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# Integral Reformulation and Direct Power-Series Collocation for Multi-Term Caputo Equations with Variable Coefficients

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**Abstract: This article develops a direct power-series collocation method for smooth multi-term Caputo initial-value problems with variable coefficients. The contribution is a basis-level fractionalintegral construction that converts the differential model into an augmented algebraic system without first deriving a problem-specific operational matrix. Application of the Riemann–Liouville integral exposes the classical initial data, while gamma- and beta-function identities provide closed expressions for the action of the transformed operator on monomials. Sufficient conditions are established for equivalence of the differential and integral formulations, uniqueness through a contraction argument, conditional convergence of the collocation approximation, and residualbased a posteriori control. The resulting algorithm is implemented symbolically in MAPLE. Two numerical tests are examined. A variable-coefficient cubic benchmark is reproduced with maximum grid errors of $1.93 \times 10^{-10}$ and $2.38 \times 10^{-10}$ at degrees three and four, respectively. In a second multi-order test, the maximum grid error decreases from $2.46 \times 10^{-3}$ at degree three to $4.03 \times 10^{-5}$ at degree seven. Evaluation of the rounded degree-ten coefficient vector gives $8.91 \times 10^{-5}$, identifying coefficient sensitivity in the global monomial basis. The method is therefore most attractive at low and moderate degree, where its transparent derivation and reproducibility outweigh the conditioning disadvantages of monomials.**

***Keywords:*** *Caputo Fractional Derivative; Multi-Term Fractional Equation; Power-Series Collocation; Variable Coefficients; Convergence Analysis; MAPLE.*

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## I. INTRODUCTION

Fractional differential equations encode hereditary and nonlocal response through operators whose present value depends on the preceding state history. The Caputo derivative is particularly convenient for initial-value problems because its data are expressed through integer-order derivatives with familiar physical interpretations [1, 2]. Multi-term equations are more demanding than single-order models: different fractional orders interact with variable coefficients, and the transformed equation contains nested weakly singular integrals.

Polynomial projection remains a major strategy for these problems. Taylor expansion provides a direct approximation but can be sensitive to truncation and derivative availability [3]. Operationalmatrix methods replace fractional operators by matrices associated with a chosen basis, while modern spectral schemes use shifted Legendre, Jacobi, Chebyshev, wavelet, or reproducing-kernel representations. Recent studies report high-order collocation for fractional integro-differential equations [4], multi-step $hp$ schemes for weakly singular Volterra kernels [5], nonlinear multi-term spectral collocation with variable coefficients [6], Legendre treatment of nonlinear Fredholm models [7], shifted-Chebyshev schemes [8], and Chebyshev-based reproducing kernels for nonlocal conditions [9]. Shifted fractional-order Chelyshkov functions have also been combined with Riemann–Liouville integration matrices for delayed equations [10]. These advances offer excellent approximation properties, but the specialised basis construction can obscure the algebraic path from the original equation to the computed coefficients.

The present article develops one focused contribution: a direct fractional integral collocation construction in a monomial basis. Every matrix entry is obtained from the transformed equation itself, and power-law data are evaluated

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through gamma-function formulas. The approach is intended for smooth solutions at low or moderate polynomial degree; it is not proposed as a replacement for graded or $hp$ methods when endpoint singularities dominate.

The article makes four specific contributions. First, it gives an explicit equivalence derivation that retains the initial data. Second, it constructs the augmented collocation system directly, without a separately tabulated fractional operational matrix. Third, it relates uniqueness, projection error, and residual error through a common contraction framework. Fourth, it regenerates all numerical values from the reported coefficient vectors, separating polynomial reproduction from genuine degree refinement and exposing the effect of coefficient rounding.

## II. RELATED LITERATURE

Well-posedness is a prerequisite for meaningful discretisation. For Caputo–Volterra equations, fixed-point theory in weighted spaces has been used to establish local and global mild solutions under Lipschitz assumptions [11]. The Banach contraction principle remains especially useful because the contraction constant also supplies an error-amplification factor [12].

Numerically, the literature can be divided into direct polynomial expansions, operational matrices, spectral collocation, and kernel-based methods. Bernstein operational matrices have been used for multi-term equations [13], and perturbed collocation has been applied to singular multi-order models [14]. Recent spectral work emphasises orthogonal bases and carefully selected nodes: shifted Legendre collocation gives high precision for smooth problems [4]; multi-step Legendre– Gauss approximation resolves weak singularities [5]; and nonlinear multi-term equations have been treated by spectral collocation with rigorous error analysis [6]. Related developments use Legendre–Gauss–Lobatto, shifted Chebyshev, reproducing-kernel, and fractional-order Chelyshkov constructions [10, 8, 9, 7].

The gap addressed here is therefore not the absence of polynomial methods. It is the need for a compact construction whose fractional action is auditable term by term and whose low-degree implementation can be reproduced in a computer algebra system. The trade-off is explicit: monomials simplify derivation but become ill-conditioned at high degree. This article treats conditioning as part of the numerical conclusion rather than concealing it behind overlapping solution curves.

## III. MATHEMATICAL FORMULATION AND PRELIMINARIES

Consider

$$C_{D^{\mu}_{0+}}y(x)=\sum_{j=0}^{r} q_j\,(x)^{C}D^{\alpha_j}_{0+}y(x)+h(x),\qquad 0\le x\le 1, \tag{1}$$

With initial conditions

$$y^{(k)}(0)=\eta k,\qquad k=0,1,\dots,m-1, \tag{2}$$

Where $m-1<\mu\le m$ and $m_j-1<\alpha_j\le m_j$. The coefficient $q_j$ and forcing $h$ are known.

For $\gamma>0$, the Riemann-Liouville fraction integral is:

$$\dot{}_0I_x^{\gamma}f(x)=\frac{1}{\Gamma\gamma}\int_0^x (x-t)^{\gamma-1}f(t)dt \tag{3}$$

For $n-1<\gamma\le n$, the Caputo derivative is:

$${}^{C}\!D^{\gamma}_{0+}f(x)=\frac{1}{\Gamma(\mathrm{n}-\gamma)}\int_0^x (x-t)^{n-\gamma-1}f^{(n)}(t)dt \tag{4}$$

The fundamental composition identity is:

$${}_{0+}I_x^{\gamma}D^{\gamma}_{0+}f(x)=f(x)-\sum_{k=0}^{n-1}\frac{f^{(k)}(0)}{k!}x^k \tag{5}$$

Lemma 1 (Monomial action). If $n\ge m_j$, then

$${}^{C}D^{\alpha j}_{0+}x^n=\frac{\Gamma(n+1)}{\Gamma(n-\alpha j+1)}x^{n-\alpha j}; \tag{6}$$

The derivative is zero for $0\le n<m_j$.

Proof

Insert the $m_j$ th derivative $x^n$ into the Caputo definition. The remaining integral is a beta integral, and cancellation of $\Gamma(m_j-\alpha_j)$ gives equation (6).

Lemma 2 (Transformed power contribution). If $(x)=x^{\sigma j}$, then:

$${}_{0+}I_x^{\mu}\left(x^{\sigma_j}CD^{\alpha_j}_{0+}x^n\right)=\frac{\Gamma(n+1)\Gamma(n-\alpha_j+\sigma_j+1)}{\Gamma(n-\alpha_j+1)\Gamma(n-\alpha_j+\sigma_j+\mu+1)}x^{n-\alpha_j+\sigma_j+\mu} \tag{7}$$

*Proof*

Combine equation (6) with

$$\int_0^x (x-s)^{a-1}s^{b-1}ds=\ x^{a+b-1}\,B(a,b) \tag{8}$$

And use

$(a,b)=\Gamma(a)\Gamma(b)\Gamma(a+b)$.

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## IV. DIRECT POWER-SERIES COLLOCATION METHOD

Applying ${}_{0+}I_x^{\mu}$ to Equation (1) gives:

$$y(x) = W(x) + \sum_{j=0}^{r} \frac{1}{\Gamma(\mu)\Gamma(m_j - \alpha_j)} \int_0^x (x-s)^{\mu-1} q_j(s)$$

$$\times \int_0^x (x-s)^{m_j-\alpha_j-1} y^{(m_j)}(t)dtds \quad (9)$$

Where

$$W(x) = \sum_{k=0}^{m-1} \frac{\eta k}{k!} x^k$$

$$+\frac{1}{\Gamma(\mu)} \int_0^x (x-s)^{\mu-1} h(s)ds, \quad (10)$$

Theorem 3 (Equivalance). Under the regularity required by the Caputo derivatives, equations (1) and (2) and equation (9) are equivalent.

*Proof*

Equation (9) follows by applying the fractional integral to equation (1), using equation (5), inserting the Caputo definition for each lower-order term, and substituting equation (2). Conversely, applying ${}^{C}D_{0^+}^{\mu}$ to equation (9) recovers equation (1); the initial polynomial in $W$ restores equation (2).

Let

$$yN(x) = \sum_{n=0}^{N} a_n x^n = \phi_N(x)a, \qquad \phi_N(x) = [1, x, \dots, x^N]. \quad (11)$$

After substituting into equation (9), defines:

$$L_j(x) = \frac{1}{\Gamma(\mu)\Gamma(m_j - \alpha_j)} \int_o^x (x-s)^{\mu-1} q_j(s)$$

$$\times \int_0^s (s-t)^{m_j-\alpha_j-1} \frac{d^{m_j}}{dt^{m_j}} \phi_N(t)dtds \quad (12)$$

The collocation row is:

$$V(x_i) = \phi_N(x_i) - \sum_{j=0}^{r} L_j(x_i), \qquad V(x_i)a = W(x_i) \quad (13)$$

The initial rows are:

$$U_k = \frac{d^k}{dx^k}\phi_N(x), \qquad U_k a = \eta_k. \quad (14)$$

For $n_c = N + 1 - m$, use the equally spaced points $x_i = i/N, i = 1, \dots, n_c$, and stack the equations:

$$\begin{bmatrix} V(x1) \\ \vdots \\ V(x_{nc}) \\ U_0 \\ \vdots \\ U_{m-1} \end{bmatrix} a = \begin{bmatrix} W(x1) \\ \vdots \\ W(x_{nc}) \\ \eta_0 \\ \vdots \\ \eta_{m-1} \end{bmatrix} \quad (15)$$

Power-law entries follow immediately from equation (7); general smooth data can be integrated symbolically or by controlled quadrature.

➢ *Algorithm 1 Direct Fractional-Integral Power-Series Collocation*

- *Require: Orders $x$; Coefficients ; Forcing $h$; Initial Data; Degree $x$.*

✓ Apply ${}_{0+}x^x$ and form $x(x)$.
✓ .set $yn(x) = \sum_{n=0}^{N} a_n x^n$.
✓ Evaluate each transformed basis contribution using equation (7) or quadrature.
✓ Assemble $x + 1 \quad - x$ collocation rows and $x$ initial-condition rows.
✓ Solve equation (14) with a linear solver; do not form the inverse explicitly.
✓ Reconstruct $x_x$ and evaluate errors and off-grid residuals.

## V. THEORETICAL ANALYSIS

Let $B$ be a nonempty admissible class that is complete in the uniform metric, invariant under the right-hand side $T$ of equation (9), and such that:

$$\|u^{(mj)} - v^{(mj)}\|\infty, \qquad u, v \in B. \quad (16)$$

Assume $\|qj\|\infty, \leq Q_j$.

Theorem 4 (sufficient uniqueness condition). *If*

$$K = \sum_{j=0}^{r} \frac{Q_j L_j}{\Gamma(\mu + m_j - \alpha_j + 1)} < 1 \quad (17)$$

Then equation (8) has a unique solution in B.

*Proof.* For $x, x \in x$, subtract $xx$ and $xx$, apply equation (16), and evaluate the nested beta integrals. Since $0 \leq x \leq 1$,

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$$\|Tu - Tv\|\infty \leq \sum_{j=0}^{r} \frac{Q_j L_j}{\Gamma(\mu + m_j - \alpha_j + 1)} \|u - v\|_\infty = K\|u - v\|_\infty \tag{18}$$

Thus $T$ is a contraction on the complete invariant set $B$, and Banach's theorem gives a unique fixed point (Zhou, 2014).

Let $x_x$ denote interpolation at the chosen nodes and let the discrete solution satisfy

$x_x = x_x x\ x_x$

Proposition 5 (Conditional collocation error). *If $P_N T$ has Lipschitz constant $\kappa_N < 1$ on a common invariant set, then:*

$$\|y - yN\|_\infty \leq \frac{\|(I - P_N)Ty\|_\infty}{1 - K_N} \tag{19}$$

Proof Using $x = x\ x$ and $x_x = \|y - yN\|_\infty \leq \|Ty - P_N Ty\|_\infty + \|P_N Ty - P_N Ty_N\|_\infty \leq \|(I - P_N)Ty\|_\infty + \kappa_N\|y - yN\|_\infty$.

Rearrangement yield equation:

Proposition 6 (Residual-based a posteriori control). *If $T$ has contraction constant $\kappa < 1$ and $z$*

*$\in B$, then*

$$\|y - z\|_\infty \leq \frac{\|z - Tz\|_\infty}{1 - k} \tag{20}$$

Proof. Since $y = Ty$, $\|y - z\|_\infty \leq \|Ty - Tz\|_\infty + \|Tz - z\|_\infty$. Apply the contraction estimate and rearrange.

Remark 7. The factor $x_x$ includes interpolation stability and is not automatically equal to $x$. Moreover, monomial coefficient recovery can become ill-conditioned even when the polynomial approximation space remains adequate. The numerical results below therefore report errors regenerated from the printed coefficients rather than infer convergence from degree alone.

## VI. NUMERICAL RESULTS AND DISCUSSION

All values were regenerated from the coefficient vectors produced by the MAPLE computations and checked against the stated exact solutions. Calculations used 30-digit working precision before coefficients were printed.

➢ *Variable-Coefficient Polynomial-Reproduction Test*

Consider

$$^{C}D_{0+}^{a} y(x) = \frac{\kappa}{x^{a-\beta}}\, {}^{C}D_{0+}^{\beta} y(x) + \frac{\kappa}{x^{a-2}} y(x) + f(x) \tag{21}$$

With y (0) =$y(0) = 0, k = -1, a = \frac{3}{2}, and\ \beta = 12 the\ forcing\ is$:

$$f(x) = \left[6x\left(\frac{\Gamma(4-\beta) + k\Gamma(4-\mathrm{a})}{\Gamma(4-\mathrm{a})\Gamma(4-\beta)} + \frac{x^2}{6}\right) - 2\left(\frac{\Gamma(3-\beta) + k\Gamma(3-\mathrm{a})}{\Gamma(3-\mathrm{a})\Gamma(3-\beta)} + \frac{x^2}{2}\right)\right] x^{2-a} \tag{22}$$

The exact solution is $y = x^3 - x^2$. this benchmark is retained from the thesis and is related to perturbed collocation tests for multi-order equations (Uwaheren et al., 2020). The computed polynomials are:

Polynomials are:

$x_3(x) = 2.5324187192 \times 10^{-13} + 6.5978333907 \times 10^{-12}x - 1.0000000002x^2 + x^3$, (23)
$x_4(x) = 1.3655743203 \times 10^{-14} - 2.5464075293 \times 10^{-12}x - 0.9999999993x^2$

$+ 0.9999999984x^3 + 6.6444272306 \times 10^{-10}x^4$. (24)

Table 1 Regenerated Values and Absolute Errors for the Variable-Coefficient Test

| $x$ | Exact | $y_3$ | $E3$ | $y4$ | $E4$ |
|---|---|---|---|---|---|
| 0.2 | -0.032000000000 | -0.032000000006 | $6.43 \times 10^{-12}$ | -0.031999999984 | $1.58 \times 10^{-11}$ |
| 0.4 | -0.096000000000 | -0.096000000029 | $2.91 \times 10^{-11}$ | -0.095999999974 | $2.56 \times 10^{-11}$ |
| 0.6 | -0.144000000000 | -0.144000000068 | $6.78 \times 10^{-11}$ | -0.144000000009 | $9.00 \times 10^{-12}$ |
| 0.8 | -0.128000000000 | -0.128000000122 | $1.22 \times 10^{-10}$ | -0.128000000101 | $1.01 \times 10^{-10}$ |
| 1.0 | 0.000000000000 | $-1.93 \times 10^{-10}$ | $1.93 \times 10^{-10}$ | $-2.38 \times 10^{-10}$ | $2.38 \times 10^{-10}$ |

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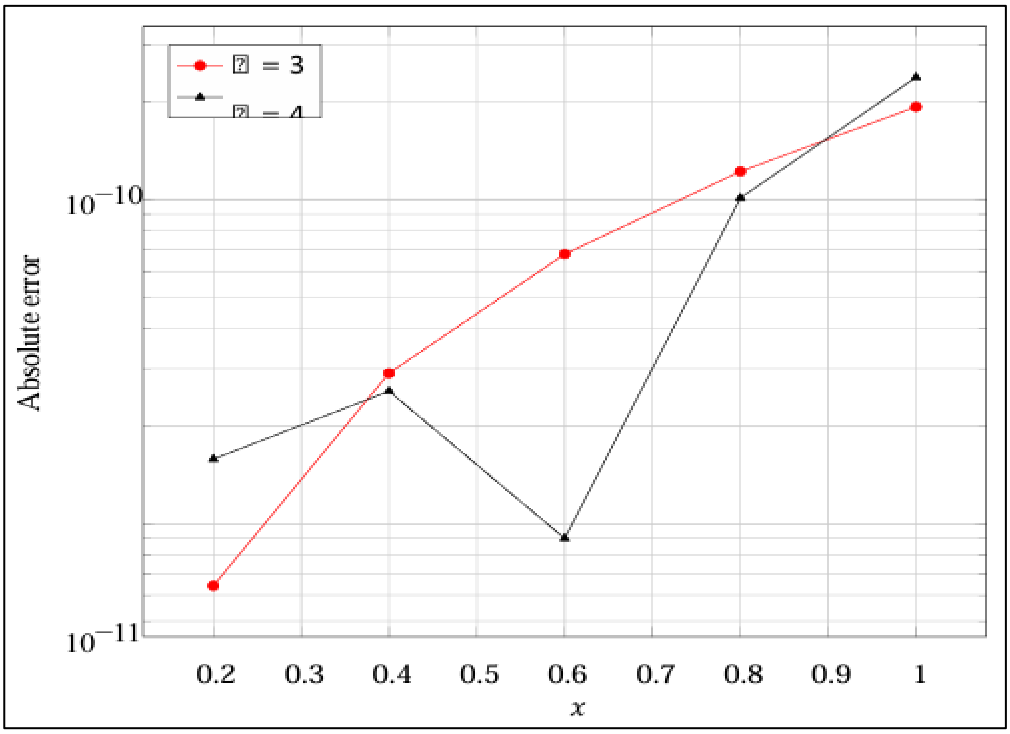


Fig 1 Pointwise Absolute Errors for the Variable-Coefficient Test.

The exact cubic belongs to the degree-three trial space, so this experiment primarily tests algebraic consistency rather than asymptotic convergence. Both approximations reproduce the solution to roughly ten decimal places on the grid. The degree-four result is not uniformly better because it introduces an unnecessary coefficient and an additional conditioning channel. Accordingly, the appropriate conclusion is exact-space reproduction at modest degree, not a claimed universal convergence rate.

➢ *Degree Refinement and Coefficient Sensitivity*

Consider the benchmark associated with Bernstein operational-matrix testing (Rostamy et al., 2013):

$$y''(x) = x^{2c} D_{0+v}^{3/2}(x) + x^{1/2c} D_{0+}^{1/2} y(x) + x^{1/3} y(x) + f(x) \quad (25)$$

Where $(0) = x'(0) = 0$ and

$$f(x) = 6\sqrt{\pi}x - 8x^{7/2} - \frac{16}{5}x^3 - \sqrt{\pi}x^{\frac{10}{3}} \quad (26)$$

The exact solution is $(x) = \sqrt{\pi}x^3$ The regenerated errors for $x = 3, 5, 7, 10$ are shown in table 2.

Table 2 Absolute Errors Under Polynomial-Degree Refinement

| $x$ | $E_3$ | $E_5$ | $E_7$ | $E_{10}$ |
|---|---|---|---|---|
| 0.1 | $7.22380 \times 10^{-5}$ | $1.44791 \times 10^{-6}$ | $1.47124 \times 10^{-6}$ | $1.26064 \times 10^{-6}$ |
| 0.3 | $9.76986 \times 10^{-5}$ | $1.12047 \times 10^{-5}$ | $1.11360 \times 10^{-5}$ | $9.45111 \times 10^{-6}$ |
| 0.5 | $3.56687 \times 10^{-4}$ | $2.67238 \times 10^{-5}$ | $2.49883 \times 10^{-5}$ | $2.20359 \times 10^{-5}$ |
| 0.7 | $1.07076 \times 10^{-3}$ | $4.48625 \times 10^{-5}$ | $3.69507 \times 10^{-5}$ | $4.05719 \times 10^{-5}$ |
| 0.9 | $2.46146 \times 10^{-3}$ | $6.32204 \times 10^{-5}$ | $4.02933 \times 10^{-5}$ | $8.91207 \times 10^{-5}$ |
| $E_{N,\infty}$ | $2.46146 \times 10^{-3}$ | $6.32204 \times 10^{-5}$ | $4.02933 \times 10^{-5}$ | $8.91207 \times 10^{-5}$ |

The maximum grid error decreases by a factor of approximately 38.9 between $x = 3$ and $x = 5$, and by a further factor of 1.57 between $x = 5$ and $x = 7$. The rounded $x = 10$ coefficient vector produces a maximum error about 2.21 times that at $x = 7$. This does not negate the approximation result in equation (18); it shows that coefficient recovery and coefficient reporting are distinct from approximation-space accuracy. A shifted orthogonal basis, QR-based solve, or retention of more coefficient digits would be preferable beyond moderate degree.

Taken together, the examples demonstrate two different properties. The first confirms that the transformed fractional terms and initial rows reproduce a solution already contained in the trial space. The second supplies genuine degree-refinement evidence and identifies the onset of monomial sensitivity. Presenting both prevents near-machine-precision polynomial recovery from being misinterpreted as a general spectral-rate result.

## VII. CONCLUSION

A direct power-series collocation method has been formulated for smooth multi-term Caputo initial-value problems with variable coefficients. The method begins with an equivalent fractionalintegral equation, acts directly on monomial basis functions, and produces an augmented algebraic system containing both collocation and initial-condition rows. Theoretical results give sufficient conditions for uniqueness, conditional convergence, and residual-based error control. The numerical evidence supports a qualified conclusion. At low degree, the method is transparent, accurate, and straightforward to reproduce in MAPLE. Exact-space polynomial tests are recovered to approximately $10^{-10}$ on the reported grid. Degree refinement in the second test reduces the maximum error by nearly two orders of magnitude before rounded high-degree coefficients reveal the conditioning limitations of the monomial basis. Future work should retain the integral construction while replacing monomials with shifted orthogonal or piecewise bases, adopting stable factorizations, and reporting condition numbers and off-grid residuals as can be seen in the figure below.

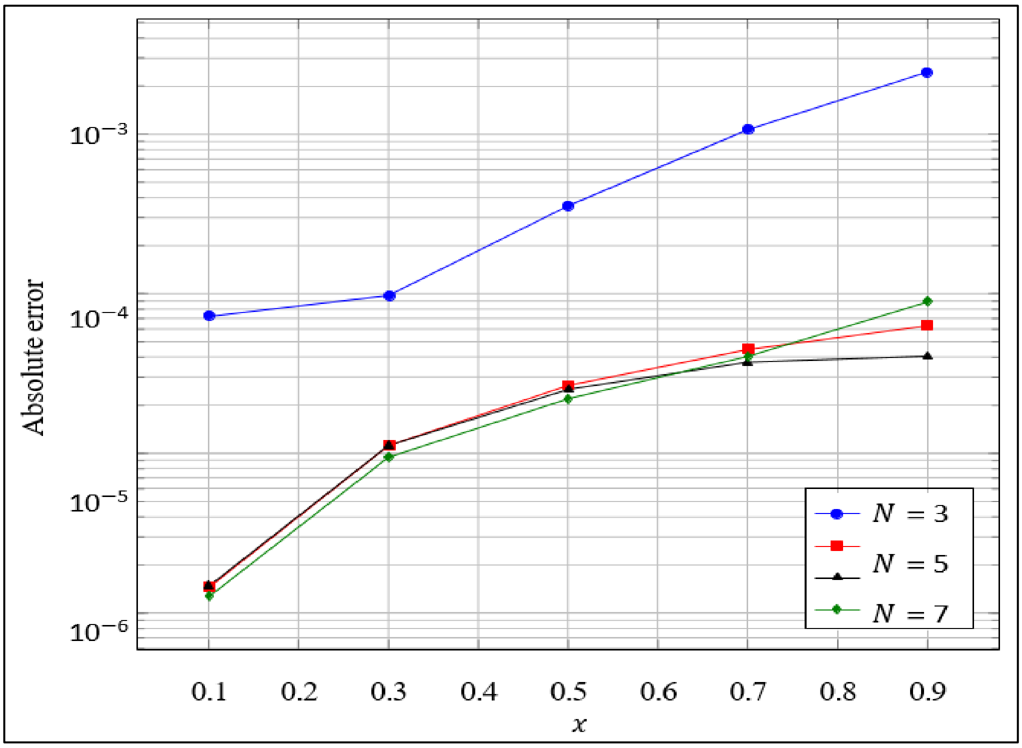


Fig 2 Error Behaviour Under Polynomial-Degree Refinement.

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> *Author Contributions*

J.C.A. developed the mathematical formulation, derived the theoretical results, and performed the numerical computations. A.M.A. supervised the research, verified the analysis, and contributed to the interpretation of results. I.A.O assisted with the numerical implementation and the preparation of the manuscript. All authors reviewed and approved the final manuscript.

> *Competing Interests*

The authors declare no competing interests.

## REFERENCES

[1]. Diethelm, K. (2010). The analysis of fractional differential equations. Springer. https://doi.org/10.1007/978-3-642-14574-2
[2]. Kilbas, A. A., Srivastava, H. M., & Trujillo, J. J. (2006). Theory and applications of fractional differential equations. Elsevier.
[3]. Huang, L., Li, X.-F., Zhao, Y., & Duan, X.-Y. (2011). Approximate solution of fractional integro-differential equations by Taylor expansion method. Computers & Mathematics with Applications, 62(3), 1127–1134. https://doi.org/10.1016/j.camwa.2011.03.037
[4]. Wu, C., & Wang, Z. (2022). The spectral collocation method for solving a fractional integrodifferential equation. AIMS Mathematics, 7(6), 9577–9587. https://doi.org/10.3934/math.2022532

[5]. Wang, C., & Chen, B. (2023). An hp-version spectral collocation method for fractional Volterra integro-differential equations with weakly singular kernels. AIMS Mathematics, 8(8), 19816– 19841. https://doi.org/10.3934/math.20231010
[6]. Kang, Y.-S., & Jo, S.-H. (2024). Spectral collocation method for solving multi-term fractional integro-differential equations with nonlinear integral. Mathematical Sciences, 18, 91–106. https://doi.org/10.1007/s40096-022-00487-9
[7]. Tedjani, A. H., Amin, A. Z., Abdel-Aty, A.-H., Abdelkawy, M. A., & Mahmoud, M. (2024). Legendre spectral collocation method for solving nonlinear fractional Fredholm integrodifferential equations with convergence analysis. AIMS Mathematics, 9(4), 7973–8000. https://doi.org/10.3934/math.2024388
[8]. Hamood, M. M., Sharif, A. A., & Ghadle, K. P. (2025). A numerical approach to fractional Volterra–Fredholm integro-differential problems using shifted Chebyshev spectral collocation. Scientific Reports, 15, 29678. https://doi.org/10.1038/s41598-025-13732-7
[9]. Saldır, O. (2025). Numerical solution of fractional integro-differential equations with non-local conditions using reproducing kernel method based on Chebyshev polynomials. Journal of Applied Mathematics and Computing, 71(Suppl 1), 1403–1431. https://doi.org/10.1007/s12190-02502518-9
[10]. Ahmed, A. I. (2026). A numerical method based on shifted fractional-order Chelyshkov functions for fractional delay integro-differential equations. Numerical Algorithms. https://doi.org/10.1007/s11075-026-02397-6
[11]. Nguyen, H. T., Nguyen, H. C., Wang, R., & Zhou, Y. (2021). Initial value problem for fractional Volterra integro-differential equations with Caputo derivative. Discrete and Continuous Dynamical Systems - B, 26(12), 6483–6510. https://doi.org/10.3934/dcdsb.2021030
[12]. Zhou, Y. (2014). Basic theory of fractional differential equations. World Scientific. https://doi.org/10.1142/9069
[13]. Rostamy, D., Alipour, M., Jafari, H., & Baleanu, D. (2013). Solving multi-term orders fractional differential equations by operational matrices of BPs with convergence analysis. Romanian Reports in Physics, 65(2), 334–349.
[14]. Uwaheren, O. A., Adebisi, A. F., & Taiwo, O. A. (2020). Perturbed collocation method for solving singular multi-order fractional differential equations of Lane–Emden type. Journal of theNigerian Society of Physical Sciences, 2(3), 141–148. https://doi.org/10.46481/jnsps.2020.69