

Batch-Service Queueing Model with Multiple Working Vacation, Change Over Time and Reneging

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Abstract: This work comprises a model with the multiple working vacations, reneging, and change over time of the server in an individual arrival-batch service queue. The server performs three types of service modes, such as working vacation, batch service in platform-I, and batch service in platform-II according to (a,c,d) policy. The system activates only when the accumulation of customers reaches 'a', and handles 'b' or less than 'b' units at a time in a batch with normal service rates in Platform -I. If the queue length exceeds 'b', the server shifts to Platform-II and serves maximum 'd' units in a batch with a high rate. Queue size below 'a' incites the server to take multiple working vacations at a slower rate. Reneging of units is considered when the server opts for vacation. Change over time is also deemed when a-1 and b units are in the queue. Steady-state probabilities have been obtained and performance measures like expected queue length, expected busy period, idle time of the server, expected waiting time are developed. Finally, some particular cases and numerical illustrations have been discussed.

Keywords: Markovian Arrival and Bulk Service, (a, b, d) Control Limit Policy, Reneging, Multiple Working Vacation, Change Over Time.

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I. INTRODUCTION

Bulk service models appear in powerful areas like semiconductors manufacturing systems, computer networks, transportation, etc. The queue size below a limit or above a limit is not always practicable to serve them in a batch at a time, considering the server capacity. Also, activation of the server with high capacity is not profitable for a small queue size. It is far better to operate two service platforms based on the queue size, such as a general and an additional server. Queija[10] prepared a queueing model in which the server switches service rate concerning the finite and infinite storage queues. Andrews et.al.[1] analyzed a discrete-time queueing model with varying service rates and depicted the stability of the system in connection with arrival rate. Chakravarthy et.al.[5] analyzed the MAP queueing model with a single server at the beginning of the service, and the additional servers added based on the queue size.

When the queue size drops below a limit, the server opts for vacation until the queue size reaches its minimum. Serving units with a lower rate in the vacation stage came into practice at the beginning of the twentieth century. Servi and Finn[12], Baba[2], Banik[4], Zhang and Hou[19] etc. presented so many papers on working vacation in different

models. Tian et.al.[13] surveyed all working vacation models and demonstrated that the effective tool for analyzing working vacation is the matrix analytical approach. Lin and Ke[8] studied varying service rates of R servers in a Markovian model with the single working vacation of the server using matrix analytical method. The bulk service with policy (a,c,d) in a discrete-time queueing model, is investigated by Baburaj[3]. Yu et.al.[18] searched optimum service rate in the general input model with server's change over time and working vacation under a given cost structure employing parabolic method. Vijay-alaxmi and Suchitra[17] studied G/M/1 queue with state-dependent and change over time of the server employing the supplementary variable approach. Vijayalaxmi and Seleshi[16] analyzed the steady-state probability and performance measures of batch service model with policy (a, c, d) and Bernoulli scheduled vacation interruption using the supplementary variable method. They [11] considered a model for batch service system which performs according to the (a, c, d) policy with multiple working vacations and changes over time. Recently Chandrashekar et.al[6] gathered information about working vacation models motivating analysts, industry people who require working vacation models for removing congestion in their systems.

Impatience of customers negatively affect the performance of the system in case of profit, time, cost, and reneging behavior of the customers, because reneging of one customer motivates others to do the same. Zhao et.al.[20] considered balking and reneging in the M/M/1/N queue with multiple working vacations using the matrix so-lution method. Maheshwari et.al.[9] studied the reneging effect in machine repair with multiple vacations of repairmen using the Matrix recursive approach. Goswami [7] is concerned with reneging and balking in buffer arrival queueing model with single or multiple vacations. Vijayalaxmi and Jyothsna[14] discussed balking and exponentially distributed reneging when the server is on different vacation in a single server model. They modified this study for heterogeneous servers, namely server-I and server-II, which performs as, if the queue size is null then server-II goes on working vacation whereas server-I stays idle in the system[15].

This paper represents two different rated balk service platforms, namely, Platform-I with normal service rate and Platform-II with faster service rate that is activating according to the queue size. Any one of the two Platforms is activating at a time, and Platform-II has high capacity, rate, and maintenance cost than Platform-I. It is considered that, serving in Platform-II for a small queue size will badly affect the performance and maintenance of the system. The reneging concept is applied when the server is on complete vacation. The steady-state probabilities of this model have been obtained employing the recursive method. The measures of performance are procured, and some of the particular cases are studied. Numerically showed the effect of parameters on queue size and cost.

II. MODEL DESCRIPTION AND NOTATIONS

This model describes, an infinite capacity single arrival with (a, b, d) policy where $a \leq b \leq d$ and exponentially distributed services(Platform-I, Platform-II, and working vacation), multiple^ vacations, change over time and reneging of the customer. It executes as follows,

- The Markovian arrivals with rate λ enter the system and get into service according to FCFS rule. The customer goes into a batch service when the number of customers reaches a, and the server attends to a maximum of b units in Platform-I with service rate μ_1 .
- When the queue size exceeds a point 'b', the sever shifts the service platform to platform-II with a maximum capacity of d, with exponential service rate μ_2 and serves all units together. Once a batch is served in Platform-II, and the server finds the queue size is empty or below the lower control limit, or greater than b, the server opts for vacation or serves in Platform-I or continues in Platform-II, respectively.
- If the number of customers drops below the lower threshold(a), the server opts for vacation. The server on vacation may encourage the customer to renege from the system at a rate α before starting the working vacation. Suppose, if there are 'n' customers waiting for the service in the queue, then the average reneging rate is measured as $n\alpha$, $n = 1, 2, \dots, a$.
- Whenever the server finds the number of customers in the waiting room is less than a, he continuous vacation, otherwise he gets into normal service. Once the server enters vacation, he executes low-rated multiple services which are exponentially distributed with the rate η until he finds the queue size more than the lower control limit(a).
- When the server returns from vacation and finds queue size equals $a - 1$, he will wait for arrival(Change over time), with exponential rate v_1 . If he gets a unit in the change over time, his busy period starts with a normal service rate. Otherwise, he will back to working vacation with the exponential rate ϕ .
- As well as at b queue size, and the server is in Platform-I, the server will wait for arrival (Change over time) with exponential waiting rate v_2 . If the server finds customers during this change over time, he shifts to Platform-II and, serves all units together in a batch with a rate of μ_2 . Otherwise, he will continue working on Platform-I itself.

The parameters used in this models are tabulated as below.

Table 1 Notations

Notations	Descriptions
λ	Arrival rate
μ_1	Rate of Platform-I Service
μ_2	Rate of Platform-II Service
α	Reneging rate
η	Working vacation rate
β	Multiple vacation rate
v_1	Change over time rate at $n=a-1$
v_2	Change over time rate at $n=b$

III. STEADY STATE PROBABILITIES AND ANALYSIS

Let us define the steady states P_n , Q_n , W_n and $(T_{(a-1)}, T_b)$ which represents the busy state of server in Platform-I for $n \geq 0$, busy state of server in Platform-II for $n \geq 0$, working vacation state for $n \geq 0$ and change over time of server at $n = (a - 1)$ and $n = b$ respectively, where, 'n' be the number of customers waiting for the service. The transition diagram of this model is shown below.

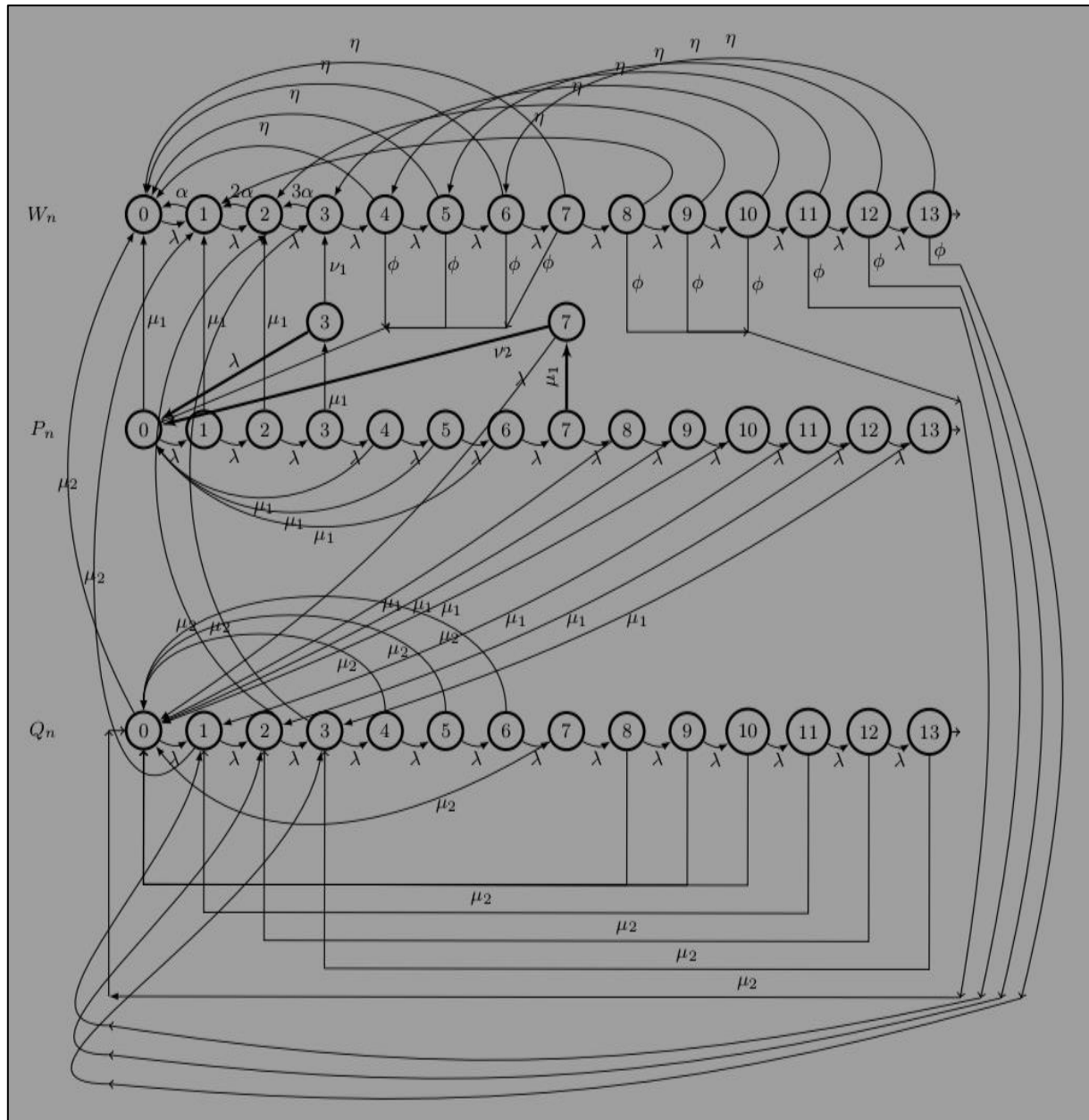


Fig 1 Transition Diagram

The Chapman-Kolmogorov forward difference equations of this system are defined as follows.

$$\lambda W_0 = \mu_1 P_0 + \mu_2 Q_0 + \alpha W_1 + \eta \sum_{n=a}^b W_n \quad (1)$$

$$(\lambda + n\alpha)W_n = \mu_1 P_n + \mu_2 Q_n + \lambda W_{n-1} + (n+1)\alpha W_{n+1} + \eta W_{n+b}, \quad 1 \leq n \leq a-2 \quad (2)$$

$$\begin{aligned} (\lambda + (a-1)\alpha + \phi)W_{a-1} &= \nu_1 T_{a-1} + \mu_2 Q_{a-1} + \lambda W_{a-2} + \eta W_{a-1+b} \\ (\lambda + \phi + \eta)W_n &= \lambda W_{n-1} + \eta W_{n+b}, \end{aligned} \quad (3)$$

$$n \geq a \quad (4)$$

$$(\lambda + \mu_1)P_0 = \lambda T_{a-1} + \mu_1 \sum_{n=a}^b P_n + \phi \sum_{n=a}^b Q_n \quad (5)$$

$$(\lambda + \mu_1)P_n = \lambda P_{n-1} \quad n \geq 1 \quad (6)$$

$$(\lambda + \mu_2)Q_0 = \lambda T_b + \mu_2 \sum_{n=b+1}^d Q_n + \phi \sum_{n=b+1}^d P_n \quad (7)$$

$$(\lambda + \mu_2)Q_n = \lambda Q_{n-1} + \phi W_{n+d} \quad n \geq 1 \quad (8)$$

$$(\lambda + \nu_1)T_{a-1} = \mu_1 P_{a-1} \quad (9)$$

$$(\lambda + \nu_2)T_b = \mu_1 P_b \quad (10)$$

Using recursive method, let us solve the steady state probabilities as follows.

Equation (3.6) $\Rightarrow$

$$P_n = m_1^n P_0, \quad m_1 = \frac{\lambda}{\lambda + \mu_1} \quad (11)$$

Equation (3.9) $\Rightarrow$

$$T_{a-1} = m_2 m_1^{a-1} P_0, \quad m_2 = \frac{\mu_1}{\lambda + \nu_1} \quad (12)$$

Equation (3.10) $\Rightarrow$

$$T_b = m_3 m_1^b P_0, \quad m_3 = \frac{\mu_1}{\lambda + \nu_2} \quad (13)$$

Applying shift operator z in equation (3.4) gives equation as, $K(z) = \eta z^{b+1} - (\lambda + \phi + \eta)z + \lambda = 0$ which can be solved by Rouché's Theorem as:

$$W_n = W_0 r^n, \quad n \geq a \quad (14)$$

Here, root of the equation $K(z)$ is 'r' which falls in between 0 and 1. Applying the values of T_{a-1} , P_n , W_n in (3.5), we get,

$$W_0 = P_0 C_1, \quad C_1 = \frac{(1-r)[1-\lambda m_1^a [m_2+1-m_1^{b-a}]]}{\phi m_1 r^a (1-r^{b-a})} \quad (15)$$

Let us consider,

$$R = \frac{1-\lambda m_1^a (m_2+1-m_1^{b-a})}{m_1 (1-r^{b-a})},$$

Then

$$C1 = \frac{(1-r)R}{\phi r^a}$$

Equating (3.8), we obtain for $n \geq 1$,

$$Q_n = m_4^n Q_0 + C_2(m_4^n - r^n)P_0 \quad (16)$$

$$C2 = \frac{(1-r)m_4 r^{d+1-a} R}{m_4 - r} \text{ and, } m_4 = \frac{\lambda}{\lambda + \mu_2} \quad (16)$$

Solving equation (3.7) yields,

$$Q_0 = C_3 P_0 \quad (17)$$

Where,

$$C_3 = \frac{m_4 \{ \lambda m_3 m_1^b + \lambda m_4^b (1 - m_4^{d-b}) C2 - r^{b+1} (1 - r^{d-b}) R \left[\frac{\mu_2 m_4 r^{d+1-a}}{m_4 - r} - \frac{\lambda}{r^a} \right] \}}{\lambda (1 - m_4^{b+1} (1 - m_4^{d-b}))}$$

Substituting the value of Q_0 in (3.16) gives,

$$Q_n = m_4^n C_3 + C_2(m_4^n - r^n)P_0 \quad (18)$$

Solving (3.2) gives,

$$W_n = \left\{ \begin{array}{l} C_1 f(n, \alpha, \lambda) - \frac{1}{\lambda} f(n, \alpha, \lambda) S_{(0, n-1)} \left[\mu_1 (1 + C_3) + \eta r^a \frac{(1-r^{b-a})}{1-r} \right] \\ - \frac{1}{\lambda} f(n, \alpha, \lambda) [\mu_1 S^*(m_1, n-1) + \mu_2 (C_2 + C_3) S^*(m_4, n-1) + (\eta C_1 - \mu_2 C_2) S^*(r, n-1)] \end{array} \right\} P_0 \quad (19)$$

$$\text{Where, } f(n, \alpha, \lambda) = \frac{\lambda^n}{n! \alpha^n}, \quad S(0, n) = \sum_{j=0}^n j! \left(\frac{\alpha}{\lambda}\right)^j, \quad S^*(x, n-1) = \sum_{i=1}^{n-1} x^i S(i, n-1).$$

For $n=c-2$,

$$W_{c-2} = K P_0, \text{ where}$$

$$K = \left\{ \begin{array}{l} C_1 f(a-2, \alpha, \lambda) - \frac{1}{\lambda} f(a-2, \alpha, \lambda) S_{(0, a-3)} \left[\mu_1 (1 + C_3) + \eta r^a \frac{(1-r^{b-a})}{1-r} \right] \\ - \frac{1}{\lambda} f(a-2, \alpha, \lambda) [\mu_1 S^*(m_1, a-3) + \mu_2 (C_2 + C_3) S^*(m_4, a-3) + (\eta C_1 - \mu_2 C_2) S^*(r, a-3)] \end{array} \right\}$$

Substituting this value in (3.3) gives,

$$W_{a-1} = \frac{P_0}{(\lambda + (a-1)\alpha + \phi)} [r^{a-1} (\eta r^b C_1 - \mu_2 C_2) + \nu_1 m_2 m_1^{a-1} + \mu_2 m_4^{a-1} [C_2 + C_3] + \lambda K] \quad (20)$$

$$= P_0 C_4$$

Finally, P_0 can be obtained by the normalizing condition

$$P_0 = \frac{1}{\left\{ 1 + C_3 + C_4 + \frac{m_1}{1-m_1} + \frac{m_4}{1-m_4} [C_2 + C_3] - \frac{r}{1-r} [C_2 - r^{a-1} C_1] + m_2 m_1^{a-1} + m_3 m_1^b + C_1 E_{a-2} \left(\frac{\lambda}{\alpha} \right) \right\} - \frac{1}{\alpha} S \left[\mu_1 (1 + C_3) + \eta r^a \frac{(1-r^{b-a})}{1-r} \right] - [\mu_1 S^{**}(m_1) + \mu_2 (C_2 + C_3) S^{**}(m_4) + (\eta C_1 - \mu_2 C_2) S^{**}(r)] \}} \quad (21)$$

Where,

$$E_{\alpha-2}\left(\frac{\lambda}{\alpha}\right) = \frac{\lambda}{\alpha} + \frac{1}{2!} \left(\frac{\lambda}{\alpha}\right)^2 + \dots + \frac{1}{(a-2)!} \left(\frac{\lambda}{\alpha}\right)^{a-2}$$

$$S = \frac{\lambda}{\alpha} \sum_{i=1}^{a-2} \frac{1}{i} + \frac{\lambda^2}{\alpha} \sum_{i=2}^{a-2} \frac{1}{i(i-1)} + \dots + \left(\frac{\lambda}{\alpha}\right)^{a-2} \frac{1}{(a-2)!}$$

$$S^{**}(x) = \sum_{k=1}^{a-3} \frac{x^k}{\alpha} \left[\sum_{i=k+1}^{a-2} \frac{1}{i} + \frac{\lambda}{\alpha} \sum_{i=k+2}^{a-2} \frac{1}{i(i-1)} + \dots + \left(\frac{\lambda}{\alpha}\right)^{a-3-k} \frac{1}{(a-2)(a-3) \dots (k+1)} \right]$$

➤ Performance Measures

• Queue-Length of the Model:

The L_q , queue-length of this model is equated as:

$$L_q = E(QL) = \frac{\lambda}{\alpha} C_1 E_{\alpha-3} \left(\frac{\lambda}{\alpha}\right) - \frac{1}{\alpha} \hat{S} [\mu_1(1 + C_3) + \eta r^a \frac{(1 - r^{b-a})}{1 - r}]$$

$$- [\mu_1 \hat{S}^{**}(m_1) + \mu_2(C_2 + C_3) \hat{S}^{**}(m_4) + (\eta C_1 - \mu_2 C_2) \hat{S}^{**}(r)] + (a-1) C_4 \frac{C_1 r^a}{1 - r} + \left(\frac{m_1}{1 - m_1}\right)^2 + (C_2 + C_3)$$

$$\frac{m_4}{(1 - m_4)^2} - C_2 \frac{r}{(1 - r)^2} + (a-1) m_2 m_1^{a-1} + b m_3 m_1^b$$

Where,

$$E_{\alpha-2}\left(\frac{\lambda}{\alpha}\right) = \frac{\lambda}{\alpha} + \frac{1}{2!} \left(\frac{\lambda}{\alpha}\right)^2 + \dots + \frac{1}{(a-2)!} \left(\frac{\lambda}{\alpha}\right)^{a-2}$$

$$\hat{S} = \frac{\lambda}{\alpha} (a-2) + \frac{\lambda^2}{\alpha} \sum_{i=1}^{a-2} \frac{1}{i} + \dots + \left(\frac{\lambda}{\alpha}\right)^{a-2} \frac{1}{(a-3)!}$$

$$\hat{S}^{**}(x) = \sum_{k=1}^{a-3} \frac{x^k}{\alpha} \left[(a-2) + \frac{\lambda}{\alpha} \sum_{i=k+1}^{a-2} \frac{1}{i} + \dots + \left(\frac{\lambda}{\alpha}\right)^{a-3-k} \frac{1}{(a-3)(a-4) \dots (k+1)} \right]$$

• Waiting Time:

Average Waiting-Time, W_q of customer is given by:

$$E(W) = Q.L/\lambda \quad (22)$$

• Idle Time:

Average length of idle time is:

$$E(I) = \frac{a}{\lambda} + \frac{1}{v_1} + \frac{1}{v_2} \quad (23)$$

• Probability of Server is Idle:

$$P(\text{Idle} - \text{Server}) = \sum_{n=0}^{a-1} W_n + T_{a-1} + T_b \quad (24)$$

• Busy Cycle:

Busy cycle covers server's both the idle and busy duration. On the other way, the probability of idle-server can be expressed as the fraction of expected idle time and expected busy cycle. Hence:

$$E(\text{busy} - \text{Cycle}): E(c) = \frac{\frac{a}{\lambda} + \frac{1}{v_1} + \frac{1}{v_2}}{\sum_{n=0}^{a-1} W_n + T_{a-1} + T_b} \quad (25)$$

Busy period:

Let us define,

$$P(\text{The server is busy in platform-I}): = \frac{E(B_1)}{E(c)}$$

$$P(\text{The server is busy in platform-II}): = \frac{E(B_2)}{E(c)}$$

$$P(\text{The server is busy on working vacation}) = \frac{E(V)}{E(C)} \text{ and hence,}$$

$$E(\text{busy period in platform-I}): E(B_1) = \frac{\frac{a}{\lambda} + \frac{1}{v_1} + \frac{1}{v_2}}{\sum_{n=0}^{a-1} W_n + T_{a-1} + T_b} \sum_{n=1}^{\infty} P_n$$

$$E(\text{busy period in platform-II}): E(B_2) = \frac{\frac{a}{\lambda} + \frac{1}{v_1} + \frac{1}{v_2}}{\sum_{n=0}^{a-1} W_n + T_{a-1} + T_b} \sum_{n=1}^{\infty} Q_n$$

$$E(\text{The server is busy on working vacation}): E(V) = \frac{\frac{a}{\lambda} + \frac{1}{v_1} + \frac{1}{v_2}}{\sum_{n=0}^{a-1} W_n + T_{a-1} + T_b} \sum_{n=1}^{\infty} W_n$$

- *Optimum Policy:*

It is a method of finding the perfect values of control limits (a, v_1, v_2) which maximize the total profit and improves the performance of the system. Let us consider $C_h, C_{st}, C_1, C_2, C_3$ are holding cost of units, the startup cost of the server before starting the batch service, cost of service in Platform-I, cost of service in Platform-II and cost of working vacation service respectively according to unit time and C_s is the set up charge of the system per busy cycle. The linear cost function is expressed as:

$$Cost(a, v_1, v_2) = C_h L_q + C_s \frac{1}{E(C)} + C_{st} \frac{E(I)}{E(C)} + C_1 \frac{E(B_1)}{E(C)} + C_2 \frac{E(B_2)}{E(C)} + C_3 \frac{E(V)}{E(C)}$$

We can obtain the optimum values of the parameters numerically by minimizing the total cost.

➤ *Particular Cases:*

- *Case (i):*

When $a = b = d = 1$ and $\alpha = 0$ gives a single server model without a second platform ($\mu_2 = 0$), reneging and change over time. Then the required steady state probabilities are:

$$\lambda W_0 = \mu_1 P_0 + \eta W_1 \quad (26)$$

$$(\lambda + \phi) W_n = \lambda W_{n-1} + \eta W_{n+1}, \quad n \geq 1 \quad (27)$$

$$(\lambda + \mu_1) P_0 = \mu_1 P_1 + \phi W_1, \quad (28)$$

$$(\lambda + \mu_1) P_n = \mu_1 P_{n+1} + \lambda P_{n-1} + \phi W_{n+1} \quad n \geq 1 \quad (29)$$

$$P_0 = \frac{1}{\left\{ 1 + \frac{k}{1-k} + \frac{\mu_1}{\lambda - \eta r} \left(1 + \frac{r}{1-r} - \frac{\phi r^2}{(1-r)K(r)} \right) \right\}}$$

$$P_n = P_0 \left[k^n - \frac{\phi r^{n+1} \mu_1}{K(r)(\lambda - \eta r)} \right], \quad n \geq 1, \quad W_n = \frac{r^n \mu_1}{\lambda - \eta r} P_0, \quad n \geq 0$$

Where k is the root of the equation $K(z) = \mu_1 z^2 - (\lambda + \mu_1)z' + \lambda = 0$ of equation (3.29) and only on possible value of k is, $k = \frac{\lambda}{\mu_1}$. Hence this model indicates a single service model with infinite limits and multiple vacations without reneging.

• *Case (ii):*

Applying, $b = d, v_1 = v_2$ and $\alpha = 0$ in the model, generates a batch service model without additional platform ($\mu_2 = 0$) and reneging. The model represents $M/M^{(a,b)}/1/M/V$ model along with change over time.

• *Case(iii):*

Taking $a = b = d = 1$, $\alpha = 0$, $\eta = 0$ and no change over time gives a single service model without the additional

platform ($\mu_2 = 0$) and reneging. The steady-state probabilities of this model are, $P_0 = 1 - \frac{\lambda}{\mu_1}$ and $P_n = (\frac{\lambda}{\mu_1})^n (1 - \frac{\lambda}{\mu_1})$ for, $n \geq 1$ represents $M/M/1/\infty$ model.

IV. NUMERICAL ILLUSTRATIONS

The numerical results of this model are carried out in this section.

- Table-2 represents the probabilities of each states, for assumed values of the parameters, using the equations (3.11) to (3.21). The parameters are, $\lambda = 2.1$, $\mu_1 = 3$, $\mu_2 = 4.5$, $\alpha = 0.8$, $v_1 = 1.2$, $v_2 = 1.3$, $\phi = 1.7$, $\eta = 2.3$, $a=4$, $b=8$, $d=12$

Table 2 P_n , Q_n , and W_n

n	P_n	Q_n	W_n
0	0.0002738	2.249968e-07	0.146358
1	1.127643e-04	7.212870e-08	3.685071e-01
2	4.643237e-05	2.308475e-08	4.831496e-01
3	1.911921e-05	7.378823e-09	1.463580e-01
4	7.872616e-06	2.356226e-09	3.509591e-02
5	3.241665e-06	7.518131e-10	8.773976e-03
6	1.334803e-06	2.397394e-10	2.193494e-03
7	5.496249e-07	7.641228e-11	5.483734e-04
8	2.263161e-07	2.434588e-11	1.370933e-04
9	9.318900e-08	7.754640e-12	3.427333e-05
10	3.837194e-08	2.469441e-12	8.568331e-06
11	1.580021e-08	7.862450e-13	2.142082e-06
12	6.505969e-09	2.502973e-13	5.355205e-07
13	2.678929e-09	7.967217e-14	1.338801e-07
14	1.103088e-09	2.535827e-14	3.347003e-08
15	4.542128e-10	8.070546e-15	8.367505e-09
16	1.870288e-10	2.568403e-15	2.091876e-09
17	7.701186e-11	8.173446e-16	5.229690e-10
18	3.171077e-11	2.600955e-16	1.307422e-10
19	1.305737e-11	8.276551e-17	3.268555e-11
20	5.376566e-12	2.633644e-17	8.171387e-12
...	...	...	...
		and	
$T_{a-1} = 1.504711e - 05$, $T_b = 8.90571e - 08$			

- Table-3 represents the probability of each states for $\alpha = 0$, $\eta = 0$, considering the values of the parameters, $\lambda = 2.1$, $\mu_1 = 3$, $\mu_2 = 4.5$, $v_1 = 1.2$, $v_2 = 1.3$, $\phi = 1.7$, $a=4$, $b=8$, $d=12$.

Table 3 P_n , Q_n and W_n for $\alpha = 0$, $\eta = 0$

n	P_n	Q_n	W_n
0	0.06957069	8.315461e-06	0.09940452
1	2.864676e-02	2.645828e-06	0.1403341
2	1.179572e-02	8.418545e-07	0.1571870
3	4.857062e-03	2.678628e-07	0.4804802
4	1.999967e-03	8.522907e-08	2.645828e-06
5	8.235158e-04	2.711834e-08	8.418545e-07
6	3.390947e-04	8.628563e-09	2.678628e-07
7	1.396272e-04	2.745452e-09	8.522907e-08
8	5.749357e-05	8.735529e-10	2.711834e-08

9	2.367382e-05	2.779486e-10	8.628563e-09
10	9.748045e-06	8.843820e-11	2.745452e-09
...	...	...	...

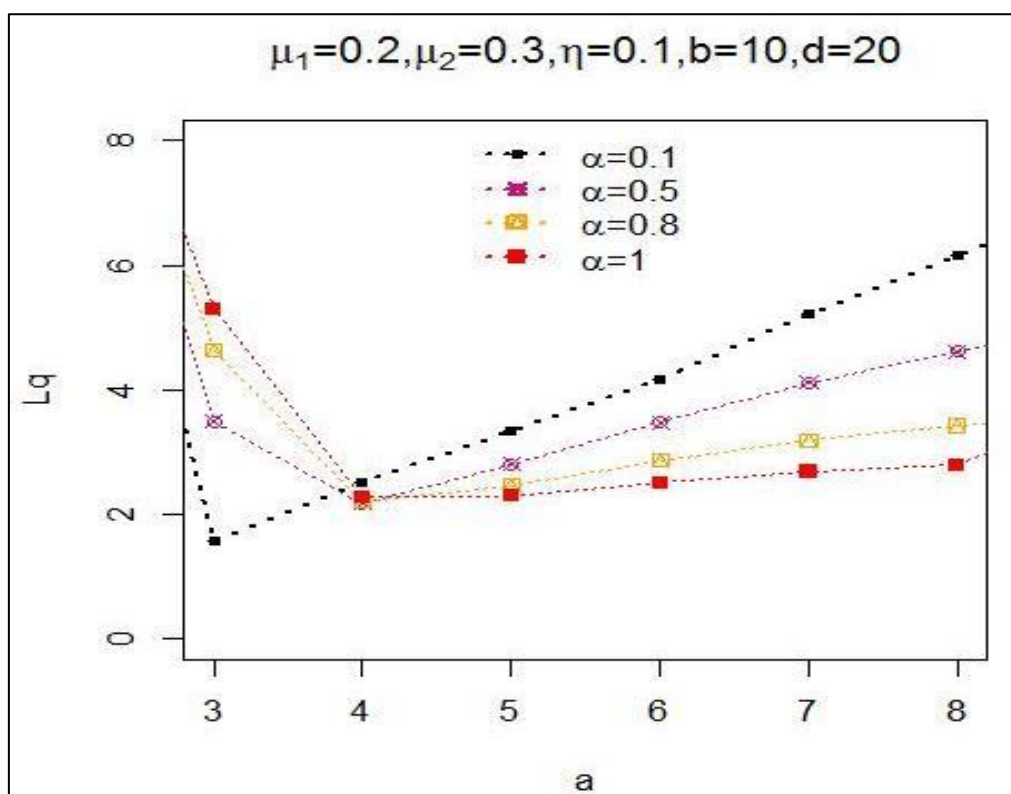
- Table-4 represents the optimal values of a^* , L_q^* , W_q^* and $cost^*$ for various values of μ_1 , μ_2 considering $\lambda = 2$, $\alpha = 0.1$, $v_1 = 1.1$, $v_2 = 1.7$, $\phi = 0.8$, $\eta = 0.09$, $b=10$, $d=15$, and $C_h = 35$, $C_s = 70$, $C_{st} = 60$, $C_1 = 20$, $C_2 = 60$, $C_3 = 70$.

Table 4 Optimum Values

μ_1, μ_2	a^*	L_q^*	W_q^*	$cost^*$
0.7,0.87	3	1.098049	0.9150412	186.3398
0.3,0.47	3	1.343657	1.119714	187.8106
0.2,0.31	3	1.724194	1.436829	197.5232
0.1, 0.27	4	2.460538	1.230269	283.5964
0.1,0.2	4	2.464044	1.232022	319.9376
0.13,0.14	4	2.482839	1.24142	371.4579
0.11,0.12	4	2.471606	1.235803	389.0866
0.09,0.11	4	2.472062	1.236031	399.0741
0.16, 0.17	4	2.528219	1.26411	406.1132

The table is formed according to the traffic intensity of the model and it demonstrates that the optimum values of a^* , L_q^* , W_q^* and $cost^*$ increases as the traffic intensity increases.

- Figure-2 shows the effect of lower limit on queue size(L_q) for different rates of reneging, taking the other parameters as, $\lambda = 4$, $v_1 = 1.3$, $v_2 = 1.3$, $\phi = 0.7$, $b=15$, $d=20$.



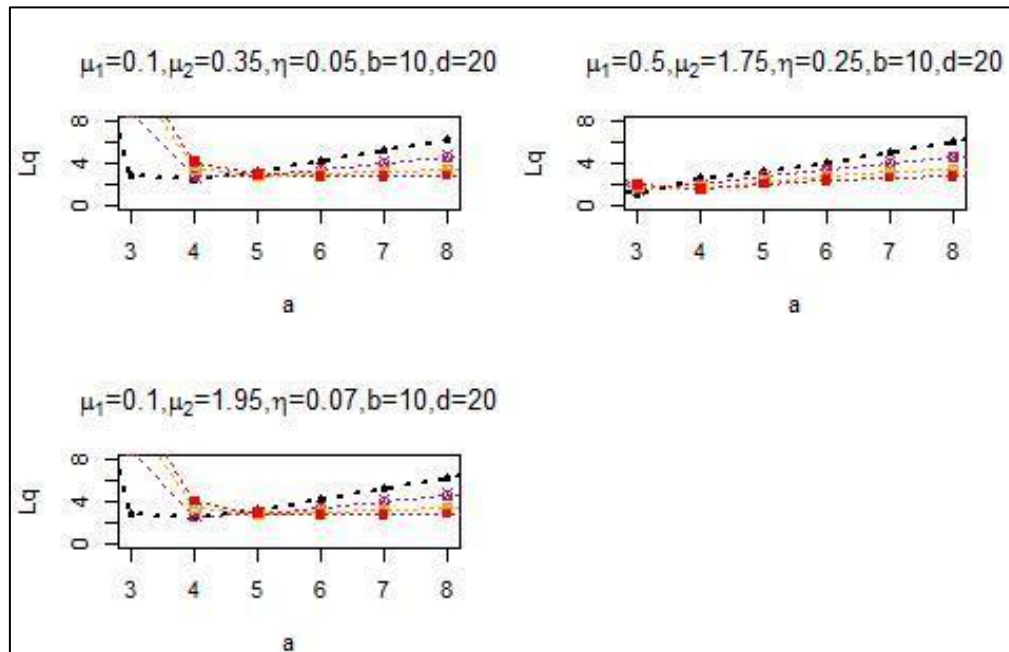


Fig 2 Effect of a on L_q

Figure-2 shows the effects of lower limit a on the queue size for various reneging rates. As would be expected, the figure depicts that up to a particular value of a , the queue size decreases as a increases, and then, the queue size increases suddenly for low rated reneging queues whereas

gradually increases for high rated reneging queues. Because ' a ' will be the minimum threshold to start a service and the accumulation of customers decreases as reneging increases, hence high-rated reneging queues have slowly increasing queue size.

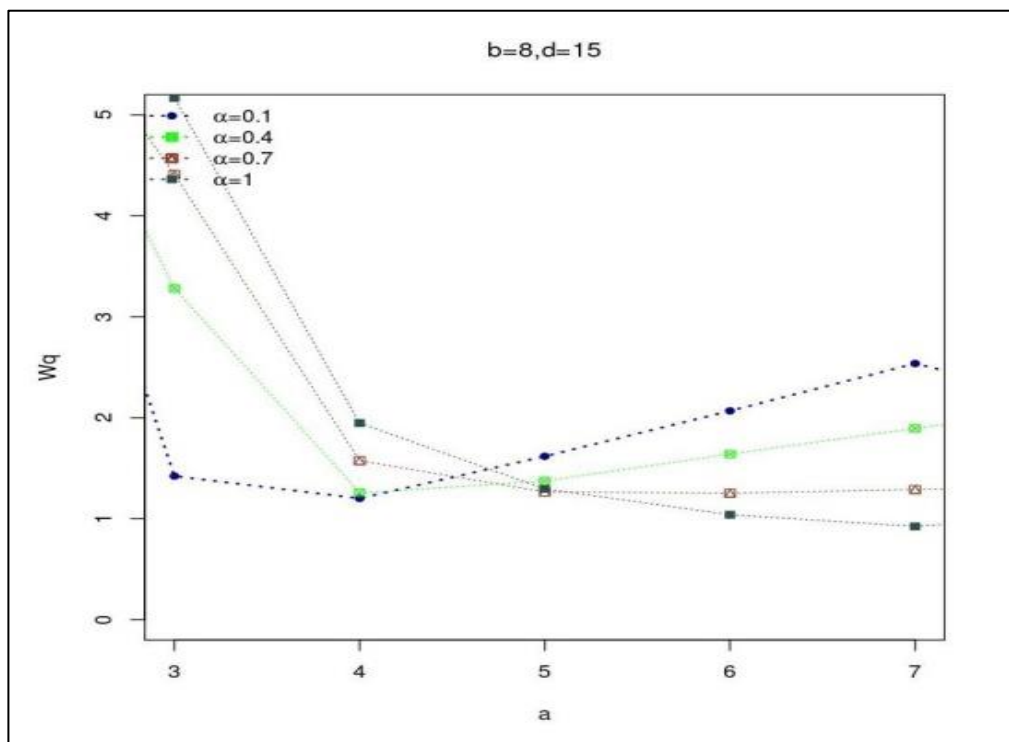


Fig 3 Change in Waiting Time

Figure-3 indicates the changes in waiting time with respect to the value of lower limit ' a ' of customers for $b = 8$ and $d = 15$. The graph depicts that the waiting time increases as ' a ' increases after a particular ' a ', but waiting time decreases as reneging rate increases.

- Figure-4 shows the effect of ' a ' on the expected cost for the different rates of reneging with, $\lambda = 3.6$, $\mu_1 = 0.1$, $\mu_2 = 0.35$, $v_1 = 1$, $v_2 = 1.3$, $\phi = 0.9$, $\eta = 0.05$, $b=13$, $d=18$, and cost parameters $C_s = 70$, $C_{st} = 40$, $C_1 = 20$, $C_2 = 50$, $C_3 = 75$.

Figure-4 depicts that the optimum value of lower limit change as the holding cost changes. Because as the holding cost increases, the optimum value of a also gradually increase, and it will be the same for all rated reneging

queues. But, obviously slow rated reneging queue has minimum expected costs than the high-rated reneging queues, hence the server has to minimize the rate of reneging to decrease the expected cost.

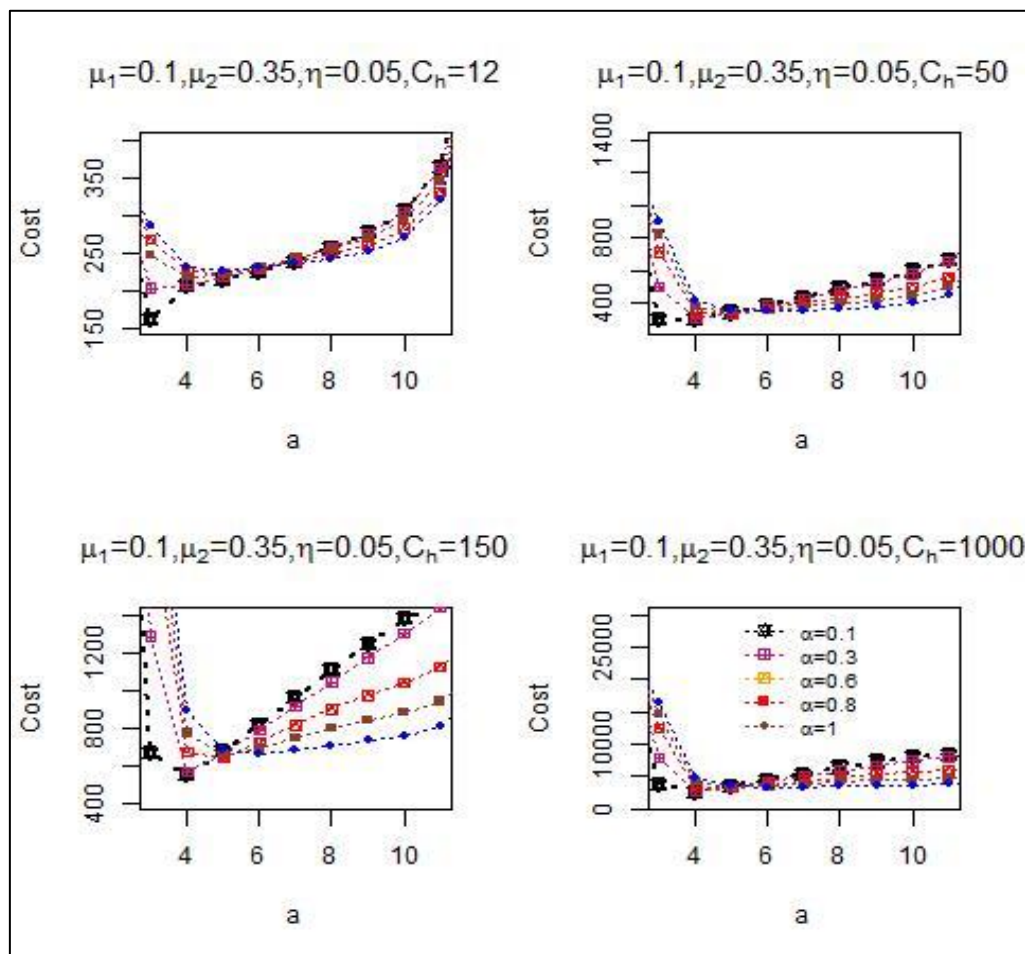


Fig 4 Effect of a on Cost

V. CONCLUSION

This paper comprises the model of a single arrival queue with two different rated batch services. The reneging of the customer before the server starts working vacation, and change over time of the server are included in this model. The steady-state conditions are discussed and attained the probabilities of the different states. The performance measures namely, queue size, waiting time of the customer, busy period, busy cycle, idle time, and optimum policy are obtained. Numerically showed the effect of lower limit a on total cost, waiting time, and queue size. Particular cases are identified by changing the condition of the system.

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