ISSN No:-2456 – 2165

Trivial Fuzzy Topology of the Set of Fuzzy Graphs

[1]P.Jayalakshmi,
Assistant Professor in Mathematics,
Sri GVG Visalakshi College for Women (Autonomous),
Udumalpet,

[2]N.Ramya Priya,
Research Scholar in Mathematics,
Sri GVG Visalakshi College for Women (Autonomous),
Udumalpet

Abstract:-In this paper an attempt is made to study about Fuzzy Topological Graph Theory and the trivial fuzzy topologies of the set of fuzzy graphs. The Suspension and Amalgamation of two fuzzy topological graphs are explained. Furthermore a discussion is made on Bouquets of circle. Various properties of edge contraction and connectivity of Fuzzy Topological Graph are established.

Keywords:-Fuzzy Topological Graph, Trivial Fuzzy Topologies, Suspension, Amalgamation, Bouquets of Circle.

I. INTRODUCTION

In 1965, Zadeh introduced the notion of fuzzy set which is characterized by a membership function that assigns a grade of membership ranging from 0 to 1to each of its objects. The first definition of fuzzy graph was introduced by Kaufmann (1973), based on Zadeh's fuzzy relations (1971). The extension of the concepts of ordinary topological space is the fuzzy topology was introduced by C.L.Chang in 1968. Fuzzy topological graph is an important concept in the field of graph theory dealing with uncertainty. This paper is focused on trivial fuzzy topology of the set of fuzzy graphs.In section 2, the classical and basic definitions of fuzzy graph is reviewed. Section 3 deals with the definitions of Trivial topologies of the set of fuzzy graphs. In section 4 ,a study is made on discrete and indiscrete fuzzy topological graph. In Section 5 the concepts such as Amalgamations, edge contraction, connectivity and bouquet of circle of fuzzy graphs are generalized.

II. PRELIMINARIES

A. Fuzzy Topology

A family $\delta \subseteq I^X$ of fuzzy sets is called a fuzzy topology for *X* if it satisfies the following three axioms

1. $\forall \alpha \in I, \alpha \in \delta.$
2. $\forall A, B \in \delta \Rightarrow A, A \wedge B \in \delta.$
3. $\forall (A_j)_{j \in J} \in \delta \Rightarrow \vee_{j \in J} A_j \in \delta.$

The pair (X, δ) is called a fuzzy topological space or fts, for short. The elements of δ are called fuzzy open sets. A fuzzy set *K* is called fuzzy closed set if $K^c \in \delta.$

B. Fuzzy Graph

A fuzzy graph is a pair $G:(\sigma, \mu)$ where σ is a fuzzy subset of S, μ is a symmetric fuzzy relation on σ. The elements of *S* are called the nodes or vertices of *G* and the pair of vertices edges in *G*.

C. Fuzzy Cut Node

A node is a fuzzy cut node of $G:(\sigma, \mu)$ if removal of it reduces the strength of connectedness between some other pair of nodes.

III. TRIVIAL TOPOLOGIES OF THE SET OF FUZZY GRAPHS

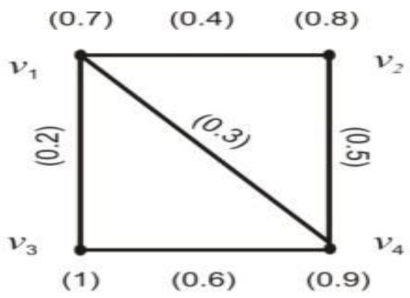
A. Fuzzy Topological Graph

Let *G* be a fuzzy topological graph and *I* the unit interval.

A family of $\chi \subseteq I^G$ of fuzzy graphs is called a fuzzy topological graph of *G*. If it satisfies the following conditions.

i) $[0, 1] \in \chi$

ii) For every $\mu, \sigma \in \chi \Rightarrow \mu(u_{i,} v_j) \leq (\sigma(u), \sigma(v))$



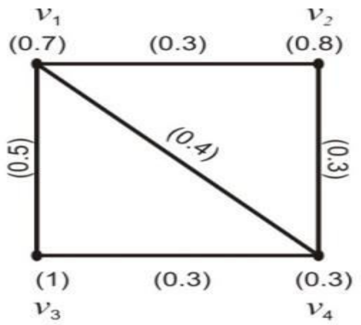
(G)

Fig 4.1

Figure of Fuzzy Topological Graph

B. Open And Closed of Fuzzy Graphs

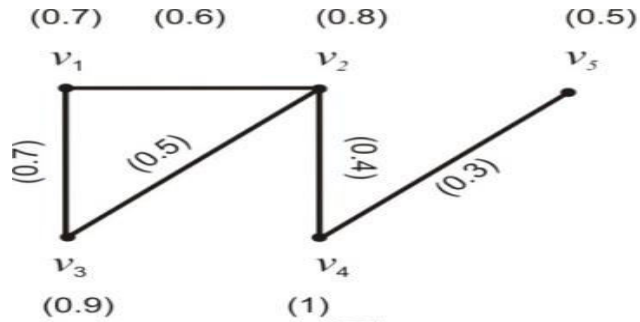
A fuzzy topological graph in χ are called open fuzzy topological graph. A fuzzy topological graph $\mu \in I^G$ is called closed if and only if μ^c is open.



(G^c)

Fig 4.2

Figure of Open Fuzzy Topological Graph



(G)

Fig 4.3

Figure of Closure and Interior

C. *Closure And Interior of Fuzzy Graphs*

The closure and interior of fuzzy topological graph $\mu \in I^G$ are defined respectively as,

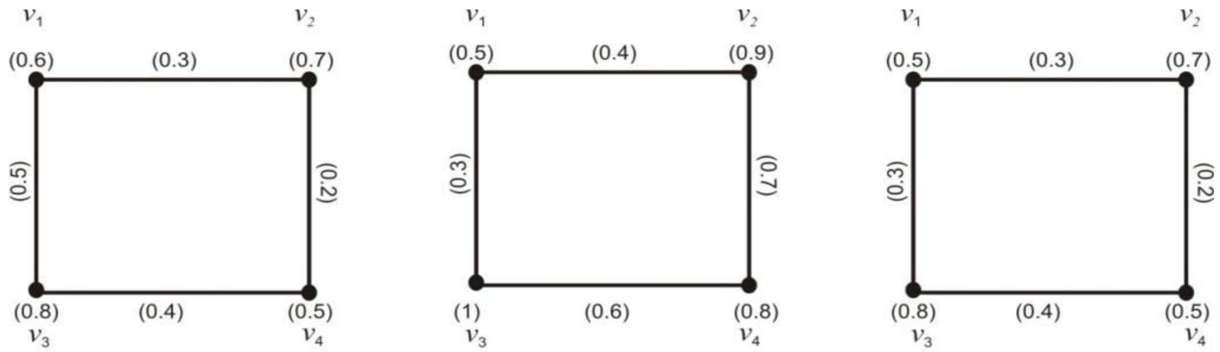
$$\overline{\mu} = \inf\left\{\sigma(v_i) : \sigma(v_i) \geq \mu(v_i, v_j) ; \sigma(v_i)^c \in \chi\right\}$$

$$\overset{\circ}{\mu} = \sup\left\{\sigma(v_i) : \sigma(v_i) \leq \mu(v_i, v_j) ; \sigma(v_i) \in \chi\right\}$$

From the above graph $\overline{\mu} = 0.5 \,\&\, \overset{\circ}{\mu} = 1$

D. *Intersection of A Fuzzy Topological Graph*

Two fuzzy topological graphs *H* and *F* in *G* are said to be intersecting if and only if there exists a $\sigma(v_i) \in G$ such that $(H \wedge F)(\sigma(v_i)) \neq 0$ then, *H* and *F* intersect at $\sigma(v_i)$



(I) H (ii) F (iii) (H^F)

Fig 4.5

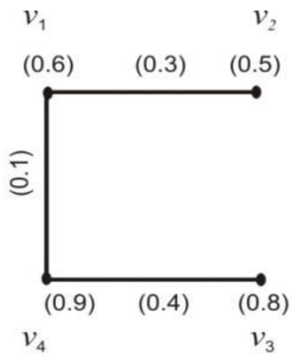
Figure of Intersection of Fuzzy Topological graph

E. *Fuzzy Topological Fuzzy Sub Graph*

Let (G, χ) be a fuzzy topological sub graph. A fuzzy sub graph G' of χ is called a fuzzy topological sub graph if

$$\sigma'(v_i) \subseteq \sigma(v_i) \text{ and } \mu'(u_i, v_j) \subseteq \mu(u_i, v_j)$$

(i.e.) if $\sigma'(v_i) \leq \sigma(v_i)$ for every $u \in V$ and $\mu'(u_i, v_j) \leq \mu(u_i, v_j)$ for every $\mu(u_i, v_j) \in G$



(G')

Fig 4.6

Figure of Fuzzy Topological Fuzzy Subgraph

F. Suspension of Fuzzy Graphs

The suspension of a fuzzy topological graph *G* from a fuzzy topological graph G' is obtained by adjoining every vertex of *G* to every vertex of G' and is denoted as $G + G'$. Its vertex set is $\sigma(v_i) = \max\{\sigma(v_i), \sigma(v_i')\}$ and the edge set

$$\mu(v_i, v_j) = \left[\mu(v_i, v_j) \vee \mu'(v_i, v_j)\right] \wedge \left(\sigma(v_i) \wedge \sigma(v_i')\right)$$

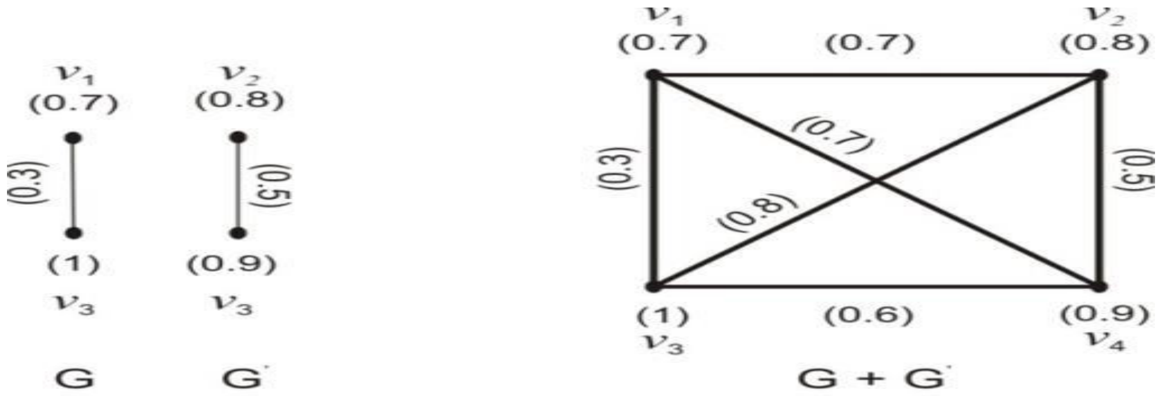


Fig 4.7
Figure of Suspension

IV. TRIVIAL TOPOLOGIES OF THE SET OF FUZZY GRAPHS

A. Introduction:

Topological ideas are present in almost all areas of today's mathematics.
There are two kinds of trivial topologies

1. Discrete topology
2. Indiscrete topology

a). Discrete Topology
For a given non-empty set $\mu(u_i, v_j)$ & χ be a family of all subsets defined over the edge set (μ, χ) is fuzzy topological graph in I^G as χ is a fuzzy topology on $\mu(u_i, v_j)$ in I^G

Example
Let $\mu(u_i, v_j) = \{\mu(u_1, u_2), \mu(u_1, u_3)\}$ be an edge set. show that χ is a class of all the edges subsets of the set $\chi = \{0, \phi, \{\mu(u_1, u_2)\}, \{\mu(u_1, u_3)\}, \mu(u_i, v_j), 1\}$

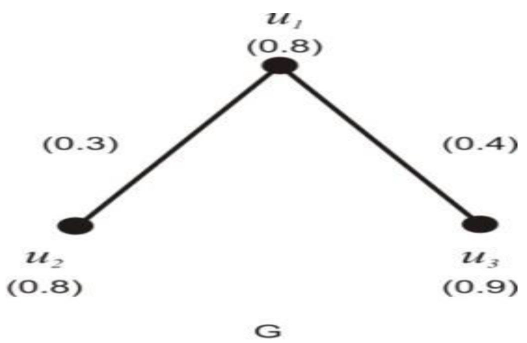


Fig 4.20

To verify that the class χ is a discrete fuzzy topology graph and (μ, χ) is a fuzzy topological graph it is enough to show that the axioms of topology are satisfied.

Axiom 1
The null edge set and complete edge set $\mu(u_i, v_j)$ must belong to the topology χ.

Fig 4.21

In figure G_1

$$\mu(u_1,u_2)=\mu(u_2,u_3)=\mu(u_1,u_3)=0\in\chi \text{ and}$$

In figure G_2

$$\mu(u_1,u_2)=0.3, \mu(u_2,u_3)=0.5, \mu(u_1,u_3)=0.4\in\chi$$

As $\phi, \mu(u_i,v_j)\in\chi$, first axiom of topology is satisfies.

Axiom 2

To satisfy this axiom of topology, we have to check that the union of all the sub sets of the class χ lies in χ itself.

It is quite obvious that the union of a null edge set to any other edge set is the edge set it.

$$\phi\bigcup\{\mu(u_1,u_2)\}\in\phi, \phi\bigcup\{\mu(u_1,u_3)\}\in\chi,$$
$$\phi\bigcup\{\mu(u_i,v_j)\}\in\chi,$$
$$\phi\bigcup\{\mu(u_1,u_2)\}\bigcup\{\mu(u_1,u_3)\}\bigcup\mu(u_i,v_j)\in\chi.$$

Similarly the union of the edge set $\mu(u_i,v_j)$ with any set will be the set $\mu(u_i,v_j)$ itself.

Hence,

$$\{\mu(u_1,u_2)\}\bigcup\mu(u_i,v_j)\in\chi,$$
$$\{\mu(u_1,u_3)\}\bigcup\mu(u_i,v_j)\in\chi,$$
$$\{\mu(u_1,u_2)\}\bigcup\{\mu(u_1,u_3)\}\bigcup\mu(u_i,v_j)\in\chi.$$

We shall check using diagram whether

$$\{\mu(u_1,u_2)\}\bigcup\{\mu(u_1,u_3)\}.$$

Belongs to χ Or not.

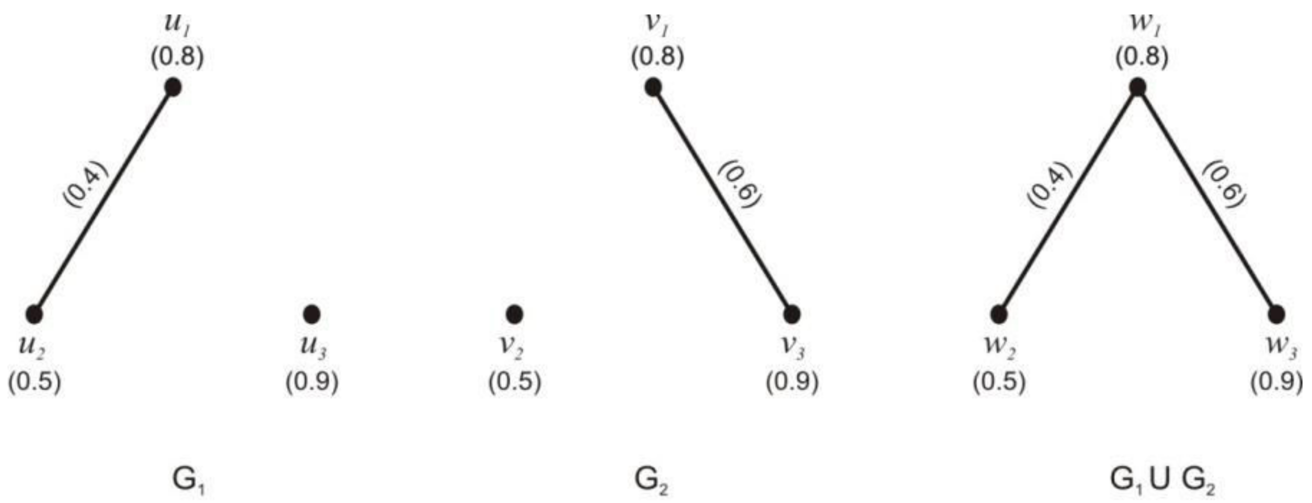


Fig 4.22

Axiom 3

We have to show that the intersection of all the subsets must lie in χ itself.

The intersection of null edge set to any other edge subsets is a edge set. Hence G must lie in the class χ itself.

$$\phi\bigcap\{\mu(u_1,u_2)\}\in\phi, \phi\bigcap\{\mu(u_1,u_3)\}\in\chi,$$
$$\phi\bigcap\{\mu(u_1,u_2)\}\bigcap\{\mu(u_1,u_3)\}\in\chi,$$
$$\phi\bigcap\{\mu(u_1,u_2)\}\bigcap\{\mu(u_1,u_3)\}\bigcap\mu(u_i,v_j)\in\chi.$$

The intersection of any edge set with the edge set $\mu(u_i,v_j)$ is the set itself.

$$\therefore\{\mu(u_1,u_2)\}\bigcap\mu(u_i,v_j)\in\chi,$$
$$\{\mu(u_1,u_3)\}\bigcap\mu(u_i,v_j)\in\chi,$$
$$\{\mu(u_1,u_2)\}\bigcap\{\mu(u_1,u_3)\}\bigcap\mu(u_i,v_j)\in\chi.$$

We now show that

$$\{\mu(u_1,u_2)\}\bigcap\{\mu(u_1,u_3)\}\in\chi$$

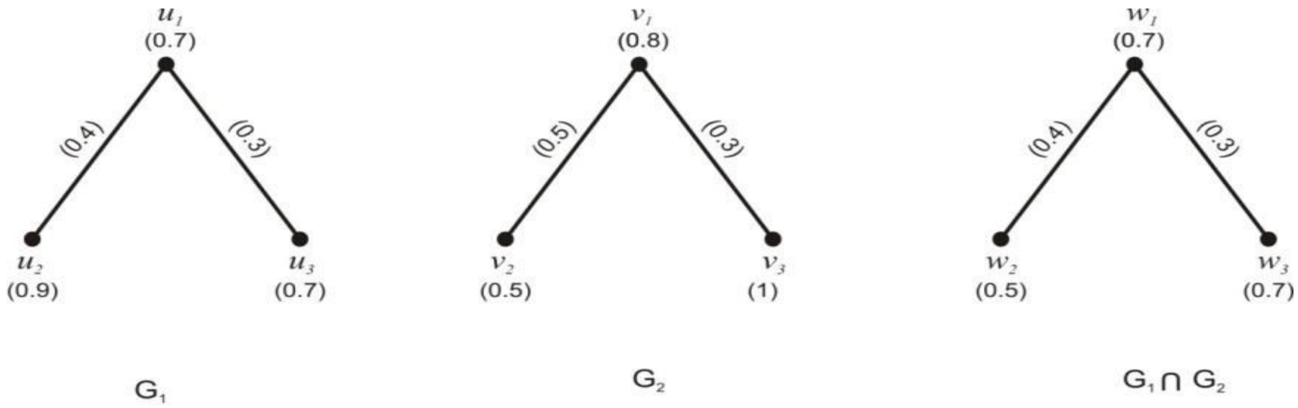


Fig 4.23

Hence intersection on subsets of χ lies in χ itself.

Therefore, the class D is discrete topology on the edge set $\mu(u_i, v_j)$ and $(\mu(u_i, v_j), \chi)$ is a discrete fuzzy topological graph.

B. In Discrete Topology

Let $\mu(u_i, v_j)$ be any edge set, then the class $\tau = \{\phi, \mu(u_i, v_j)\}$ consisting of the empty set and the set

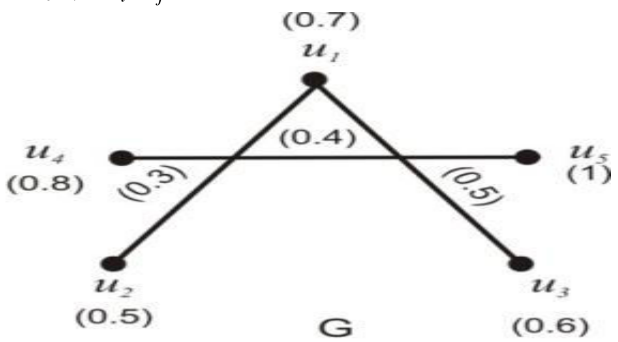


Fig 4.24

$\mu(u_i, v_j)$ is always a topology on $\mu(u_i, v_j)$ called indiscrete fuzzy topological graph.

The pair $(\mu(u_i, v_j), \tau)$ is called an indiscrete fuzzy topological graph.

Example

Given edge set

$$\mu(u_i, v_j) = \{\mu(u_1, v_2), \mu(u_1, v_3), \mu(u_4, v_5)\}$$

And a topology on it $\tau = \{\phi, \mu(u_i, v_j)\}$

To verify that τ is an indiscrete fuzzy topological graph.(i.e.) it satisfies all the axioms of topology.

Axiom 1

It is satisfied as the null edge set, $(i.e)\phi$ and the complete edge set $\mu(u_i, v_j)$ belongs to the τ .

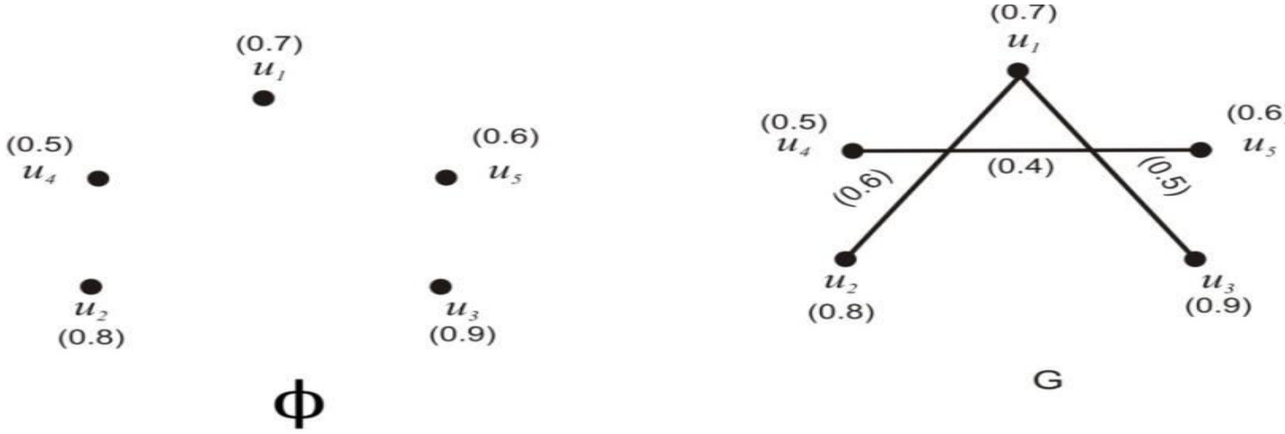


Fig 4.25

Axiom 2

It is also satisfied as the union of all the edge subsets of τ lies in itself.

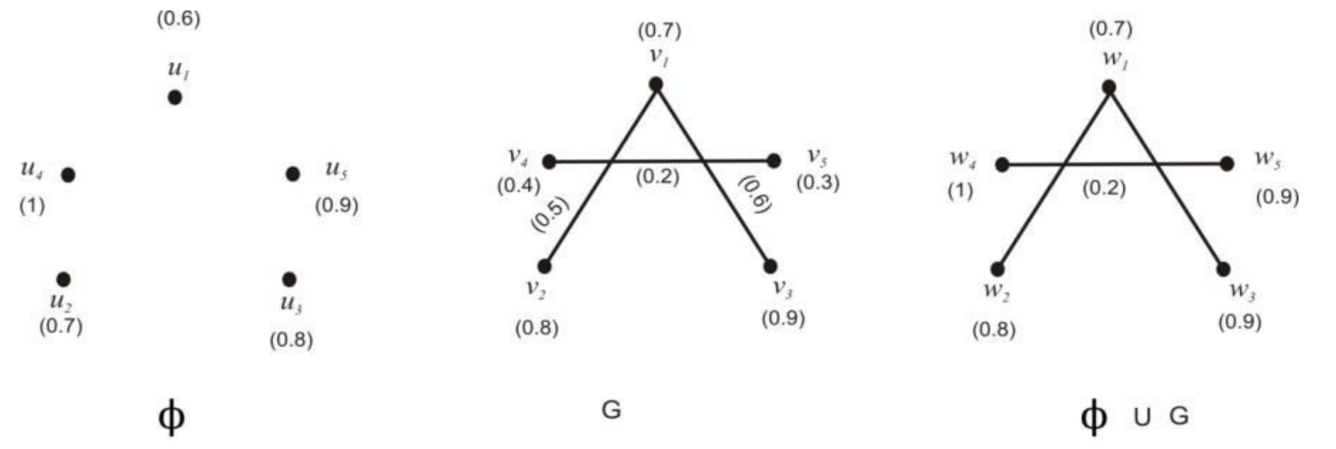


Fig 4.26

Axiom 3

The last axiom of topology also holds true (i.e.) the intersection of all the edge subsets also lies in τ itself.

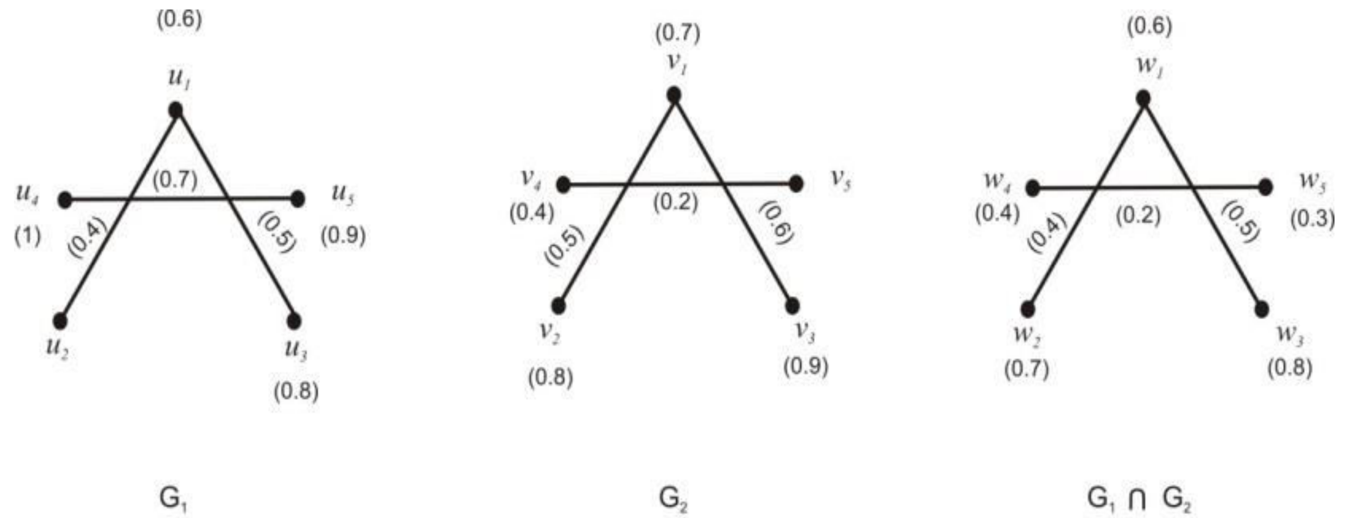


Fig 4.27

V. AMALGAMATIONS,EDGE CONTRACTION, CONNECTIVITY AND BOUQUET OF CIRCLE OF FUZZY GRAPHS

A. Amalgamations of Fuzzy Graphs

Lets G and G' be two fuzzy topological graphs and let $f : H \rightarrow H'$ be an isomorphism from a fuzzy topological sub graph H and G to a fuzzy topological sub graph H' of G'. The amalgamation $G *_f G'$ is obtained from the union of the vertex set $\sigma(v_i) = (\sigma_1(v_i) \vee \sigma_2(v_j))$ and $\mu(v_i, v_j) = [\mu_1(v_i, v_j) \vee \mu_2(v_i, v_j)]$

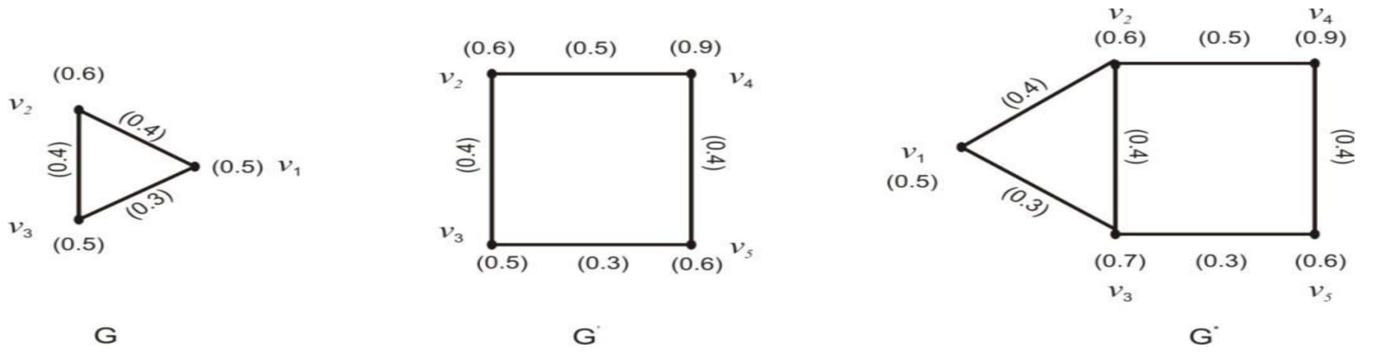


Fig 4.9

Figure of Amalgamation

Theorem

Let *A* and *B* be two planar fuzzy topological planar graphs. Then the fuzzy topological graph obtained by amalgamating *A* and *B* either at a single vertex *v* or along a single edge $\mu(u_i, v_j)$ is planar.

Proof:

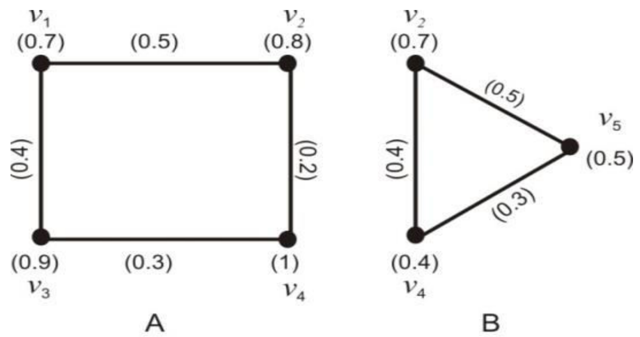
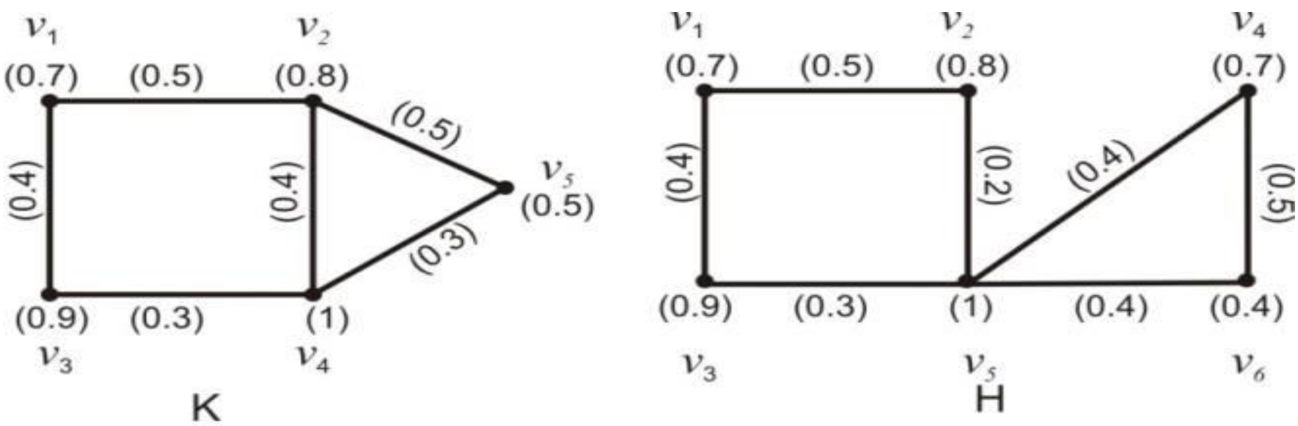


Fig 4.12

Let the fuzzy topological graphs *A* and *B* are imbedded disjointly in the plane. Hence, both the copies of vertex $\sigma(v_i)$ or of the edges $\mu(u_i, v_j)$ lie on the exterior region. It would be easy to construct a planar imbedding of the amalgamation for a vertex amalgamation, by pulling each copy of the vertex $\sigma(v_i)$ of the fuzzy graph *A* and *B*. Then the amalgamation of the fuzzy topological graphs *A* and *B* by the vertices forms a new fuzzy topological graph *H*.
H is the combination of two cycles and it is planar.



Amalgamation by an Edge Amalgamation by a Vertex

Fig 4.13

Let the value of the amalgamation vertex be

$\sigma(v) = (\sigma(v_i) \wedge \sigma(v_j))$

For an edge amalgamation of each copy of an edge $\mu(u_i, v_j)$ does the same pulling away as with a vertex.

To get the respective copies of $\mu(u_i, v_j)$ to the exterior region, let *A* and *B* be first imbedded in separate copies of the 2-cycle, and select in each copy a region that contains $\mu(u_i, u_j)$ & $\mu(v_i, v_j)$ on its boundary. Then paste the two resulting cycles together by the edges. Thereby forming a new 2-cycle in which the graphs *A* and *B* are imbedded so that both copies of $\mu(u_i, u_j)$ & $\mu(v_i, v_j)$ lie on the boundary of the same region.

Hence, the amalgamation of two edges of the fuzzy topological graphs *A* and *B* forms a new fuzzy topological graph *K*.

Therefore, *K* is also planar.

B. Edge Contraction And Connectivity of Fuzzy Graphs

Suppose that the fuzzy topological graph G' is obtained from the fuzzy topological graph *G* by contracting the edges $\mu(v_i, v_j)$ with end points $\sigma(u)$ and $\sigma(v)$ to the single vertex $\sigma(v')$.

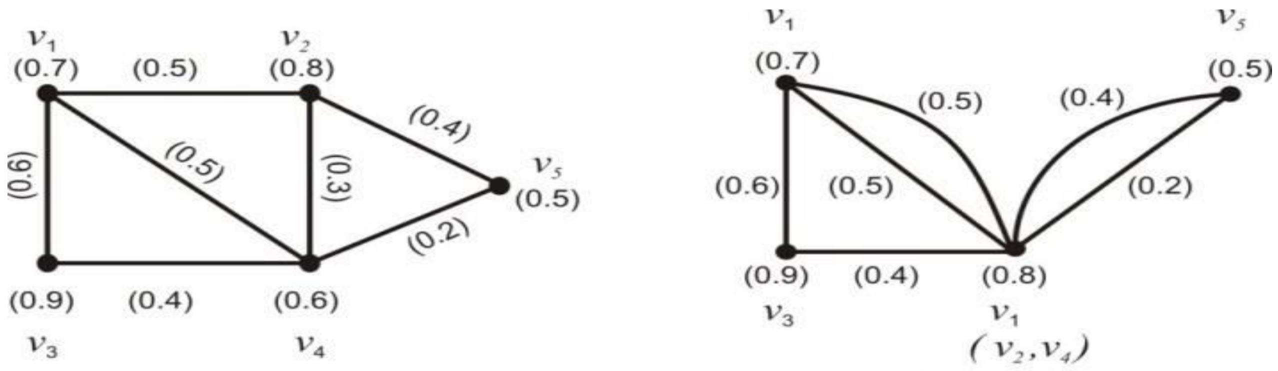


Fig 4.14

Figure of Edge Contraction and Connectivity

Theorem

Let *G* be a fuzzy topological graph that contains no homeomorphic of k_5 or $k_{3,3}$, and let $\mu(u_i, v_j)$ be any edge in *G*. Then the result of simplicially (simple graph) contracting the fuzzy topological graph *G* an edge $\mu(u_i, v_j)$ is a graph G' that contains no homeomorphic of K_5 or of $K_{3,3}$.

Proof

Let *G* be a fuzzy topological graph that contains no homeomorphic of K_5 or $K_{3,3.}$ we would imbed *G* in the plane, then simplicially contract $\mu(u_i, v_j)$ within the imbedding and there by obtain an imbedding of the contracted fuzzy topological graph G'. Now let us take a contradiction suppose that the contracted fuzzy topological graph G' contains a homeomorphic of K_5 or $K_{3,3}$. Then *G* would contain a fuzzy topological sub graph *H* such the contraction of a single edge $\mu(u_i, v_j)$ of H produces a homeomorphic of K_5 or $K_{3,3}$.

The fuzzy sub graph *H* itself contains a homeomorphic of K_5 or $K_{3,3}$. Contradicting the hypothesis for the original fuzzy topological graph *G*. Then the sub graph *H* is homeomorphic to K_5 or $K_{3,3}$.

Therefore, G' contains no homeomorphic of K_5 or $K_{3,3.}$

Theorem

Let *G* be a connected fuzzy topological graph with five or more vertices. Then there is some edge $\mu(u_i, v_j)$ of *G* such that the fuzzy topological graph $G / \mu(u_i, v_j)$ obtained by contracting $\mu(u_i, v_j)$ is also connected.

Proof

Suppose by way of contradiction that, for every edge $\mu(u_i, v_j)$, the contradiction fuzzy topological graph $G / \mu(u_i, v_j)$ has a set of two vertices that disconnects it.

ISSN No:-2456 – 2165

One of those two vertices must be the vertex obtained by identifying the two end points of the edge $\mu(\mathrm{u}_i, v_j)$; otherwise, the same set of two vertices would also disconnect G, there by contradicting the connectivity of *G*.

Thus for an edge $\mu(u_i, v_j)$ the end points $\sigma(u_i) \& \sigma(v_k)$ of $\mu(u_i, v_j)$ together with the vertex $\sigma(u_i)$ will give a new fuzzy topological graph *H*.

Let us choose an edge $\mu(u_i, v_j)$ and a vertex $\sigma(u_i)$ such that the largest component k of the graph $G - \{\sigma(u_i), \sigma(v_j)\}$ is the largest possible for any disconnecting set consisting of two vertices, two of which are adjacent. Deleting an edge and contraction of the vertices $\sigma(v_k)$ to $\sigma(u_i)$ with all the edges related to $\sigma(v_k)$ of *G* gives a new fuzzy topological graph *H*. This will reduces the strength of the connectedness of *G*. Therefore *H* is also connected.

Example

A connected fuzzy topological graph G with five or more vertices then contraction of any edge to the fuzzy topological graph G is also connected.

Solution

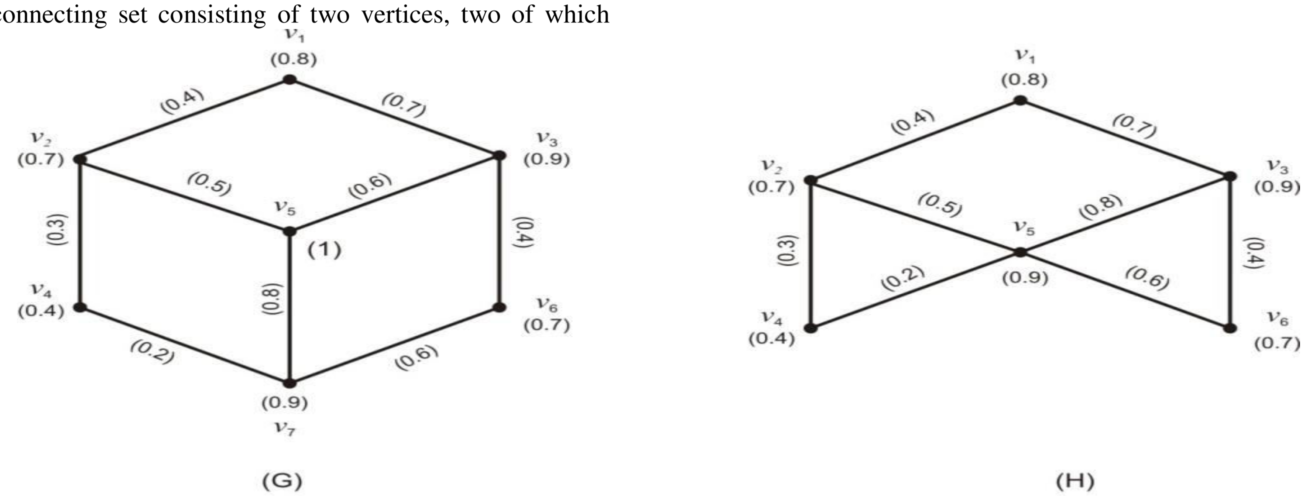


Fig 4.16

Let *G* be a connected fuzzy topological graph with seven vertices let the vertex set be

$$V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$$

Now the contraction of edge $\mu(v_5, v_7)$ at the vertex $\sigma(v_5)$ will form a new fuzzy topological graph *H*. Let us calculate the strength of the connectedness.

$$\mu(v_1, v_2) = 0.5; \mu(v_1, v_3) = 0.7; \mu(v_1, v_4) = 0.3; \mu(v_1, v_5) = 0.$$
$$\mu(v_1, v_6) = 0.4; \mu(v_2, v_3) = 0.4; \mu(v_2, v_4) = 0.4; \mu(v_2, v_5) = 0$$
$$\mu(v_2, v_6) = 0.5; \mu(v_3, v_4) = 0.3; \mu(v_3, v_5) = 0.8; \mu(v_3, v_6) = 0$$
$$\mu(v_4, v_5) = 0.3; \mu(v_4, v_6) = 0.3; \mu(v_5, v_6) = 0.6.$$

$$\mu^{\infty}(\mathrm{u}, v) = 0.8 > 0$$

Therefore *H* is connected.
From that we conclude,

If $\mu^{\infty}(u, v) = \mu(v_5, v_6)$ then the strength of the connectedness will be changed.

If $\mu^{\infty}(u, v) \neq \mu(v_5, v_6)$ then the strength of the connectedness will not be changed.

C. Minor of A Fuzzy Topological Graph

A fuzzy topological graph *H* is a minor of a fuzzy topological graph *G* if a copy of *H* can be obtained from *G* by deleting "or" contracting edges of *G*.

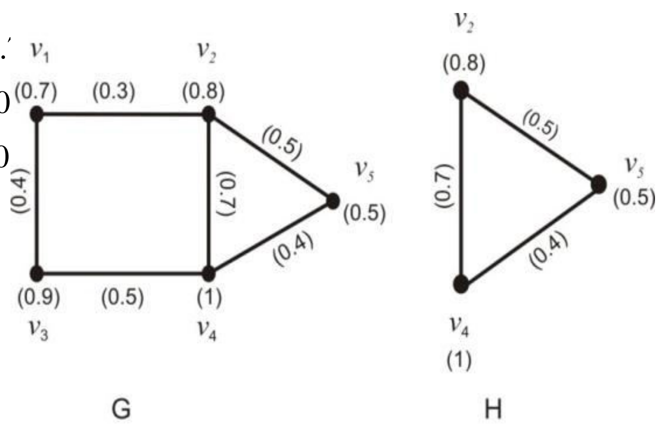


Fig 4.17

Figure of Minor of Fuzzy Topological Graph

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Theorem
A fuzzy topological graph G is planar if and only if neither K_5 nor $K_{3,3}$ is a minor of G

Proof
Let G be planar and K_5 nor $K_{3,3}$
Suppose K_5 or $K_{3,3}$ is a minor of G
We know that,If a fuzzy graph G is planar if and only it does not contain a sub graph which is homeomorphic from K_5 or $K_{3,3}$

Which is a contradiction to G is planar
G does not contains neither K_5 or $K_{3,3}$ as a minor of G
Conversely

Assume on the contrary that let G be a graph that does not contain K_5 or $K_{3,3}$ as a minor.
If a fuzzy graph G is planar if and only it does not contain a sub graph which is homeomorphic from K_5 or $K_{3,3}$
Therefore, G is planar.
Hence the theorem.

D. Bouquets of Circles of Fuzzy Graphs

A "*Bouquet of n circles*" is a fuzzy topological graph with one vertex and n loops. The standard model is denoted B_n.

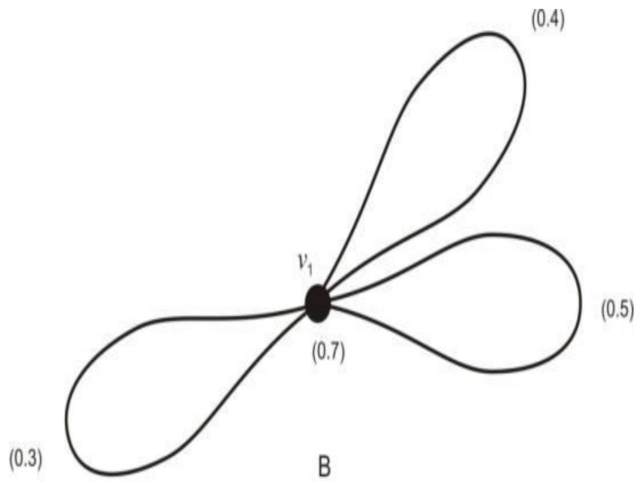

Fig 4.18

Figure of Bouquet of Circles

Example

The bouquet B_{n+k} is an amalgamation of the bouquets B_n and B_k at a vertex.

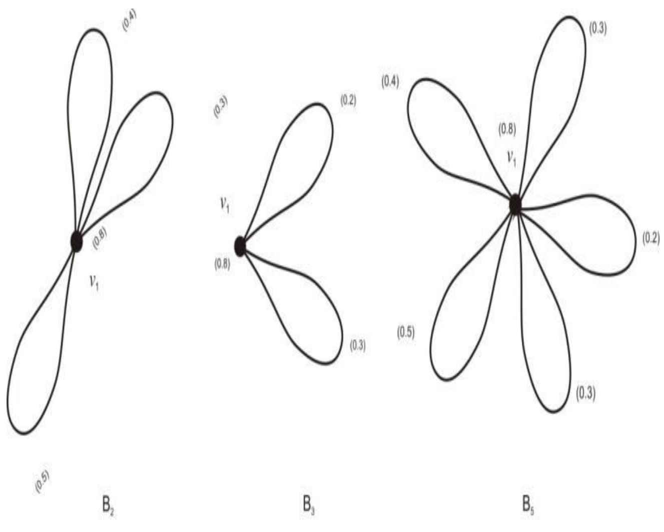

Fig 4.19

VI. OBSERVATIONS

- Every complement of a fuzzy topological graph is open,
- The complete fuzzy topological graph K_{n+1} is isomorphic to the suspension of the complete fuzzy topological graph K_n from the fuzzy topological graph K_1.
- The suspension of two complete fuzzy topological graph will become a complete bipartite fuzzy topological graph.
- *(i.e.)* $K_m + K_n = K_{mn}$
- $(K_m + K_n)^C = K_{mn}$ (Membership value become 0)
- $((K_m + K_n)^C)^C = K_{mn}$.
- Suspension of two fuzzy topological planar graphs need not be a planar fuzzy topological graph.
- Amalgamation of two fuzzy topological planar graphs will be afuzzy topological planar graph.
- The edge complement K_n^C of the complete fuzzy topological graph has no edges at all but has n vertices

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