated a plan-execute-check cycle throughout the remainder of the solution process. Within the planning phase, they also engaged in a sub-cycle of conjecturing, imagining the consequences of an approach, and evaluating whether to accept or reject it. The framework further identifies resources, affect, heuristics, and monitoring as attributes that operate within every phase.

Research with school students points in the same direction. Rott et al. [10] developed a descriptive phase model from analyses of students' problem-solving processes and documented that many processes departed from a linear sequence, with students returning to earlier phases when they encountered difficulties. Research on mathematical modelling reports comparable findings. Borromeo Ferri [17] reconstructed individual students' modelling processes and found that students did not traverse the phases of the modelling cycle in a fixed order; the study also highlighted the difficulty of assigning students' statements and actions to discrete phases at all. Verschaffel et al. [15] likewise describe how students solving word problems revise their interpretation of the situation, re-represent the given information, and adjust their strategies as understanding develops.

An illustrative vignette, constructed for analytic purposes, clarifies the point. A Grade 10 student receives the

following task: a rectangular garden is 3 metres longer than it is wide, a path 1 metre wide surrounds the garden, and the area of the path is 38 square metres; the student must find the dimensions of the garden. The student draws a diagram, lets the width be w , and writes the outer dimensions as $(w + 2)$ and $(w + 5)$. The student then sets the outer area equal to 38 and obtains a quadratic equation whose roots are not whole numbers. The unexpected result prompts the student to reread the problem, which reveals that 38 square metres is the area of the path alone. The student returns to the understanding phase, revises the plan to subtract the garden's area from the outer area, obtains $4w + 10 = 38$, and finds that the garden measures 7 metres by 10 metres. The student then verifies the result by recalculating the path area.

The vignette shows that monitoring during execution triggered a return to understanding, which in turn produced a new plan. A checklist that treats understanding as complete before planning begins cannot represent this pathway, yet this pathway is characteristic of productive problem solving. The conclusion is not that Pólya's phases are invalid categories. The phases remain useful descriptions of the kinds of activity that problem solving involves. The conclusion is that the phases describe states between which solvers move recursively rather than steps through which they pass once.

➤ *Metacognition as Continuous Regulation*

Pólya's guidance to check each step during execution shows that he did not confine monitoring to the final phase. The checklist enactment, however, typically locates reflection in the looking-back phase, and even Pólya's own formulation offers little explicit guidance on the managerial decisions that determine whether a solution attempt succeeds, such as when to abandon a plan, how to allocate effort, or how to judge whether progress is being made.

Flavell [18] introduced the distinction between metacognitive knowledge and the monitoring and regulation of cognition, and this distinction has since been widely adopted in mathematics education. Garofalo and Lester [11] built on it to propose a cognitive–metacognitive framework for mathematical performance with four categories of activity: orientation, organisation, execution, and verification. The central contribution of their framework was to show that metacognitive decisions occur within every category rather than in a separate terminal stage. Schoenfeld [7], [8] reached a compatible conclusion through his analysis of control, which he identified alongside resources, heuristics, and beliefs as a determinant of problem-solving success. His protocols showed that the absence of control, rather than the absence of knowledge, frequently explained failure.

Broader research on self-regulated learning reinforces this view. Veenman et al. [19] review evidence that metacognitive skilfulness contributes to learning performance beyond intellectual ability. Zimmerman [20] describes self-regulation as a cyclical process of forethought, performance, and self-reflection, in which feedback from each cycle informs the next. This cyclical structure differs from a linear sequence in which reflection occurs once at the end.

Instructional research indicates that metacognition can be taught and that such teaching improves mathematics achievement. Mevarech and Kramarski [21] designed the IMPROVE method, which combines metacognitive questioning, peer interaction, and systematic feedback, and found that seventh-grade students taught with the method outperformed control groups on several measures of mathematics achievement. Kramarski et al. [22] reported that metacognitive instruction also improved students' performance on authentic mathematical tasks. These studies suggest that metacognitive prompts embedded throughout problem solving, rather than attached to its end, offer a productive direction for instruction.

The implication for Pólya's model is therefore specific. Pólya recognised the importance of checking, and his questions contain the seeds of metacognitive regulation. The model as commonly enacted, however, does not represent regulation as a continuous layer that governs transitions between phases. A revised framework should make that layer explicit.

IV. LEARNER DIVERSITY: FROM WHERE STUDENTS STRUGGLE TO WHY

The checklist enactment of Pólya's model implicitly assumes a generic solver who possesses the resources required to move through each phase. Research on students' difficulties challenges that assumption. Tambychik and Meerah [23] asked students in Malaysia to explain their difficulties with mathematical problem solving; the students attributed their difficulties to gaps in specific mathematical skills and to cognitive demands such as processing the information in a problem. Phonapichat et al. [24] reported that elementary students experienced difficulties in comprehending problem statements and in translating them into mathematical representations. These findings indicate that many difficulties originate in conceptual and representational resources rather than in failure to follow a sequence.

Language constitutes a further source of variation. Abedi and Lord [25] showed that the linguistic complexity of mathematics test items affected students' performance and that simplifying the language of items improved the performance of students with limited proficiency in the language of instruction. Affect constitutes yet another source. Ramirez et al. [26] found that mathematics anxiety among first- and second-grade students was associated with reduced use of advanced problem-solving strategies, which in turn was associated with lower achievement, and that this relationship was strongest among children with high working memory capacity.

A careful reading of this evidence requires caution. The finding that students experience difficulties at every phase does not by itself show that Pólya's model is inadequate. Indeed, the phases can serve as a useful diagnostic lens that helps teachers locate *where* a student's attempt breaks down. The limitation lies elsewhere: the model offers little account of *why* a breakdown occurs. A student who cannot understand

a problem may lack the relevant concept, may be unable to parse the language of the task, or may be too anxious to persist long enough to make sense of it. Each cause calls for a different instructional response. A framework that aims to guide teaching must therefore attend to the conceptual, linguistic, and affective conditions under which students engage with each phase.

➤ *Problem Solving in Technology-Rich Environments*

Pólya developed his model before digital technology became part of mathematics classrooms, and the model therefore makes no provision for tools that act on the solver's reasoning. Contemporary classrooms increasingly include dynamic geometry environments, computer algebra systems, spreadsheets, graphing tools, programming environments, and, more recently, artificial intelligence applications [12]. These tools change the texture of problem solving in ways that bear directly on the phase structure.

Dynamic tools allow students to test a conjecture almost as soon as they form it, which compresses planning, execution, and checking into rapid cycles. Kuzle [28] examined the metacognitive processes of a preservice teacher solving a non-routine geometry problem in a dynamic geometry environment and found that the software supported strategies that were not available with paper and pencil. The study also found that the effectiveness of solution paths depended on managerial decisions and on the coordinated use of knowledge and technology, and it questioned whether every metacognitive behaviour was productive. Barrera-Mora and Reyes-Rodríguez [29] similarly examined the cognitive processes that students developed when solving problems within technological environments. Hegedus and Otálora [30] studied fourth-grade students working collaboratively in a multi-touch dynamic geometry environment and reported that interactions among peers, the interviewer, and the software fostered socially mediated metacognitive processes and the co-construction of mathematical strategies.

Research on computational thinking extends this argument. Wing [31], in an influential position paper, argued that computational thinking constitutes a fundamental skill for all students rather than a specialism of computer scientists. Weintrop et al. [32] subsequently developed a taxonomy of computational thinking practices for mathematics and science classrooms that includes data practices, modelling and simulation practices, computational problem-solving practices, and systems thinking practices. These practices involve iterative cycles of designing, testing, and refining solutions, which align more closely with a recursive model than with a linear one.

Technology does not guarantee improved problem solving. Drijvers [27] analysed six cases from leading studies and concluded that the success of digital technology in mathematics education depends on the design of the tool and the tasks that exploit its pedagogical potential, on the role of the teacher, and on the educational context. This conclusion supports a mediated rather than a deterministic view: tools reshape the phase structure of problem solving, but the direction of that reshaping depends on task design and

teaching. A revised framework should therefore represent digital tools as mediating conditions that interact with the solver's regulation of the process.

➤ *Problem Solving as a Socially Mediated Activity*

Socio-constructivist theory holds that higher mental functions originate in social interaction before they are internalised by the individual [16]. Wood et al. [33] introduced the concept of scaffolding to describe how a tutor supports a young learner in completing a task beyond the learner's unassisted capability, and the concept has since been extended to classroom settings. Cobb and Yackel [34] proposed an emergent perspective that coordinates individual cognition with the social norms and practices of the classroom, arguing that neither can be understood in isolation.

Pólya's model addresses the individual solver. The dialogues in *How to Solve It* do represent a form of social mediation, but the mediation is asymmetric: an expert teacher poses questions to guide a single student. Contemporary classrooms, in contrast, frequently organise problem solving around small-group work and whole-class discussion. Yackel et al. [35] showed that small-group interactions in second-grade mathematics created learning opportunities that did not arise in individual work. Webb [36] reviewed studies of verbal interaction in small groups and found that giving elaborated explanations was positively related to mathematics achievement. Walshaw and Anthony [37] reviewed research on mathematics classroom discourse and concluded that the teacher's role in shaping explanation, argumentation, and the defence of ideas is central to the quality of students' mathematical learning.

The most direct challenge to the individualist framing comes from research on socially mediated metacognition. Goos et al. [13] conducted a three-year study of small-group problem solving in senior secondary mathematics classrooms. Unsuccessful problem solving was characterised by poor metacognitive decisions compounded by a lack of critical engagement with peers' thinking, whereas successful outcomes were associated with students challenging and discarding unhelpful ideas and endorsing useful strategies. The study reconceptualised metacognition as a social practice. This finding implies that, in collaborative settings, the monitoring function that Pólya assigned to the individual can be distributed across members of a group.

The model therefore requires extension rather than replacement. A revised framework should represent peer discourse as a condition that shapes how students understand problems, generate and evaluate plans, and verify results, and it should recognise that regulation may be shared as well as individual.

V. CREATIVITY, INSIGHT, AND HIGHER-ORDER THINKING

A common criticism holds that Pólya's model leaves little room for creativity. This criticism requires qualification. Pólya's treatment of plausible reasoning explicitly valued guessing, analogy, and generalisation as sources of

mathematical discovery [6]. The concern is more accurately directed at the checklist enactment, which tends to emphasise a single convergent route to a correct answer rather than the generation and comparison of multiple approaches.

Research on mathematical creativity describes a broader set of processes. Silver [38] argued that creativity can be productively viewed as an orientation or disposition that can be fostered in the general school population, rather than as a trait of exceptional individuals. Silver identified fluency, flexibility, and novelty as core dimensions of creativity and argued that instruction rich in problem solving and problem posing can develop these dimensions. Mann [39] argued that creativity lies at the heart of mathematics as a discipline and that school mathematics often neglects it. Leikin and Pitta-Pantazi [40] reviewed the state of research on creativity in mathematics education and described the creative product, process, person, and environment as distinct foci of study. Liljedahl and Cai [1] further documented the growth of research on problem posing as a complement to problem solving.

The conjecture–imagine–evaluate sub-cycle identified by Carlson and Bloom [9] illustrates how creative generation and critical evaluation interweave within the planning phase. A checklist that treats devising a plan as the selection of a known strategy obscures this generative activity. Alternative creative problem-solving models are, however, also typically presented as sequences of stages, such as generating ideas, evaluating them, and implementing the most promising option. The lesson is therefore not that stages as such suppress creativity. The lesson is that any staged model, including Pólya's, constrains creativity when teachers present it rigidly, and supports creativity when teachers present it as a set of recursively accessible moves that include divergent generation as well as convergent verification.

➤ *Counterarguments: The Case for Structured Heuristics*

A balanced reassessment must engage the strongest case in favour of Pólya's model as a structured scaffold. Cognitive load theory provides that case. Kirschner et al. [41] argued that minimally guided instruction is less effective than explicit guidance for novice learners, because novices lack the organised knowledge in long-term memory that would allow them to manage the working-memory demands of unstructured problem solving. From this perspective, a clear phase structure can reduce the burden of deciding what to do next and can direct attention to productive activity. The critique developed in Sections IV to IX must therefore be reconciled with evidence that structure supports beginners.

Research on heuristic instruction adds nuance rather than settling the question. Lester [42] reported that instruction focused narrowly on heuristics produced limited gains. Lester [43] later summarised principles about problem-solving instruction that had emerged since the early 1970s, including the principle that problem-solving ability develops slowly and requires sustained instructional attention over an extended period. Schoenfeld [8] showed that heuristic instruction became effective when strategies were taught with sufficient specificity and combined with explicit attention to control.

These findings suggest that the value of a structured model depends on how teachers use it rather than on whether they use it.

Research on productive failure introduces a further consideration concerning timing. Kapur [44] reported two randomised studies in which students who attempted to solve problems before receiving instruction developed greater conceptual understanding and transfer than students who received instruction first, without any loss of procedural knowledge. This evidence does not contradict cognitive load theory directly; rather, it suggests that the sequencing of guidance and exploration matters for different learning outcomes.

The counterarguments sharpen rather than overturn the critique. They support retaining Pólya's phases as an explicit scaffold for novices, and they locate the problem in rigidity rather than in structure. A defensible position therefore holds that the phases should be taught explicitly, presented as recursive rather than sequential, accompanied by continuous metacognitive prompts, and gradually faded as students develop the resources and regulation to manage problem solving independently.

VI. DISCUSSION: FROM HEURISTIC MODEL TO HISTORICAL ARTEFACT?

The six dimensions examined in this paper do not operate independently, and their interactions carry the most important implications of the analysis. Digital tools amplify recursion, because rapid feedback invites students to cycle through planning, execution, and checking many times within a single attempt. Collaboration externalises metacognition, because peers who challenge and endorse one another's ideas perform the monitoring function that Pólya assigned to the individual [13], [30]. Language and affect determine where recursion stalls, because a student who cannot parse a problem or who avoids advanced strategies under anxiety cannot return productively to earlier phases [25], [26]. Creativity depends on recursion, because the generation and evaluation of conjectures requires repeated movement between planning and checking [9].

These interactions suggest that the principal limitation of the checklist enactment is not any single omission but the representation of problem solving as a sequence performed by an isolated individual. That representation obscures the continuous regulation, the social and digital mediation, and the learner-specific conditions that research identifies as central.

The question posed in the heading of this section can now be answered directly. Pólya's model has not become a historical artefact. The four phases continue to describe the principal kinds of activity that problem solving involves, and they continue to underpin contemporary process models, including models that explicitly reject linearity [4], [9], [10]. The checklist enactment of the model, however, has become outdated as a description of how students solve problems and as a guide to how teachers should teach problem solving. The

appropriate response is therefore reconceptualisation rather than replacement.

➤ *Towards a Recursive, Regulated and Mediated Framework*

This section proposes a framework that preserves Pólya's phases while addressing the limitations identified above. The framework comprises three nested layers, which Fig. 1 depicts and Table 2 summarises.

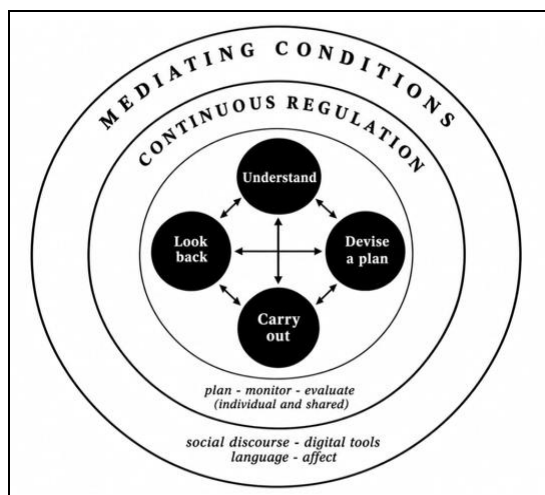


Fig 1 The Recursive, Regulated, and Mediated Problem-Solving Framework

Table 2 Components of the Recursive, Regulated, and Mediated Framework

Layer	Components	Theoretical grounding	Classroom indicator
Recursive core	Understand, devise a plan, carry out, look back, as mutually accessible states	Pólya [3]; Carlson & Bloom [9]; Rott et al. [10]	Students return to earlier phases when monitoring reveals a problem
Continuous regulation	Planning, monitoring, and evaluation across all phases; individual and shared	Garofalo & Lester [11]; Schoenfeld [7]; Zimmerman [20]; Goos et al. [13]	Students justify decisions to continue, revise, or abandon a plan
Mediating conditions	Social discourse; digital tools and tasks; language and concepts; affect	Webb [36]; Walshaw & Anthony [37]; Drijvers [27]; Abedi & Lord [25]; Ramirez et al. [26]	Teachers design discussion, tool use, language support, and affective support deliberately

The framework yields six design principles for instruction. First, teachers should present the four phases as moves that students can make at any point rather than as steps to complete in order. Second, teachers should embed metacognitive prompts in every phase, for example by asking students during execution whether their current approach is making progress and what evidence supports that judgement. Third, teachers should sequence exploration and explicit guidance deliberately, allowing productive struggle where conceptual understanding is the goal and providing structured guidance where novices would otherwise be overwhelmed [41], [44]. Fourth, teachers should structure collaboration so that students explain their reasoning and challenge one another's ideas, rather than simply dividing labour [13], [36]. Fifth, teachers should select digital tools for an explicit didactical function and design tasks that exploit that function [27]. Sixth, teachers should anticipate linguistic and affective barriers and should fade the phase scaffold as students' regulation develops.

The first layer is a *recursive core*. The core retains Pólya's four phases but represents them as mutually accessible states rather than sequential steps. A solver may move from any phase to any other, and transitions are triggered by the outcomes of monitoring, as in the plan–execute–check cycles described by Carlson and Bloom [9] and the phase transitions documented by Rott et al. [10].

The second layer is *continuous regulation*. Regulation comprises planning, monitoring, and evaluation, and it operates across all phases rather than within the looking-back phase alone [11], [20]. Regulation may be exercised by an individual or shared among members of a group, as research on socially mediated metacognition demonstrates [13].

The third layer comprises *mediating conditions*. These conditions include social discourse among peers and with the teacher [36], [37], digital tools and the tasks designed around them [27], [32], learners' linguistic and conceptual resources [23], [25], and affect [26]. The conditions shape how each phase unfolds and how readily students can move between phases.

The framework also generates propositions that empirical research can test. Proposition 1 holds that students taught the phases as recursive moves will display more phase transitions and more productive abandonment of unsuccessful plans than students taught the phases as a sequence. Proposition 2 holds that metacognitive prompts distributed across all phases will improve problem-solving performance more than prompts confined to the looking-back phase. Proposition 3 holds that structured collaboration will increase the frequency of socially shared monitoring and that this increase will mediate gains in problem-solving success. Proposition 4 holds that the benefits of digital tools for problem solving will depend on whether tasks are designed to exploit the tools' capacity for rapid conjecture and testing.

VII. IMPLICATIONS

➤ *Implications for Classroom Practice*

Classroom teachers can retain the familiar vocabulary of Pólya's phases while changing how they present and use it.

Displays and worked examples that show solution pathways looping back to earlier phases can communicate that returning to understanding is a sign of good problem solving rather than failure. Questioning routines can attach monitoring prompts to every phase, and group tasks can assign roles that require students to explain and to challenge.

➤ *Implications for Teacher Education*

Teacher education programmes can help preservice and in-service teachers distinguish Pólya's heuristic programme from its checklist enactment. Programmes can also develop teachers' capacity to diagnose why a student's attempt breaks down, rather than only where it breaks down, and to orchestrate productive mathematical discourse [37]. Lester [43] identifies the proficiencies that teachers need to support problem solving as an important focus for research and development.

➤ *Implications for Curriculum and Assessment*

Curriculum documents that reference Pólya's model can describe the phases as recursive and can specify metacognitive and collaborative competencies alongside the phases. Assessment practices can reward evidence of monitoring, revision, and justification rather than only correct final answers, and can include tasks that invite multiple solution methods and problem posing [38].

➤ *Implications for Research*

The propositions in Section XII offer a starting point for empirical investigation. Classroom-based design research could examine how teachers enact the framework and how students' phase transitions change as a result. Studies in multilingual and resource-constrained settings would be particularly valuable, because the linguistic and affective conditions identified in Section VI may operate differently across contexts. Research on artificial intelligence tools in mathematics classrooms, which Engelbrecht and Borba [12] identify as an emerging area, could examine whether such tools support or short-circuit students' regulation of problem solving.

VIII. CONCLUSION

This paper has reassessed Pólya's four-phase model of problem solving in light of contemporary research. The analysis distinguished Pólya's heuristic programme from the checklist into which it has frequently been reduced. In answer to the first research question, the analysis found that the checklist enactment misrepresents problem solving in six respects: it implies linearity where research shows recursion, it locates reflection at the end where research shows continuous regulation, it assumes a generic solver where research shows diverse conceptual, linguistic, and affective conditions, it overlooks the mediating role of digital tools, it frames problem solving as individual where classrooms make it social, and it tends to privilege convergent routes over creative generation.

The paper also found that evidence from cognitive load theory supports the explicit teaching of a structured scaffold for novices. The synthesis therefore rejects both uncritical

retention and wholesale replacement of the model. In answer to the second research question, the proposed recursive, regulated, and mediated framework retains Pólya's phases as an iterative core, adds a continuous layer of individual and shared regulation, and situates problem solving within the social, digital, linguistic, and affective conditions of contemporary classrooms. The framework offers teachers a practical way to build on Pólya's enduring contribution and offers researchers a set of propositions to test.

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