

Analysis of Multi Criteria Decision Making and Fuzzy Multi Criteria Decision Making

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Abstract:-Today's environment every individual is facing the global competition. They have multi task and managerial decision at several stages. A manual method of decision making becomes complex and it requires intelligent modeling techniques to get the desired results. It also helps us to select the best suitable element from a set of alternatives by minimizing the effort and maximizing the benefit. If the decision makers feel that the data are in imprecise form, involving vague and linguistic descriptions then the fuzzy set theory is a modeling tool for solving such complex systems. Fuzzy set theory is suitable to use when the modeling of human knowledge and human evaluations are needed. Fuzzy program considers random parameters as fuzzy numbers and constraints are treated as fuzzy sets. In fuzzy mathematical program, membership function is used to represent the degree of satisfaction of the decision makers' expectations about the objective function level and the range of uncertainty of coefficients.

In this paper, laptop features are considered for choosing the best one. Purchasing of laptop is challenge due to wide variety of sizes, compact to carry, versatile enough to run demanding applications, features and prices etc. It is a best tool for doing research work or play at home. To overcome this selection process, a suitable technique is required. TOPSIS is one of the selection procedure techniques which provide a base for decision –making processes where there is limited number of choices but each has large number of attributes.

Thus, it presents a comparative analysis of MCDM and FMCDM methods in the context of laptop selection decision making.. However, the comparative analysis has shown that the Fuzzy TOPSIS method is better suited to the problem of laptop selection in regard to changes of alternatives and criteria, agility and number of criteria and alternative laptops. Thus, this comparative study contributes to helping researchers and practitioners to choose more effective approaches for laptop selection. Suggestions of further work are also proposed so as to make these methods more adequate to the problem of laptop selection.

I. INTRODUCTION

Decision-making can be regarded as the cognitive process resulting in the selection of a belief or a course of action among several alternative possibilities. It is the study of identifying and choosing alternatives based on the values and preferences of the decision maker. Multi-criteria decision making (MCDM) is the process of finding the best alternative from all of the feasible alternatives where all the alternatives can be evaluated according to a number of criteria or attribute (Tan & Chen, 2010). It refers to screening, prioritizing, ranking, or selecting a set of alternatives under usually independent, incommensurate or conflicting attributes (Hwang & Yoon, 1981) .

The goal of MCDM is to help the decision maker (DM) to make a choice among a finite number of alternatives or to sort or rank a finite set of alternatives in terms of multiple criteria. The widely used MCDM methods are Analytic Hierarchy Process (AHP), Grey Relational Analysis (GRA), ELECTRE (Elimination EtChoixTraduisant la REalite, Technique for the Order of Prioritization by Similarity to Ideal Solution (TOPSIS), VIKOR, etc. Among the different MCDM techniques, the most widely applied method is TOPSIS . It is one of the widely applied MCDM techniques to solve multi criteria decision making problem (Hwang and Yoon, 1981). It is based on the principle that the chosen alternative should have the longest distance from the negative-ideal solution, i.e. the solution that maximizes the cost criteria and minimizes the benefits criteria, and the shortest distance from the positive-ideal solution, i.e., the solution that maximizes the benefit criteria and minimizes the cost criteria. It is applied to find the better alternative when more number of conflicting criteria is available.

In this paper, TOPSIS and Fuzzy TOPSIS has been applied to find the better laptop. Today different kinds of laptops are available. Identification of difference between this laptop is very difficult. In this research Cost, Warranty, Size, Battery life, Specification (RAM, Processor, graphics card, speed), genuine operating system, weight of the laptop, Wi-Fi and Touch pad are considered as criteria to find the better laptop. It is difficult to select better laptop because relatively all

laptops are seems to be similar. By applying TOPSIS and Fuzzy TOPSIS method to the alternatives based on the criteria the laptops can be differentiated. MCDM evaluation metrics are applied to evaluate the laptop selection problem. A comparative analysis of TOPSIS and Fuzzy TOPSIS methods are illustrated.

Based on Dickson's (1966) empirical study, 23 criteria were identified which purchasing managers generally consider when selecting a supplier. Dempsey (1978) identified quality, delivery capability, and technical capability as imperative in supplier selection. Ellram (1990) emphasized the need not only to base supplier selection decision on the traditional price and quality criteria but also on longer term and qualitative attributes such as strategic match and evaluation of future manufacturing capabilities. A study by Moynihan et al. (2006), states that about 60% of the manufacturer's sales dollars are paid to the supplier for purchased materials . Most of those costs occur in the first stage of supply chain i.e., supplier selection. As mentioned by Venkata Rao (2007), supplier performance is a key issue which affects the success or failure of organizations. Any supply chain is initiated with the selection of right suppliers for the raw materials Parthiban et al. (2010) Supplier selection decisions are complicated by the fact that various criteria must be considered in the decision making process.

The analysis of criteria for selection and measuring the performance of suppliers has been the focus of many academicians and purchasing practitioners since 1960s. Deng yong (2005) emphasize that a new TOPSIS approach for selecting plant location under linguistic environments is presented, where the ratings of various alternative locations under various criteria, and the weights of various criteria are assessed in linguistic terms represented by fuzzy numbers. . Tracey and Tan (2001) note that one of the key elements essential to supply chain success is effective purchasing function. Lee et al. (2001) and Kumara et al. (2003) emphasize that selection of the best supplier is an essential strategic issue imperative for supply chain effectiveness and efficiency. Kumara et al. (2003) contend that strategic partnership with the right supplier must be integrated within the supply chain to contain costs, improve quality and flexibility to meet end-customers' value and reduce lead time at different stages of the supply chain. Fabio J.J.Santos and Heloisa A.Camargo(2010) states that extensions to the original Fuzzy TOPSIS, exploring two distinct versions: to increase the method with the necessary resources for the mathematics process to consider the membership values of the input data in more than one fuzzy set and to aggregate to method the empiric knowledge of an expert represented through fuzzy rules.

II. TOPSIS METHOD

TOPSIS is one of the useful Multi Attribute Decision Making techniques that are very simple and easy to implement, so that

it is used when the user prefers a simpler weighting approach. On the other hand, the AHP approach provides a decision hierarchy and requires pair wise comparison among criteria (Lee et al., 2001). TOPSIS method was firstly proposed by Hwang & Yoon (1981). According to this technique, the best alternative would be the one that is nearest to the positive ideal solution and farthest from the negative ideal solution (Benitez et al., 2007). The positive ideal solution is a solution that maximizes the benefit criteria and minimizes the cost criteria, whereas the negative ideal solution maximizes the cost criteria and minimizes the benefit criteria (Wang & Chang, 2007; Wang & Elhag, 2006; Wang & Lee, 2007; Lin et al., 2008). In other words, the positive ideal solution is composed of all best values attainable of criteria, whereas the negative ideal solution consists of all worst values attainable of criteria (Ertuğrul&Karakasoğlu, 2009).

A MADM problem with m alternatives (A_1, A_2, \dots, A_m) that are evaluated by n attributes (C_1, C_2, \dots, C_n) can be viewed as a geometric system with m points in n -dimensional space. An element x_{ij} of the matrix indicates the performance rating of the i th alternative, A_i , with respect to the j th attribute, C_j , as shown in Eqs. (1).

The terms used in the present study are briefly defined as follows:

Attributes: Attributes ($C_j, j = 1, 2, \dots, n$) should provide a means of evaluating the levels of an objective. Each alternative can be characterized by a number of attributes.

$$D = \begin{matrix} & \begin{matrix} C_1 & C_2 & C_3 & \dots & C_n \end{matrix} \\ \begin{matrix} A_1 \\ A_2 \\ A_3 \\ \dots \\ A_m \end{matrix} & \begin{bmatrix} x_{11} & x_{12} & x_{13} & \dots & x_{1n} \\ x_{21} & x_{22} & x_{23} & \dots & x_{2n} \\ x_{31} & x_{32} & x_{33} & \dots & x_{3n} \\ \dots & \dots & \dots & \dots & \dots \\ x_{m1} & x_{m2} & x_{m3} & \dots & x_{mn} \end{bmatrix} \end{matrix} \dots(1)$$

Alternatives: These are synonymous with 'options' or 'candidates'. Alternatives ($A_i, i = 1, 2, \dots, m$) are mutually exclusive of each other. **Attribute weights:** Weight values (w_j) represent the relative importance of each attribute to the others.

$$W = \{w_j | j = 1, 2, \dots, n\}.$$

Normalization: Normalization seeks to obtain comparable scales, which allows attribute comparison. The vector normalization approach divides the rating of each attribute by its norm to calculate the normalized value of x_{ij} as defined in Eqs. (2):

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^m x_{ij}^2}}, i = 1,2,\dots,m; j = 1,2,\dots,n \dots (2)$$

Given the above terms, the formal TOPSIS procedure is defined as follows:

Step 1: Construct normalized decision matrix. This step transforms various attribute dimensions into non-dimensional attributes, which allows comparisons across criteria.

Step 2: Construct the weighted normalized decision matrix. Assume a set of weights for each criteria w_j for $j = 1, \dots, n$. Multiply each column of the normalized decision matrix by its associated weight. An element of the new matrix is:

$$v_{ij} = w_j r_{ij}, \text{ for } i = 1,2,\dots, m; j = 1,2,\dots,n \dots (3)$$

Step 3: Determine the positive ideal (A^*) and negative ideal (A^-) solutions. The A^* and A^- are defined in terms of the weighted normalized values, as shown in Eqs. (4) and (5), respectively:

Positive Ideal Solution:

$$A^* = \{v_1, v_2, v_3, \dots, v_n\}$$

Where

$$v_j^* = \{\max(v_{ij}) \text{ if } j \in J; \min(v_{ij}) \text{ if } j \in J'\} \dots (4)$$

Negative Ideal Solution:

$$A^- = \{v_1, v_2, v_3, \dots, v_n\}$$

Where

$$v_j^- = \{\min(v_{ij}) \text{ if } j \in J; \max(v_{ij}) \text{ if } j \in J'\} \dots (5)$$

Where J is a set of benefit attributes (larger-the-better type) and J' is a set of cost attributes (smaller-the-better type).

Step 4: Calculate the separation measures for each alternative.

The separation of each alternative from the positive ideal alternative is:

$$S_i^+ = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^+)^2}; i = 1,2,\dots,m \dots (6)$$

Similarly, the separation of each alternative from the negative ideal alternative is:

$$S_i^- = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^-)^2}; i = 1,2,\dots,m \dots (7)$$

Step 5: Calculate the relative closeness to the ideal solution or similarities to ideal solution C_i^*

$$C_i^* = S_i^- / (S_i^+ + S_i^-), 0 < C_i^* < 1 \dots (8)$$

Note that when $A_i = A^*$ where $C_i^* = 1$

When $A_i = A^-$ where $C_i^* = 0$

Step 6: By comparing C_i^* values, the ranking of alternatives are determined. Choose an alternative with maximum C_i^* or rank alternatives according to C_i^* in descending order.

A. Fuzzy TOPSIS Model

It is often difficult for a decision-maker to assign a precise performance rating to an alternative for the attributes under consideration. The merit of using a fuzzy approach is to assign the relative importance of attributes using fuzzy numbers instead of precise numbers. This section extends the TOPSIS to the fuzzy environment (Yang & Hung, 2007). This method is particularly suitable for solving the group decision-making problem under fuzzy environment. The rationale of fuzzy theory has been reviewed before the development of fuzzy TOPSIS. The mathematics concept borrowed from Ashtiani et al. (2009), Buyukozkan et al. (2007) and Wang & Chang (2007), Kabir et al. (2011); Bahram and Asghari, 2011; Kalpande et al., 2010; Tadic et al., 2010):

Step 1: Choose the linguistic ratings ($x_{ij}, i = 1,2,\dots, m, j = 1, 2, \dots, n$) for alternatives with respect to criteria and the appropriate linguistic variables ($w_j, j = 1, 2, \dots, n$) for the weight of the criteria

The fuzzy linguistic rating (x_{ij}) preserves the property that the ranges of normalized triangular fuzzy numbers belong to $[0, 1]$; thus, there is no need for a normalization procedure.

Step 2: Construct the weighted normalized fuzzy decision matrix. The weighted normalized value v^* is calculated by Eqs. (9).

Step 3: Identify positive ideal (A^*) and negative ideal (A^-) solutions. The fuzzy positive-ideal solution (FPIS, A^*) and the fuzzy negative-ideal solution (FNIS, A^-) are shown as Eqs. (9) and (10):

Positive Ideal solution

$$A^* = \{v_1^*, v_2^*, \dots, v_n^*\},$$

Where

$$v_j^* = \{(\max v_{ij} / i = 1,2,\dots,m), j = 1,2,\dots,n\} \dots (9)$$

Negative Ideal Solution:

$$A^- = \{v_1^-, v_2^-, \dots, v_n^-\},$$

Where

$$v_j^- = \{(\max v_{ij} / i = 1,2,\dots,m), j = 1,2,\dots,n\} \dots (10)$$

Step 4: Calculate separation measures. The distance of each

alternative from A^* and A^- can be currently calculated using Eqs. (11) and (12).

$$d_i^* = \sum_{j=1}^n d(\tilde{v}_{ij}, \tilde{v}_j^*) \quad i=1,2,\dots,m \quad \dots\dots (11)$$

$$d_i^- = \sum_{j=1}^n d(\tilde{v}_{ij}, \tilde{v}_j^-) \quad i=1,2,\dots,m \quad \dots\dots (12)$$

Step 5: Calculate similarities to ideal solution. This step solves the similarities to an ideal solution by Eqs.

$$CC_i^* = d_i^*/(d_i^*+d_i^-) \quad \dots\dots (13)$$

Step 6: Rank preference order. Choose an alternative with maximum CC_i^* or rank alternatives according to CC_i^* in descending order

III. NUMERICAL CALCULATIONS

In this paper, we are comparing MCDM method such as TOPSIS and Fuzzy MCDM method such as Fuzzy TOPSIS to analyze the best methodology. For this analysis, we are considering ten different branded laptops. Among this, we are finding the best quality laptop based on comparing various conflicting criteria's with respect to i_5 processor. The Alternatives are Toshiba, Sony, LG, Acer, Samsung, Apple, HP, Dell, Lenovo, HCL and the criteria's are cost, specifications, warranty, screen size, battery life, weight, key pad, Wi-Fi and it is shown in Table 1.

criteria/ Alternatives	cost	specifications	warranty	Sreen size	Battery life	With Os or without Os	weight	Key pad	Wi-fi
Toshiba	52,500	I5 processor, 4Gb ram,2.5ghz	1 year	13.3"	6 cell li- ion	Windows7	1.39kg	Spill resistant	WLAN 802.11b/g/n
Sony	48,591	I5 processor,4GB ram,2.6 ghz	1 year	14"	Li-ion 6hrs	Windows 8	2.4kg	Spill resistant	Wlan 802.11b/g/n
LG	47,750	I5 processor, 4Gb ram,2.5ghz	1 year	15.6"	Li-ion 3hrs	Windows7	2.16kg	Spill resistant	WLAN 802.11b/g/n
Acer	41,000	I5 processor, 4Gb ram,2.5ghz	1 year	15.6"	Li-ion 2.5hrs	Windows7	2.8kg	Spill resistant	WLAN 802.11b/g/n
Samsung	45,250	I5 processor, 4Gb ram,2.5ghz	1 year	15.6"	Li-ion 2.5hrs	Windows7	2.56kg	Spill resistant	WLAN 802.11b/g/n
Apple	60,152	I5 processor, 4Gb ram,2.5ghz	1 year	17.6"	Li-ion 5hrs	OSXLION	1.08kg	Spill resistant	WLAN 802.11b/g/n
HP	52,990	I5 processor, 4Gb ram,2.5ghz,HDT- 720	1 year	15.6"	Li-ion 5hrs	Windows7	2.4kg	Spill resistant	WLAN 802.11b/g/n
Dell	52,150	I5 processor, 4Gb ram,2.5ghz	1 year	15.6"	Li-ion 5hrs	Windows7	1.8kg	Spill resistant	WLAN 802.11b/g/n
Lenova	48,500	I5 processor, 4Gb ram,2.5ghz	1 year	14"	Li-ion 3hrs	Windows7	2.2kg	Spill resistant	WLAN 802.11b/g/n
HCL	36,803	I5 processor, 4Gb ram,2.5ghz,HDT- 320	1 year	14"	Li-ion 3hrs	Windows7	2.2kg	Spill resistant	WLAN 802.11b/g/n

Table 1: Criteria's based for Laptops.

A. Empirical Illustrations for TOPSIS Method

The Decision Matrix table is used for the TOPSIS analysis and Fuzzy TOPSIS analyses are shown in Table 2.

TOSHIBA	SONY	LG	ACER	SAMSUNG	HP	HCL	APPLE	LENOVO	DELL	ALTERNATIVES	CRITERIA
7	5	5	3	5	7	5	9	5	7	COST	
7	7	6	5	5	9	7	9	7	9	SPECIFICATION	
5	5	4	4	5	5	5	4	5	5	WARRANTY	
3	5	7	7	7	5	5	9	7	7	SIZE	
3	9	7	3	3	6	5	7	5	6	BATTERY LIFE	
3	5	5	3	5	6	5	7	7	7	WITH OR WITHOUT OS	
6	7	7	7	7	6	6	5	7	5	WEIGHT	
5	7	5	5	7	7	5	9	7	9	KEYBOARD & TOUCH PAD	
5	7	6	5	7	7	6	9	7	7	WI-FI	

Table 2: Decision Matrix.

Based on the first step of the TOPSIS procedure, each element is normalized by Eqs.(2). The resulting normalized decision matrix for the TOPSIS analysis is shown as Table-3.

TOSHIBA	SONY	LG	ACER	SAMSUNG	HP	HCL	APPLE	LENOVO	DELL	ALTERNATIVES	CRITERIA
0.4556	0.2575	0.2839	0.2041	0.2862	0.3562	0.3037	0.3858	0.2602	0.3322	COST	
0.4556	0.3605	0.3407	0.3402	0.2862	0.4580	0.425	0.3858	0.3644	0.4271	SPECIFICATION	
0.3254	0.2575	0.2271	0.2721	0.2862	0.2544	0.3037	0.1714	0.260	0.2372	WARRANTY	
0.1952	0.2575	0.3975	0.476	0.4008	0.2544	0.3037	0.3858	0.3644	0.3322	SIZE	
0.1952	0.4635	0.3975	0.2041	0.171	0.3053	0.3037	0.3001	0.260	0.2847	BATTERY LIFE	
0.1952	0.2575	0.2839	0.2041	0.2862	0.3053	0.3037	0.3001	0.3644	0.3322	WITH OR WITHOUT OS	
0.3905	0.3605	0.3975	0.476	0.4008	0.3053	0.3644	0.2143	0.3644	0.2372	WEIGHT	
0.3254	0.3605	0.2839	0.3402	0.4008	0.3562	0.3037	0.3858	0.3644	0.4271	KEYBOARD & TOUCH PAD	
0.3254	0.3605	0.3407	0.3402	0.4008	0.3562	0.3644	0.3858	0.3644	0.3322	WI-FI	

Table 3: Normalized Decision Matrix

The second step requires the attribute weight information to calculate the weighted normalized ratings. The relative importance of each criterion can be obtained from the AHP method. The corresponding definitions for the importance ratios are shown in Table 4.

Level of importance (a_{ij})	Linguistic Definition for comparison of the i^{th} and the j^{th} items
1	The i^{th} item is equal important as the j^{th} item
3	The i^{th} item is slightly more important than The j^{th} item
5	The i^{th} item is more important than the j^{th} item
7	The i^{th} item is strongly more important than the j^{th} item
9	The i^{th} item is extremely more important than the j^{th} item
2,4,6,8	The intermediate values Between two adjacent judgments
$1/a_{ij} = a_{ji}$	The inverse comparison between the i^{th} and the j^{th} items

Table 4: Linguistic Definition for Importance Ratios of Two Selected Items

For instance, the weights of three criteria can be figured out as Table-5.

Criteria	C_{11}	C_{12}	C_{13}	Weight
Cost C_{11}	1	3	5	0.63
Specification C_{12}	1/3	1	3	0.78
Warranty C_{13}	1/5	1/3	1	0.31

Table 5: The Pair wise Comparison Table of the Relative Importance

The weights of the all Evaluation objectives can be obtained in the same manner as shown in Table 6.

Criteria	Weight
C11	0.63
C12	0.78
C13	0.31
C21	0.59
C22	0.28
C23	0.12
C31	0.66
C32	0.24
C33	0.08

Table 6: Weight of All Criteria.

Then, weighted normalized matrix is formed by multiplying each value with their corresponding weights. Table 7 shows the normalized weighted decision matrix for each alternative with respect to the criterion. Positive and negative ideal solutions are determined by taking the maximum and minimum values for each criterion using Eqs. (4) and (5). Then the distance of each alternative from PIS and NIS with respect to each criterion are calculated with help of Eqs. (6) and (7). Table 8 shows the separation measure of each alternative from PIS and NIS. The closeness coefficient of each logistics service provider is calculated by using Eqs. (8) And the ranking of the alternatives are determined according to these values in Table 7.

Criteria	LA1	LA2	LA3	LA4	LA5	LA6	LA7	LA8	LA9	LA10	Vj+	Vj -
C11	0.2870	0.1622	0.1789	0.1285	0.1803	0.2244	0.1913	0.2430	0.1639	0.2092	0.2870	0.1285
C12	0.3554	0.2812	0.2658	0.2653	0.2233	0.3573	0.3316	0.3009	0.2842	0.3331	0.3573	0.2233
C13	0.1008	0.0798	0.0704	0.0843	0.0887	0.0788	0.0941	0.0531	0.0806	0.0735	0.1008	0.0531
C21	0.1152	0.1519	0.2345	0.2810	0.2364	0.1501	0.1792	0.2276	0.2149	0.1960	0.2810	0.1152
C22	0.0546	0.1297	0.1113	0.0571	0.0480	0.0855	0.0850	0.0840	0.0728	0.0797	0.1297	0.0480
C23	0.0234	0.0309	0.0340	0.0244	0.0343	0.0366	0.0364	0.0360	0.0437	0.0398	0.0437	0.0234
C31	0.2577	0.2379	0.2623	0.3143	0.2645	0.2015	0.2405	0.1414	0.2405	0.1566	0.3143	0.1414
C32	0.0781	0.0865	0.0681	0.0815	0.0961	0.0855	0.0728	0.0926	0.0874	0.1025	0.1025	0.0681
C33	0.0260	0.0288	0.0272	0.0272	0.0320	0.0285	0.0291	0.0308	0.0291	0.0265	0.0320	0.0260
Si+	0.1933	0.2115	0.1658	0.1998	0.2018	0.1912	0.1691	0.2058	0.1848	0.2050		
Si-	0.2418	0.1513	0.1939	0.2457	0.1863	0.1857	0.1812	0.1840	0.1639	0.1679		
Ci+	0.5557	0.4173	0.5390	0.5514	0.4800	0.4928	0.5171	0.4720	0.4700	0.4502		

Table 7: Topsis Analysis Results

Finally, the sixth step ranks the alternatives according to Table 7. The order of ranking the alternatives using TOPSIS method results as follows:

$$LA_1 > LA_4 > LA_3 > LA_7 > LA_6 > LA_5 > LA_8 > LA_9 > LA_{10} > LA_2$$

B. Empirical Illustrations for Fuzzy TOPSIS Method

The concept of a linguistic variable is very useful in dealing with situations which are too complex or poorly defined to be reasonably described in conventional quantitative expressions. Linguistic variable are expressed in words, sentences or artificial languages, where each linguistic value can modeled by a fuzzy set. Here, the importance weights of various criteria and the ratings of qualitative criteria are considered as linguistic variables. These linguistic variables can be expressed in positive triangular fuzzy numbers as shown in Table-8.

Fuzzy number	Alternative Assessment	Weights
(1,1,3)	Very Poor (VP)	Very Low (VL)
(1,3,5)	Poor (P)	Low (L)
(3,5,7)	Fair (F)	Medium (M)
(5,7,9)	Good (G)	High (H)
(7,9,9)	Very Good (VG)	Very High (VH)

Table 8: Fuzzy Ratings for Linguistic Variable

In order to transform the performance ratings to fuzzy linguistic variables, the performance ratings in decision Matrix Table are normalized into the range of [0,1] by Eqs.(14) and (15) (Cheng, 1999):

(i) The larger the better type

$$r_{ij} = [x_{ij} - \min\{x_{ij}\}] / [\max\{x_{ij}\} - \min\{x_{ij}\}] \dots\dots (14)$$

(ii) The smaller the better type

$$r_{ij} = [\max\{x_{ij}\} - x_{ij}] / [\max\{x_{ij}\} - \min\{x_{ij}\}] \dots\dots (15)$$

C. Fuzzy Membership Function

The decision makers use the linguistic variables to evaluate the importance of attributes and the ratings of alternatives with respect to various attributes. The present study has only precise values for the performance ratings and for the attribute weights. In order to illustrate the idea of fuzzy MADM, the

existing precise values has been transformed into seven-levels, fuzzy linguistic variables - Very Low (VL), Low (L), Medium Low (ML), Medium (M), Medium High (MH), High (H) and Very High (VH). The purpose of the transformation process has two folds as: (i) to illustrate the proposed fuzzy TOPSIS method and (ii) to benchmark the empirical results with other precise value methods in the later analysis Among the commonly used fuzzy numbers, triangular and trapezoidal fuzzy numbers are likely to be the most adoptive ones due to their simplicity in modeling and easy of interpretation. Both triangular and trapezoidal fuzzy numbers are applicable to the present study. As triangular fuzzy number can adequately represent the seven-level fuzzy linguistic variables and thus, is used for the analysis hereafter. A transformation table can be found as shown in Table 10. For example, the fuzzy variable - Very Low has its associated triangular fuzzy number with minimum of 0.00, mode of 0 and maximum of 0.1. The same definition is then applied to the other fuzzy variables Low, Medium Low, Medium, Medium High, High and Very High. Figure 6 illustrates the fuzzy membership functions.

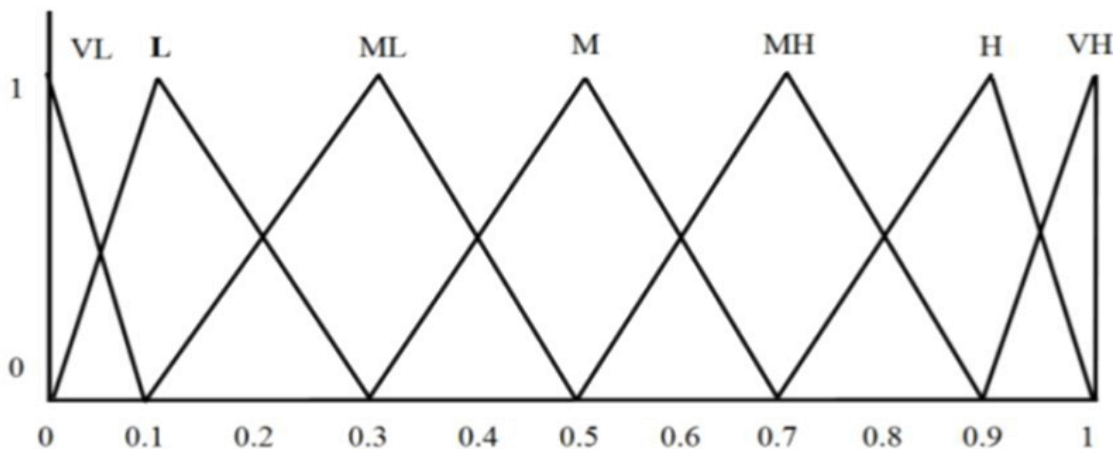


Fig.1: Fuzzy Triangular Membership Functions

The next step uses the fuzzy membership function to transform Table 9 into Table 11 as explained by the following example. If the numeric rating is 0.67, then its fuzzy linguistic variable is ‘‘MH’’. This transformation is also applied to the attributes respectively. Then, the resulting fuzzy linguistic variables are show as Table 11.

	(LA1)	(LA2)	(LA3)	(LA4)	(LA5)	(LA6)	(LA7)	(LA8)	(LA9)	(LA10)
(C11)	0.6	0.3	0.3	0	0.3	0.6	0.3	1	0.3	0.6
(C12)	0.5	0.5	0.25	0	0	1	0.5	1	0.5	1
(C13)	1	1	0	0	1	1	1	0	1	1
(C21)	0	0.3	0.6	0.6	0.6	0.3	0.3	1	0.6	0.6
(C22)	0	1	0.6	0	0	0.5	0.3	0.6	0.3	0.5
(C23)	0	0.5	0.5	0	0.5	0.75	0.5	1	1	1
(C31)	0.5	1	1	1	1	0.5	0.5	0	1	0
(C32)	0	0.5	0	0	0.5	0.5	0	1	0.5	1
(C33)	0	0.5	0.25	0	0.5	0.5	0.25	1	0.5	0.5

Table 9: Normalized Decision Matrix for Fuzzy TOPSIS Analysis

Linguistic Variable	Triangular Fuzzy Number
Very Low (VL)	0,0,0.1
Low (L)	0,0.1,0.3
Medium Low (ML)	0.1,0.3,0.5
Medium (M)	0.3,0.5,0.7
Medium High (MH)	0.5,0.7,0.9
High (H)	0.7,0.9,1
Very High (VH)	0.9,1,1

Table10: Linguistic Variable and the Fuzzy Triangular Membership Functions.

	(LA1)	(LA2)	(LA3)	(LA4)	(LA5)	(LA6)	(LA7)	(LA8)	(LA9)	(LA10)
(C11)	MH	L	L	VL	L	MH	L	VH	L	MH
(C12)	M	M	ML	VL	VL	VH	M	VH	M	VH
(C13)	VH	VH	VL	VL	VH	VH	VH	VL	VH	VH
(C21)	VL	L	MH	MH	MH	ML	ML	VH	MH	MH
(C22)	VL	VH	MH	VL	VL	M	ML	MH	ML	M
(C23)	VL	M	M	VL	M	MH	M	VH	VH	VH
(C31)	M	VH	VH	VH	VH	M	M	VL	VH	VL
(C32)	VL	M	VL	VL	M	M	VL	VH	M	VH
(C33)	VL	M	L	VL	M	M	L	VH	M	M

Table 11: Decision Matrix Using Fuzzy Linguistic Variables

The fuzzy linguistic variable is then transformed into a fuzzy triangular membership function as shown in Table 12. This is the first step of the fuzzy TOPSIS analysis. The fuzzy attribute weight is also collected in Table 12.

	(LA1)	(LA2)	(LA3)	(LA4)	(LA5)	(LA6)	(LA7)	(LA8)	(LA9)	(LA10)	Weight
(C11)	(0.5,0.7,0.9)	(0,0.1,0.3)	(0,0.1,0.3)	(0,0,0.1)	(0,0.1,0.3)	(0.5,0.7,0.9)	(0,0.1,0.3)	(0.9,1,1)	(0,0.1,0.3)	(0.5,0.7,0.9)	(0.5,0.7,0.9)
(C12)	(0.3,0.5,0.7)	(0.3,0.5,0.7)	(0.1,0.3,0.5)	(0,0,0.1)	(0,0,0.1)	(0.9,1,1)	(0.3,0.5,0.7)	(0.9,1,1)	(0.3,0.5,0.7)	(0.9,1,1)	(0.5,0.7,0.9)
(C13)	(0.9,1,1)	(0.9,1,1)	(0,0,0.1)	(0,0,0.1)	(0.9,1,1)	(0.9,1,1)	(0.9,1,1)	(0,0,0.1)	(0.9,1,1)	(0.9,1,1)	(0.1,0.3,0.5)
(C21)	(0,0,0.1)	(0,0.1,0.3)	(0.5,0.7,0.9)	(0.5,0.7,0.9)	(0.5,0.7,0.9)	(0.1,0.3,0.5)	(0.1,0.3,0.5)	(0.9,1,1)	(0.5,0.7,0.9)	(0.5,0.7,0.9)	(0.3,0.5,0.7)
(C22)	(0,0,0.1)	(0.9,1,1)	(0.5,0.7,0.9)	(0,0,0.1)	(0,0,0.1)	(0.3,0.5,0.7)	(0.1,0.3,0.5)	(0.5,0.7,0.9)	(0.1,0.3,0.5)	(0.3,0.5,0.7)	(0.1,0.3,0.5)
(C23)	(0,0,0.1)	(0.3,0.5,0.7)	(0.3,0.5,0.7)	(0,0,0.1)	(0.3,0.5,0.7)	(0.5,0.7,0.9)	(0.3,0.5,0.7)	(0.9,1,1)	(0.9,1,1)	(0.9,1,1)	(0,0.1,0.3)
(C31)	(0.3,0.5,0.7)	(0.9,1,1)	(0.9,1,1)	(0.9,1,1)	(0.9,1,1)	(0.3,0.5,0.7)	(0.3,0.5,0.7)	(0,0,0.1)	(0.9,1,1)	(0,0,0.1)	(0.3,0.5,0.7)
(C32)	(0,0,0.1)	(0.3,0.5,0.7)	(0,0,0.1)	(0,0,0.1)	(0.3,0.5,0.7)	(0.3,0.5,0.7)	(0,0,0.1)	(0.9,1,1)	(0.3,0.5,0.7)	(0.9,1,1)	(0.1,0.3,0.5)
(C33)	(0,0,0.1)	(0.3,0.5,0.7)	(0,0.1,0.3)	(0,0,0.1)	(0.3,0.5,0.7)	(0.3,0.5,0.7)	(0,0.1,0.3)	(0.9,1,1)	(0.3,0.5,0.7)	(0.3,0.5,0.7)	(0,0,0.1)

Table 12: Fuzzy Decision Matrix and Fuzzy Attribute Weights

The second step in the analysis is to find the weighted fuzzy decision matrix. using Eqs. (17) and the fuzzy multiplication Eqs. (13), the resulting fuzzy weighted decision matrix is shown as Table 13.

	(LA1)	(LA2)	(LA3)	(LA4)	(LA5)	(LA6)	(LA7)	(LA8)	(LA9)	(LA10)
(C11)	.25,.49, .81	0,0.07 ,.027	0,0.07,0.27	0,0,0.09	0,0.07,0.27	0.25,0.49, 0.81	0,0.07 ,.027	0.45,0.7 ,.0.9	0,0.07 ,.027	0.25,0.49 ,.0.81
(C12)	0.15,0.3 ,.0.63	0.35,0.63, 0.15	0.63,0.15, 0.35	0.15,0.35, 0.63	0.35,0.63, 0.05	0.63,0.05, 0.21	0.05,0.21, 0.45	0.21,0.45,0	0.45,0,0	0,0,0.09
(C13)	0.09,0. 3 ,.0.5	0.3,0.5 ,.0.09	0.5,0.09 ,.0.3	0.09,0.3, 0.5	0.3,0.5,0	0.5,0,0	0,0,0.05	0,0.05,0	0.05,0,0	0,0,0.05
(C21)	0,0, 0.07	0,0.07,0	0.07,0, 0.05	0,0.05,0.2 1	0.05,0.21 ,.0.15	0.21,0.15 ,.0.35	0.15,0.35 ,.0.63	0.35,0.63, 0.15	0.63, 0.15, 0.35	0.15, 0.35 ,.0.63
(C22)	0,0, 0.05	0,0.05, 0.09	0.05,0.09, 0.3	0.09,0.3 ,.0.5	0.3,0.5,0,0 5	0.5,0.05,0.2 1	0.05,0.21, 0.45	0.21,0.45, 0	0.45,0,0	0,0,0.05
(C23)	0,0, 0.03	0,0.03,0	0.03,0,0,0 5	0,0.05,0.2 1	0.05,0.21,0	0.21,0,0.05	0,0.05,0.2 1	0.05,0.21, 0	0.21,0,0	0,0,0.03
(C31)	0.09 ,.0.25, 0.49	0.25,0.49, 0.27	0.49,0.27, 0.5	0.27,0.5 ,.0.7	0.5,0.7, 0.27	0.7,0.27 ,.0.5	0.27,0.5 ,.0.7	0.5,0.7, 0.27	0.7,0.27 ,.0.5	0.27,0.5 ,.0.7
(C32)	0,0, 0.05	0,0.05,0,0 3	0.05,0.03 ,.0.15	0.03,0.15 ,.0.35	0.15,0.35,0	0.35,0,0	0,0,0.05	0,0.05,0	0.05,0,0	0,0,0.05
(C33)	0,0, 0.01	0,0.01,0	0.01,0,0	0,0,0.07	0,0.07,0	0.07,0,0	0,0,0.03	0,0.03,0	0.03,0,0	0,0,0.01

Table 13: Fuzzy Weighted Decision Matrix

Table 14 shows that, the elements v_{ij} are normalized positive triangular fuzzy numbers and their ranges belong to the closed interval [0, 1]. Thus, fuzzy positive-ideal solution (FPIS, A^*) and the fuzzy negative-ideal solution (FNIS, A^-) can be defined as $v_j^*=(1,1,1)$ and $v_j^*=(0,0,0)$, $j=1,2,3,\dots,n$. This is the third step of the fuzzy TOPSIS analysis.

For the fourth step, the distance of each alternative from A^* and A^- can be currently calculated using Eqs. (11) And (12).The resulting fuzzy TOPSIS analyses are summarized in Table 14.

	(LA1)	(LA2)	(LA3)	(LA4)	(LA5)	(LA6)	(LA7)	(LA8)	(LA9)	(LA10)	A*	A-
(C11)	.25,.49,.8	0,0,07, 0.27	0,0,07,0.27	0,0,0,09	0,0,07,0.27	0.25,0.49, 0.81	0,0,07,0.27	0.45,0.7, 0.9	0,0,07,0.27	0.25,0.49 0.81	1,1,1	0,0,0
(C12)	0.15,0.35 0.63	0.35,0.63, 0.15	0,0,07,0.27	0.15,0.35, 0.63	0.35,0.63, 0.05	0.63,0.05 ,0.21	0,0,07,0.27	0.21,0.45,0	0.45,0,0	0,0,0,09	1,1,1	0,0,0
(C13)	0.09,0. 3 ,0.5	0.3,0.5, 0.09	0.5,0.09,0. 3	0.09,0.3, 0.5	0.3,0.5,0 0.5,0,0	0.5,0,0	0,0,0,05	0,0,05,0	0.05,0,0	0,0,0,0 5	1,1, 1	0,0, 0
(C21)	0,0, 0,07	0,0,07, 0	0,07,0,0 5	0,0,05,0. 21	0,05,0,21 , 0.15	0,21,0.15,0. 35	0,15,0.35,0. 63	0,35,0,63 , 0.15	0,63,0.15,0. 35	0,15,0. 35 ,0.63	1,1, 1	0,0, 0
(C22)	0,0,0,0 5	0,0,05, 0,09	0,05,0,09 ,0,3	0,09,0,3, 0,5	0,3,0,5, 0,05	0,5,0,05, 0,21	0,05,0,21,0. 45	0,21,0,45 ,0	0,45,0,0	0,0,0,0 5	1,1, 1	0,0, 0
(C23)	0,0, 0,03	0,0,03,0	0,03,0,0 5	0,0,05, 0,21	0,05,0,21 ,0	0,21,0,0,05	0,0,05,0,21	0,05,0,21 ,0	0,21,0,0	0,0,0,0 3	1,1, 1	0,0, 0
(C31)	0,09,0. 25 ,0,49	0,25,0,4 9, 0,27	0,49,0,27, 0,5	0,27,0,5, 0,7	0,5,0,7, 0,27	0,7,0,27,0,5	0,27,0,5,0,7	0,5,0,7,0. 27	0,7,0,27,0,5	0,27,0. 5, 0,7	1,1, 1	0,0, 0
(C32)	0,0,0,0 5	0,0,05, 0,03	0,05,0,03, 0,15	0,03,0,15 , 0,35	0,15,0,35 ,0	0,35,0,0	0,0,0,05	0,0,05,0	0,05,0,0	0,0,0,0 5	1,1, 1	0,0, 0
(C33)	0,0,0,0 1	0,0,01,0	0,01,0,0	0,0,0,07	0,0,07,0	0,07,0,0	0,0,0,03	0,0,03,0	0,03,0,0	0,0,0,0 1	1,1, 1	0,0, 0
di*	7.5846	7.1795	7.5238	8.0725	7.4934	6.7388	7.5263	6.5800	7.0393	6.6648		
di-	1.7740	2.2980	1.8825	1.1591	1.9136	2.7984	1.8482	2.8743	2.4692	2.8632		
Cci*	0.1895	0.2424	0.2001	0.1255	0.2034	0.2934	0.1971	0.3040	0.2596	0.3005		

Table 14: Fuzzy TOPSIS Analysis

The fifth step solves the similarities to an ideal solution by Eqs. (13). Based on the Table 14, the order of ranking the alternatives using fuzzy TOPSIS method results as follows: $LA_8 > LA_{10} > LA_6 > LA_9 > LA_2 > LA_5 > LA_3 > LA_7 > LA_1 > LA_4$.

According to the TOPSIS and fuzzy TOPSIS methods, the preference order of the alternatives is summarized in Table 15.

Preference order	1	2	3	4	5	6	7	8	9	10
Topsis	LA1	LA4	LA3	LA7	LA6	LA5	LA8	LA9	LA10	LA2
Fuzzy topsis	LA8	LA10	LA6	LA9	LA2	LA5	LA3	LA7	LA1	LA4

Table 15: The Order of Ranking of the Alternatives for Different Methods

It is evident that both methods lead to the choice of LA_8 ; hence, Laptop of LA_8 shows the highest quality. Other than LA_8 , the preferences vary between methods. The fuzzy TOPSIS concludes with the order of ranking $LA_8 > LA_{10} > LA_6 > LA_9 > LA_2 > LA_5 > LA_3 > LA_7 > LA_1 > LA_4$ whereas TOPSIS method concludes with the order of ranking $LA_1 > LA_4 > LA_3 > LA_7 > LA_6 > LA_5 > LA_8 > LA_9 > LA_{10} > LA_2$. Due to the MCDM nature of the proposed problem, an optimal solution may not exist; however, the systematic evaluation of the MCDM problem can reduce the risk of a poor quality selection. When precise performance ratings are available, the TOPSIS method is considered to be a viable approach in solving a Laptop Selection problem. Fuzzy TOPSIS is a preferred choice for the instance of imprecise or vague performance ratings in solving the proposed quality problem.

The aim of the proposed methodology is to recommend a systematic evaluation of selecting a quality Laptop including the comparison of both TOPSIS and Fuzzy TOPSIS, to find out the best Laptop. The proposed methodology provides a systematic approach to narrow down the number of alternatives and to facilitate the decision making process.

IV. CONCLUSIONS

The present study explored the use of TOPSIS and fuzzy TOPSIS in solving a Laptop selection problem. The aim was to investigate the dimensions of Laptop quality, by adapting and extending the TOPSIS and fuzzy TOPSIS models. Moreover, the methods and experiences learned from the study can be valuable to the company's future strategic planning. Empirical results showed that the proposed methods are viable approaches in solving the proposed Laptop selection problem. TOPSIS is a viable method for the proposed problem and is suitable for the use of precise performance ratings. When the performance ratings are vague and inaccurate, then the fuzzy TOPSIS is the preferred technique. In addition, there exists other worth investigating MCDM methods for a Laptop selection problem. This becomes one of the future research opportunities in this classical yet important research area.

Sampling is a major limitation in this study. Since the survey was conducted based on a sample in Toshiba, Sony, LG, Acer, Samsung, Apple, HP, Dell, Lenovo, HCL comparison based on i_5 processor.

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