

# Application of Graph theory in Operations Research

Sanjay Kumar Bisen  
Faculty Mathematics  
Govt. P.G. College, Datia (M.P.)  
(Affiliated to Jiwaji University Gwalior) India

**ABSTRACT:-** One of the common themes in operation research is the modeling approach, many accurate model of operations research. Problems turn out to be intractable when subjected to standard technique this research paper show that how graph theory and networks may be profitably used to model certain discrete operations research problem from a different view-point effective algorithms.

**Keyword:-** Graph, Direct graph, Graph networks, Simple graphs.

## I. INTRODUCTION

Graph purpose in operating system. Processes are represented in graph theory. One of the common themes in operation research is the modeling approach. Unfortunately many accurate models of operations research problems. Turn out to be intractable when subjected to standard techniques. However certain discrete problems can be profitably analyzed using graph theoretic models.

This paper introduces useful concepts from graph theory and shows how they may be used to look at certain operations research problems from the viewpoint of the graph theory.

Many of the Indian industries making use of operations research activity are cloth mills, Indian Railway, Indian Airline and Defense organizations and other corporation in India While making use of the techniques of operations research. A mathematical model of the problem in formulated .this model is actually a simplified representation of the problems in graph theory.

Graph theoretic models part 1 and 2 discuss paper of certain operations research .problems based on graph. Directed graphs respectively in this paper.

## II. APPLICATION OF GRAPH THEORY IN OPERATIONS RESEARCH

Graph theory is a very natural and powerful tool in combinatorial operations research. Some important operations research problems that can be solved using graphs. A networks called transport network where a graph is used to model the transportation of commodity from one place to another's the objective is to maximize the flow or minimize the cost within the prescribed flow

### A. Job scheduling

Is the process of allocating system resources to many different tasks by an operating system, The system handles prioritized job queues that are awaiting CPU time and it should determine which job to be taken from which queue and the amount of time to be allocated for the job.

### B. Time table scheduling

Consider a college with M teachers  $T_1, T_2, T_3, \dots, T_n$  and n classes  $C_1, C_2, C_3, \dots, C_n$  given that teacher  $T_1$  is required to teach class  $C_j$  for  $P_{ij}$  periods schedule a complete time table in the minimum possible numbers of periods. The problem can be formulated as graph theory model in which a bipartite graph  $G = (V, E)$  is devised with  $V = V_1 \cup V_2$  where  $V_1 = \{T_1, T_2, \dots, T_n\}$  And  $V_2 = \{C_1, C_2, \dots, C_n\}$  and points  $T_1$  and  $C_j$  are joined by  $P_i$  lines.

The timetabling problem is equivalent to coloring the points in  $V_1$  so that no two adjacent points have the same colour with as few colours as possible –each Colour representing a distinct period.

### C. One more timetabling problem

The purpose of this paper is to explain, Now that Graph theory has become a systematic tool. How it can be used to yield new insights in to operations research models. One simple use of graphs in problem solving in any field including operations research is the following. It is often convenient to depict the relationship between elements of a system by means of a graph. Thus one often sees a transportation, assignment, or PERT problem represented by a graph. Such representations are often an aid in describing a problem and are useful as such. This paper is concerned not just with way of representing an operations research problem in terms of graph theory but more with using the results of graph theory to actually solve the problem. Not every conceivable graph theory applications are documented rather. Some of the major applications are presented to give an overall flavor to this area.

### D. Graph theoretic Algorithms

In this paper we adopt the following specialized definition of an algorithm. An algorithm for a problem is a scientific procedure which is guaranteed to converge to an optimal solution of the problem. This part discusses effective

graph theory algorithms for certain operations research problems.

*E. Chinese postman's problem*

In 1962, A chinese mathematician called Kuan Mei-ko was interested in a postman delivering mail to a number of streets. Such that the total distance walked by the postman was as short possible. How could the postman ensure that the distance walked was minimum.

Following example: - A postman has to start at A, walk along all 13 streets and return to A. The numbers on each edge represent the Length, in meters, of each street. The problem is to find a train that uses all the edges of a graph with minimum Length

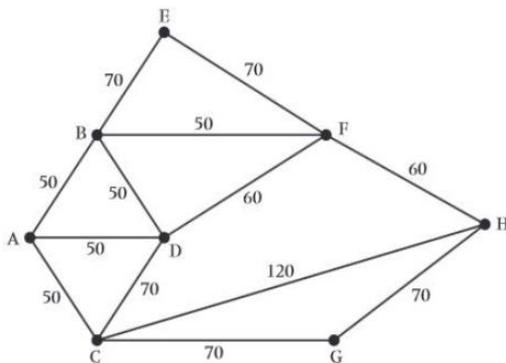
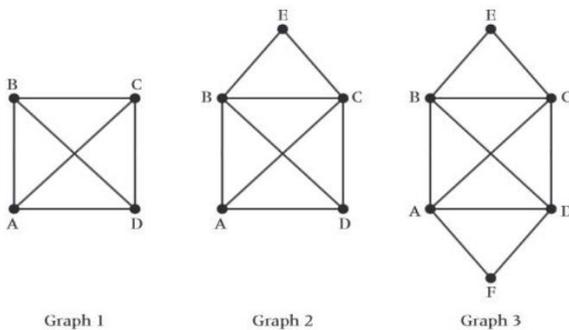


Fig. 01. Chinese postman's problem

We will return to solving this actual problem later, but initially we will look at drawing various graphs. The Chinese postman is traversable graphs given below.



From these Graph we find;

- It is impossible to draw graph 1 without either taking the pen off the paper or re- tracing an edge.
- We can draw graph 2, but only by starting at either A or D-in each case the path will end at the other vertex of D or A.
- Graph 3 can be drawn regardless of the starting position and you will always return to the start vertex.

In order to establish the differences, we must consider the order of the vertices for each graph. The following

Vertex	Order
A	3
B	3
C	3
D	3

Graph 01

Vertex	Order
A	3
B	4
C	4
D	3
E	2

Graph 02

Vertex	Order
A	4
B	4
C	4
D	4
E	2
F	2

Graph 03

When the order of all the vertices is even the graph is Traversable. When there are two odd vertices we can draw the graph but the start and end vertices are different. When there are four odd vertices the graph can't be drawn without repeating an edge.

Chinese postman algorithm :-

- An algorithm for finding an optimal Chinese postman route is.
- Step 1 :-** List all odd vertices.
  - Step 2 :-** List all possible pairing of odd vertices.
  - Step 3 :-** For each pairing find the edges that connect the vertices with the minimum weight.
  - Step 4 :-** Find the pairing such that the sum of the weights is minimized.
  - Step 5 :-** On the original graph add the edges that have been found in step 4.
  - Step 6 :-** The length of an optimal Chinese postman route is the sum of all the edges added to the total found in step 4.
  - Step 7 :-** A route corresponding to this minimum weight can then be easily found.
- Now we apply the algorithm to the original problem in fig. 01.as;
- Step 01** the odd vertices are A and H.
  - Step 02** there is only one way of pairing these odd vertices namely AH.
  - Step 03** the shortest way of joining A to H is using the path AB, BF, FH a total length of 160.
  - Step 04** these edges on to the original network in this fig

**Step 05** the length of the optimal Chinese postman route is the sum of all the edges in the original network. Which is 840 mtr Plus the answer found in step 4, which is 160 mtr., Hence the length of the optimal Chinese postman route is 100 mtr.

**Step 06** one possible route corresponding to this length is ADCGH CABDFBEFHFB A, but many other possible routes of the sum minimum length can be found.

### III. DIGRAPH METHODS IN OPERATIONS RESEARCH

A directed graph or digraph for short is a graph in which each line has been given a direction. The lines have arrows attached to them to indicate their direction many real –world systems which operations research .Analysts study can be modeled as binary relation between the elements of the system .Since this is what a digraph is .digraph theory can often be applied to analyses operations research.

#### A. Game theory

When I first skimmed the theory of games and economic behavior in 1948 I did not really understand it, but I sensed that this was the way to go in the study of multi person conscious strategic behavior. I had heard a little about operations research casually in 1944 in two applications. One concerning how to aim an anti-aircraft gun to take account of the plane’s motions during the time it took to reach it after firing and I had a vague idea that one could try to analyze the best ways for convoy defense by using some form of mathematics.

The game theory methods to operations research economics, political science, and management science. My main purpose is to cover the evolution of the relationship between game theory and operation research rather than to review all of operation research. I limit my broader comments on operation research with a few pan glossies remarks. It is my belief that operations research has been so successful that it may have put itself out of business. At least in its easy –to-recognize sense It has succeeded to the extent that it is taught in a move of less routine and watered- down manner in every business school linear programming, queuing studies, and elementary competitive models.

### IV. CONCLUSIONS

It must now be clear to the author that graph theory and its offshoots, the theory of digraphs and networks, have blossomed not only as a branch of mathematics but also as systematic tools in operations research it has been shown that graph theoretic variety of operations research .this paper is designed to benefit the students of computer science to gain depth knowledge operations research.

### REFERENCES

- [1]. Apple J. M. Plant layout and materials Handling 2<sup>nd</sup> Ronald press New York (1963).
- [2]. Bondy J.A. and U.S.R. Murty, graph theory with applications MacMillan London.
- [3]. Application of graph theory of computer science an overview by S. G. Shrinivas, S. Vetrivel and Dr. N.M. Elango. (International journal of engineering science and technology vol. 2 (9) 2010.4610-4621.
- [4]. Application of graph theory in communication networks by suman Deswal and Anita Singhrova. International journal of Application of Application or innovation in Engineering and Management Volum 1, issu 2 And October 2012.
- [5]. ORE, O, theory of graphs American mathematical society providence R.I. 1962.
- [6]. Graph theory with application to engineering and computer science Narshing Deo.
- [7]. Graph theory A survey of its use in orations research by L.R.FOULDS University of Canterbury Christchurch, N. Z jan (1982).
- [8]. Operations research by S.D. Sharma “MEERUT” publishers India.
- [9]. Operations research by R.K . Gupta “Krishna Prakashan MEERUT” India.
- [10]. Game theory and operations research 1-2002 Martin Shubik (<http://trace.tennessee.edu/utk-harlanabout>).