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δ-Open Sets and δ-Nano Continuity in δ-Nano Topological Space

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Abstract:- In this paper δ-open sets and nearly open sets in δ-nano topological space are introduced. Also we introduce some δ-nano continuous functions and the relationship between these functions are also studied. Finally we introduce two application examples in nano topology.

Keywords:-Nano δ-α open, Nano δ-Semi Open, Nano δ-pre Open, Nano δ-γ Open, Nano δ-β Open, Nano δ-Continuity. Mathematics Subject Classification 2010: 54B05.

I. INTRODUCTION

Nano topological space $(U,\tau_R(X))$ with respect to a subset X is introduced by Lellis Thivagar(1) where U is the universal set which is defined in terms of lower and upper approximations of X. The elements in the nano topological space are called nano open sets. He has defined nano closed sets, nano interior and nano closure of a set.In 2013, certain weak forms of nano open sets are introduced and studied by Lellis thivagar(2). In 2016, A. A. Nasef [a], A. I. Aggour[b,*], S. M. Darwesh[b](4) studied some near nano open sets in nano topological spaces and its properties. In this paper the relationships between some weak forms of δ-open sets in a nano topological space$(U,\tau_R(X))$ are studied. Generally the family of all nano regular open, nano α-open, nano semi-open, nano preopen, nano γ-open, nanoβ-open sets in a nano topological space(U, $\tau_R(X)$) is denoted by NRO(U, X), NαO(U, X), NSO(U, X), NPO(U, X), NγO(U, X) and NβO(U, X). In 2013, continuity of mappings in nano topological space was introduced and studied by Lellis Thivagar and Richard (3).

II. PRELIMINARIES

Definition 2.1

A non-empty finite set of objects is called the universe U. Let R be an equivalence relation on U named as the indiscernibility relation. The pair (U, R) is said to be the approximation space. Let $X\subseteq U$. Then,

(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $LR_R(X)$.
$$LR_R(X)=\cup_{x\in U}\{R(x):R(x)\subseteq X, x\in U\}$$
where R(x) denotes the equivalence class determined by $x\in U$.

(ii) The upper approximation of X with respect to R is the set of all objects which can be possibly classified as X with respect to R and it is denoted by $UR_R(X)$.
$$UR_R(X)=\cup_{x\in U}\{R(x):R(x)\cap X\neq \varphi, x\in U\}.$$

(iii) The boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not X with respect to R and it is denoted by $BR_R(X)$.
$$BR_R(X) = UR_R(X) - LR_R(X).$$

Property 2.2

If (U, R) is an approximation space and $X,Y\subseteq U$, then

1. $LR_R(X) \subseteq X\subseteq UR_R(X)$
2. $LR(\Phi)=UR_R(\Phi)=\Phi$
3. $LR_R(U)=UR_R(U)=U$
4. $UR_R(X\cup Y)=UR_R(X) \cup UR_R(Y)$
5. $UR_R(X\cap Y)\subseteq UR_R(X) \cap UR_R(Y)$
6. $LR_R(X\cup Y)\supseteq LR_R(X) \cup LR_R(Y)$
7. $LR_R(X\cap Y)= LR_R(X) \cap LR_R(Y)$
8. $LR_R(X)\subseteq LR_R(Y)$ and $UR_R(X)\subseteq UR_R(Y)$ whenever $X\subseteq Y$
9. $UR_R(X^C)=[LR_R(X)]^C$ and $LR_R(X^C)=[UR_R(X)]^C$
10. $UR_R[UR_R(X)]= LR_R[UR_R(X)]=UR_R(X)$
11. $LR_R[LR_R(X)]=UR_R[LR_R(X)]= LR_R(X)$

Definition 2.3

Let U be the universe, R be an equivalence relation on U and

$$\tau_R(X) = \{ U, \phi, LR_R(X), UR_R(X), BR_R(X)\}$$

where $X \subseteq U$. Then by Property 2.2, $\tau_R(X)$ satisfies the following axioms:

1. U and $\phi \in \tau_R(X)$
2. The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$
3. The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. (U, $\tau_R(X)$) is called the nano topological space or simply NTS. Elements of the nano topology are known as nano open sets in U. Elements of $[\tau_R(X)]^c$ are called nano closed sets.

Remark 2.4

If $\tau_R(X)$is the nano topology on U with respect to X, then the set

$B = \{ U, LR_R(X), BR_R(X)\}$ is the basis for $\tau_R(X)$.

Definition 2.5

If (U, $\tau_R(X)$) is a NTS with respect to X where $X \subseteq U$ and if $B \subseteq U$, then

(i) The nano interior of B is defined as the union of all nano open subsets of B and it is denoted by nint(B). That is, nint(B) is the largest nano open(simply N-open) subset of B.

(ii) The nano closure of B is defined as the intersection of all nano closed sets containing B and it is denoted by ncl(B). That is, ncl(B) is the smallest nano closed(simply N-closed) set containing B.

Definition 2.6

Let (X, τ) be a topological space and $B \subseteq X$. The set B is said to be

(i) regular open if B = int(ncl(B)).
(ii) α-open if $B \subseteq$ int(cl(int(B)).
(iii) Semi open if $B \subseteq$ cl(int(B)).
(iv) Preopen if $B \subseteq$ int(cl(B)).
(v) γ-open if $B \subseteq$ cl(int(B) ∪ int(cl(B)).
(vi) β-open if $B \subseteq$ cl(int(cl(B)).

Definition 2.7

A subset S of a topological space (X, τ) is called regular closed(resp., α-closed, semi closed, pre closed, γ-closed, β-closed) if its complements is regular open(res., α-open, semi open, pre open, γ-open, β-open).

Definition 2.8

Let (X, τ) be a topological space and $B \subseteq X$, then B is said to be:

(i) δ-regular open if B =int(cl_δ(B)).
(ii) δ-α-open if $B \subseteq$ int(cl(int$_\delta$(B)).
(iii) δ-semi open if $B \subseteq$ cl(int$_\delta$(B)).
(iv) δ-pre open if $B \subseteq$ int(cl_δ(B)).
(v) δ-γ-open if $B \subseteq$ cl(int$_\delta$(B) ∪ int(cl_δ(B)).
(vi) δ-β-open if $B \subseteq$ cl(int(cl_δ(B)).

Definition 2.9

A subset S of a topological space (X, τ) is called δ-regular closed (resp., δ-α-closed, δ-semi closed, δ-pre closed, δ-γ-closed, δ-β-closed) if its complements is δ-regular open (resp., δ-α-open, δ-semi open, δ-pre open, δ-γ-open, δ-β-open).

Definition 2.10

Let (U, $\tau_R(X)$) be a NTS and $B \subseteq U$. Then B is said to be

(i) N-regular open if B = nint(ncl(B)),
(ii) N-α-open if $B \subseteq$ nint(ncl(nint(B))),
(iii) N-semi open if $B \subseteq$ ncl(nint(B)),
(iv) N-pre open if $B \subseteq$ nint(ncl(B)),
(v) N-γ-open(or N-b-open) if $B \subseteq$ ncl(nint(B)) ∪nint(ncl(B)),
(vi) N-β-open(or N-semi preopen) if $B \subseteq$ ncl(nint(ncl(B))).

The family of all N-regular open sets in a NTS (U, $\tau_R(X)$) is denoted by NRO(U, X). Similarly the family of all N-α-open, N-semi open, N-pre open, N-γ-open, N-β-open sets are denoted by NαO(U, X), NSO(U, X), NPO(U, X), NγO(U, X) and NβO(U, X) respectively.

Definition 2.11

A subset S of a NTS (U, $\tau_R(X)$) is called N-regular closed (resp., N-α-closed, N-semi closed, N-pre closed, N-γ-closed, N-β-closed) if its complements is N-regular open (resp., N-α-open, N-semi open, N-pre open, N-γ-open, N-β-open).

Definition 2.12

Let (U, $\tau_R(X)$) and (V, $\tau_R(Y)$) be two NT Spaces. Let

g: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) be a mapping. Then g is said to be:

(i) N-continuous if $g^{-1}(B)$ is N-open set in U for every N-open set B in V.
(ii) N-α-continuous if $g^{-1}(B)$ is N-α-open set in U for every N-open set B in V.

(iii) N-semi continuous if $g^{-1}(B)$ is N-semi open set in U for every N-open set B in V.
(iv) N-pre continuous if $g^{-1}(B)$ is N-pre open set in U for every N-open set B in V.
(v) N-γ-continuous (or N-b-continuous) if $g^{-1}(B)$ is N-γ-open(or N-b-open) set in U for every N-open set B in V.
(vi) N-β-continuous (or N-semi pre continuous) if $g^{-1}(B)$ is N-β-open(or N semi-pre open) set in U for every N-open set B in V.

Lemma 2.13

Let (U, τ_R (X)) be a NTS, if A and B are N-open subsets of U. Then,

(i) A∩B is also N-open
(ii) ncl(A∪B) = ncl(A)∪ ncl(B)

Lemma 2.14

Let (U, τ_R (X)) be a NTS. If x∈ncl(B) iff If x∈ncl(B) iff every N-open set V containing x intersects of B.

III. δ-OPEN SETS IN δ-NANO TOPOLOGICAL SPACE

Definition 3.1

Let (U, τ_R(X)) be a NTS. A subset B is said to be N-δ-open if for each x∈B there exists a N-regular open set G such that x∈G ⊂ B. A point x∈B is called the δ-limit point of B ⊆ X if B∩I[C(U)]≠ ∅ for every N-open set U of X containing x. The set of all δ-limit point of B is called the δ-nano closure of B.

Example 3.2

Let U = {p, q, r, s, t} with U/R = {{p, r}, {q}, {s}, {t}} and X = {p, q}. Then the N topology is defined as τ_R(X) = {U, ∅, {q}, {p, q, r}, {p, r}}. N-δ-open set is τ_{R_δ}(X) = {∅, {q}, {p, q, r}, {p, r}} and $\tau_{R_\delta}(X)^C$ = {U, {p, r, s, t}, {s, t}, {q, s, t}}.

Definition 3.3

Let (U, τ_R(X)) be a NTS and B ⊆ U. Then B is said to be

(i) N-δ-regular open if A =nint(ncl_δ(A)).
(ii) N-δ-αopen if A ⊆ nint(ncl($nint_\delta$(A)).
(iii) N-δ-semi open if A ⊆ ncl($nint_\delta$(A)).
(iv) N-δ-pre open if A ⊆ nint(ncl_δ(A)).
(v) N-δ-γopen if A ⊆ ncl($nint_\delta$(A) ∪ nint(ncl_δ(A)).
(vi) N-δ-βopen if A ⊆ ncl(nint(ncl_δ(A)).

The family of all N-δ-regular open, N-δ-αopen, N-δ- semi open, N-δ-pre open, N-δ-γopen, N-δ-βopen sets in a NTS(U, τ_{R_δ}(X))are denoted by NδRO(U,X), NδαO(U,X), NδSO(U,X), NδPO(U,X), NδγO(U,X) and NδβO(U,X).

Definition 3.4

A subset S of a NTS(U, τ_R(X)) is called N-δ-regular closed(resp., N-δ-α closed, N-δ-semi closed, N-δ- pre closed, N-δ-γ closed, N-δ-β closed) if its complement is N-δ-regular open(resp., N-δ-α open, N-δ-semi open, N-δ- pre open, N-δ-γ open, N-δ-β open)

Lemma 3.5

Let (U, τ_R(X)) be a NTS and P and Q are subsets of U. Then the following hold

(i) $nint_\delta$(P) ∩ $nint_\delta$(Q) = $nint_\delta$(P∩Q)
(ii) ncl_δ(P) ∪ ncl_δ(Q) = ncl_δ(P∪Q).

Proof:

(i) Clearly $nint_\delta$(P)∩ $nint_\delta$(B) ⊃ $nint_\delta$(P∩Q).

Let x ∈ $nint_\delta$(P) ∩ $nint_\delta$(Q).

Since x∈ $nint_\delta$(P) and x∈ $nint_\delta$(Q), there exits N-closed sets C and D such that

x∈nint(C) ⊂ P and x∈nint(D) ⊂ Q and

x∈nint(C) ∩nint(D) ⊂ P∩Q

⇒x∈ $nint_\delta$(P∩Q).

(ii) The proof follows from (i) Hence the result.

Lemma 3.6

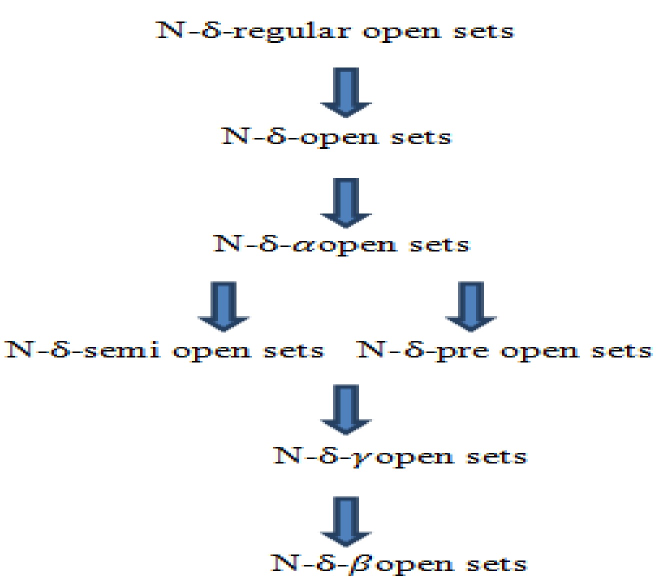
Let (U, τ_R(X)) be a NTS. Then the following hold:

(i) ncl_δ (B) is the smallest N-δ closed set containing B.
(ii) B is N-δ closed if and only if B= ncl_δ(B).
(iii) ncl_δ(B) ⊂ ncl_δ(C) whenever B ⊂ C.
(iv) ncl_δ(ncl_δ(B)) = ncl_δ(B).
(v) x∈ ncl_δ(B) if and only if B∩V≠ ∅ for every N-δ open set V containing x.
(vi) $nint_\delta$ (B) is the largest N-δopen sets contained in B.
(vii) B is N-δopen if and only if B = $nint_\delta$(B).
(viii) int_δ(B) ⊂ $nint_\delta$(C) whenever B ⊂ C.
(ix) $nint_\delta$($nint_\delta$(B)) = $nint_\delta$(B).
(x) U−$nint_\delta$(B) = ncl_δ(U−B).
(xi) U−ncl_δ(B) = $nint_\delta$(U−B).

Proof:

(i) If C is any N-δ closed set containing B, then $ncl_\delta(B) \subseteq$ C. Hence $ncl_\delta(B)$ is the smallest δ-closed set containing B.
(ii) Suppose B is N-δ closed set . Then the smallest N-δ closed set containing B is itself. Therefore B = $ncl_\delta(B)$.
(iii) The proof follows from the definition of ncl_δ and from (i).
(iv) By (i), the smallest N-δ closed set containing $ncl_\delta(B)$ is $ncl_\delta(B)$. Hence $ncl_\delta(ncl_\delta(B)) = ncl_\delta(B)$.
(v) Suppose $x \in ncl_\delta(B)$. Let G be a N-δopen set containing x. If $G \cap B = \emptyset$, then $U - G$ is a N-δ closed set containing B and so $x \notin ncl_\delta(B)$, a contradiction. Therefore, $G \cap B \neq \emptyset$. If $x \notin ncl_\delta(B)$, then there exists a N-δ closed set F containing B such that $x \notin F$. Then $G = U - F$ is a N-δopen set containing x such that $G \cap B = \emptyset$, a contradiction. Therefore, $x \in ncl_\delta(B)$.
(vi) If C is any N-δopen set contained in B, then $C \subseteq int_\delta(B)$. Hence the proof follows.
(vii) Suppose B is any N-δopen set of U. Then the largest N-δopen set contained in B is itself. Therefore B = $nint_\delta(B)$.
(viii) The proof follows from (vii).
(ix) By (vi), the largest N-δ open set containing $nint_\delta(B)$ is itself. Hence $nint_\delta(nint_\delta(B)) = nint_\delta(B)$.
(x) $ncl_\delta(B)$ is the smallest N-δ closed set containing B. The complement is the largest N-δ-open set contained in $U - B$. Therefore $U - ncl_\delta(B) = nint_\delta(U - B)$.
(xi) $nint_\delta(B)$ is the largest N-δopen set contained in B. Its complement is the smallest δ-closed set containing $U - B$. Hence $U - nint_\delta(B) = ncl_\delta(U - B)$.

The following diagram holds for a subset B of a NTS$(U, \tau_R(X))$. The following examples show that, none of these implications is reversible.



Example 3.7

Let U = {p, q, r, s} with U/R = {{p}, {r}, {q, s}} and X = {p, q}. Then the N topology is defined as $\tau_R(X)$ = {U,∅, {p}, {p, q, s}, {q, s}} and the δ-nano topology defined as $\tau_{R_\delta}(X)$ = {∅, {p}, {p, q, s}, {q, s}}. Then, we have the following:

(i) if B = {p, q, s}, then B is N-δ open set but not N-δ-regular open set.

(ii) if B = {p, r}, then B is N-δ-semi open set but not N-δ-α open set.

(iii) if B = {p, q}, then B is N-δ-pre open set but not N-δ-α open set.

(v) if B = {p, q, r}, then B is N-δ-γ open set but not N-δ-pre open set.

(vi) if B = {q, r}, then B is N-δ-β open set but not N-δ-γopen set.

(iv) if B = {q, r, s}, then B is N-δ-γ open set but not N-δ-pre open set.

Proposition 3.8

For every NTS$(U, \tau_R(X))$, N δ-P O(U,X) ∪ N δ-S O(U,X) ⊆ N δ-γ O(U,X) ⊆ N δ-β O(U,X) holds but none of these implications can be reversed.

Proof:

The proof follows from the definition 3.3 (iii, iv, v, vi)

Proposition 3.9

Let $(U, \tau_R(X))$ be a NTS then:

(i) If $P \subseteq U$ is N-open and $Q \subseteq U$ is N-δ-semi open, N-δ-pre open, N-δ-β open, N-δ-α open, then $P \cap Q$ is N-δ-semi open, N-δ-pre open, N-δ-β open, N-δ-α open.
(ii) $P \subseteq U$ is N-δ-αopen, if and only if P is the union of a N-δ-semi open set and a N-δ-preopen set.

Proof:

(i) P is N-δ open $\Rightarrow P = nint_\delta(P)$

Q is N-δ-semi open $\Rightarrow Q \subset ncl(nint_\delta(Q))$

$P \cap Q = nint_\delta(P) \cap ncl(nint_\delta(Q))$

$\subset ncl(nint_\delta(P)) \cap ncl(nint_\delta(Q))$

$\subset ncl(nint_\delta(P) \cap nint_\delta(Q))$

$\subset ncl(nint_\delta(P \cap Q))$

Similarly the other results also proved.

(iii) if P is a N-δ-γ open iff

$\Leftrightarrow P \subset nint(ncl_{\delta}(P)) \cup ncl(nint_{\delta}(P))$

$\Leftrightarrow$P is the union of N-δ-pre open and a N-δ-semi open set.

Proposition 3.10

Let $(U, \tau_R(X))$ be a NTS. The intersection of a N-δ-pre open set and a N-δ-α open set is N-δ-pre open.

Proof:

Let P∈ Nδ-PO(U,X) and Q∈ Nδ- α O(U,X),

then $P \subset nint(ncl_{\delta}(P))$, $Q \subset nint(ncl(nint_{\delta}(Q)))$.

So, $P \cap Q \subset nint(ncl_{\delta}(P)) \cap nint(ncl(nint_{\delta}(Q)))$

$\subset nint(nint(ncl_{\delta}(P)) \cap ncl(nint_{\delta}(Q)))$

$\subset nint(ncl(ncl_{\delta}(P) \cap ncl(nint_{\delta}(Q)))$

$\subset nint(ncl(ncl_{\delta}(P) \cap nint_{\delta}(Q))) \subset nint(ncl(ncl_{\delta}(P \cap Q)))$

$= nint(ncl_{\delta}(P \cap Q))$.

Hence, P∩ Qis N-δ-preopen set.

Corollary 3.11

Let $(U, \tau_R(X))$ be a NTS. The union of a N-δ-pre closed set and a N-δ-α open set is N-δ-pre closed set.

Remark 3.12

The arbitrary intersection of N-δ-β closed set is N-δ-β closed but the union of two N-δ-β closed sets may not be N-δ-β closed set. This is clearly by the following example.

Example 3.13

Let U = {p, q, r, s}. Then U/R = {{p}, {r}, {q, s}} and X = {p, q}. Then $\tau_{R_{\delta}}(X) = \{\phi, \{p\}, \{q, s\}, \{p, q, s\}\}$, the subsets A= {q} and B $= \{p, s\}$ are N-δ-β closed sets but A∪B $= \{p, q, s\}$ is not N-δ-β closed set.

Proposition 3.14

Let $(U, \tau_R(X))$ be a NTS. Each N-δ-β open set which is N-δ-semi closed is N-δ-semi open.

Proof:

Let P be a N-δ-βopen set and N-δ-semi closed.

then $P \subseteq ncl(nint(ncl_{\delta}(P)))$ and $nint(ncl_{\delta}(P)) \subseteq P$.

Therefore, $nint(ncl_{\delta}(P)) \subseteq nint_{\delta}(P)$ and

So, $ncl(nint(ncl_{\delta}(P))) \subseteq ncl(nint_{\delta}(P))$.

Hence, $P \subseteq ncl(nint(ncl_{\delta}(P))) \subseteq ncl(nint_{\delta}(P))$.

Hence P is N-δ-semi open.

Proposition 3.15

Let P be a subset of a NTS $(U, \tau_R(X))$. If B is a N-δ-β closed and N-δ-semi open, then it is N-δ-semi closed.

Proof:

Since P is N-δ- β closed and N-δ-semi open.

Then $U - P$ is N-δ-β open and N-δ-semi closed and so by Proposition 3.14

$U - P$ is N-δ-semiopen.

Therefore, P is N-δ-semi closed.

Proposition 3.16

Let $(U, \tau_R(X))$ be a NTS. Each N-δ-βopen set and N-δ-α closed set is N-δ-regular closed.

Proof:

Let P⊆U be a N-δ-β open set and N-δ-α closed set.

Then $P \subseteq ncl(nint(ncl_{\delta}(P)))$ and $ncl(nint(ncl_{\delta}(P))) \subset P$,

which implies that $ncl(nint(ncl_{\delta}(P))) \subseteq P \subseteq ncl(nint(ncl_{\delta}(P)))$.

So, $P = ncl(nint(ncl_{\delta}(P)))$.

Hence P is N-δclosed, and so it is N-δ-regular closed.

Corollary 3.17

Each N-δ-β closed set and N-δ-α open set is N-δ-regular open.

IV. SOME CLASSES OF NANO δ-CONTINUITY

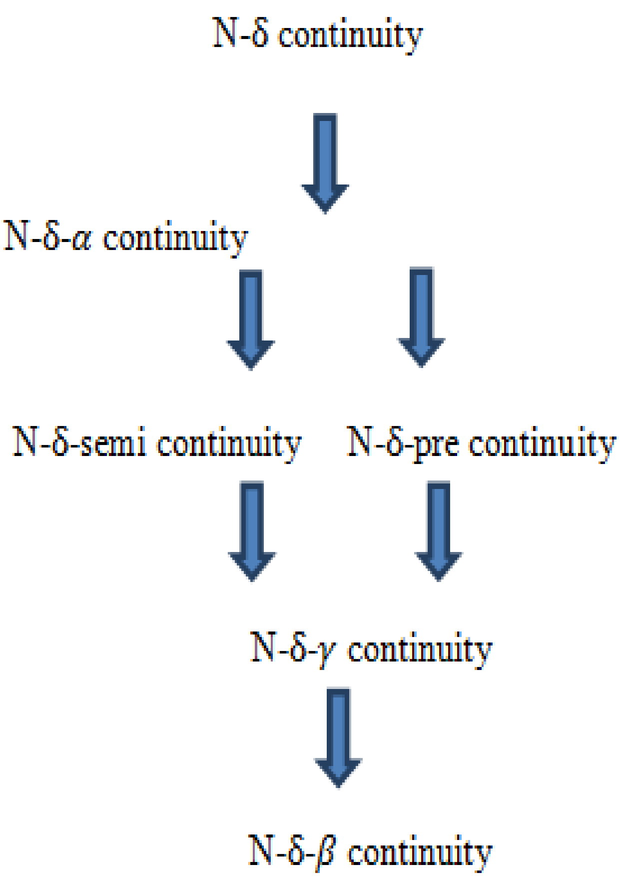
Definition 4.1

Let (U, τ_R (X)) and (V,$\tau_{R'}$(Y)) be two Nano topological spaces.

A mapping f: (U, τ_R (X)) → (V,$\tau_{R'}$(Y)) is said to be:

(i) N-δ continuous if f^{-1} (B) is N-δopen set in U for every N-open set B in V.
(ii) N-δ-α continuous if f^{-1}(B) is N-δ-αopen set in U for every N-open set B in V.
(iii) N-δ-semi continuous if f^{-1}(B) is N-δ-semi open set in U for every N-open set B in V.
(iv) N-δ-pre continuous if f^{-1}(B) is N-δ-preopen set in U for every N-open set B in V.
(v) N-δ-γ continuous if f^{-1}(B) is N-δ-γopen set in U for every N-open set B in V.
(vi) N-δ-β continuous if f^{-1}(B) is N-δ-β-open set in U for every N-open set B in V.

The relations between the above types of nano near δ continuous functions clearly by the following diagram



None of these implications is reversible as shown by the following examples.

Example 4.2

(i) Let U = {p, q, r, s} with U/R = {{p}, {s}, {q, r}} and X = {p, s}. Then τ_R (X) = {U, ϕ, {p, s}}, τ_{R_δ}(X) = { ϕ, {p, s}}. Let V = {x, y, z, w} with V/R= {{x}, {z}, {y, w}} and Y = {x, y} then τ_R(Y) = { V, ϕ, {x}, {y, w}, {x, y, z}}, $\tau_{R'_\delta}$(Y) = {ϕ, {x}, {y, w}, {x, y, z}}. Define f: (U, τ_R (X)) → (V,$\tau_{R'}$(Y)) as f(p) = y, f(q) = y, f(r) = z, f(s) = w, here {y,w} = B, f^{-1} ({ y, w}) = {q, s} then f is nano δ-α-continuous but not nano δ-continuous.

Example 4.3

(ii) Let U = {p, q, r, s} with U/R= {{p}, {r}, {q, s}} and X = {p, q}. Then τ_R(X) = { U, ϕ, {q, s}, {p, q, s}, {p}}, τ_{R_δ} (X) = {ϕ, {p}, {q, s}, {p, q, s}}. Let V = {x, y, z, w} with V/R= {{x}, {w}, {y, z}} and Y = {x, y} then τ_R(Y) = {U, ϕ, {x}, {x, y, z}, {y, z}},$\tau_{R'_\delta}$(Y) = {ϕ, {x}, {x, y, z}, {y, z}}. Define f: (U, τ_R (X)) → (V,$\tau_{R'}$(Y)) as f(p) = y, f(q) = y, f (r) = z, f (s) = w, here {x, y, z} = B, f^{-1}({x, y, z})={p, r}, then:

(iii) f is nano δ-semi-continuous but not nano δ-α-continuous.

(iv) f is nano δ-γ-continuous but not nano δ-pre-continuous.

Example 4.4

Let U = {p, q, r, s} with U/R= {{p}, {r}, {q, s}} and X = {p, q}. Then τ_R(X) = { U, ϕ, {q, s}, {p, q, s}, {p}}, τ_{R_δ} (X) = {ϕ, {p}, {q, s}, {p, q, s}}. Let V = {x, y, z, w} with V/R= {{x}, {w}, {y, z}} and Y = {x, y} then τ_R(Y) = {U, ϕ, {x}, {x, y, z}, {y, z}},$\tau_{R'_\delta}$(Y) = {ϕ, {x}, {x, y, z}, {y, z}}. Let g : (U, τ_R (X)) → (V,$\tau_{R'}$(Y)) be defined as follows g(p) = w, g(p) = y, g(r) = z, g(s) = w. Here {x, y, z} = B, g^{-1}({x, y, z}) = {q, r}. Then g is nano δ-β-continuous but not nano δ-γ-continuous.

Example 4.5

Let U = {p, q, r, s} with U/R= {{p}, {r}, {q, s}} and X = {p, q}. Then τ_R(X) = { U, ϕ, {q, s}, {p, q, s}, {p}}, τ_{R_δ} (X) = {ϕ, {p}, {q, s}, {p, q, s}}. Let V = {x, y, z, w} with V/R= {{x}, {w}, {y, z}} and Y = {x, y} then τ_R(Y) = {U, ϕ, {x}, {x, y, z}, {y, z}},$\tau_{R'_\delta}$(Y) = {ϕ, {x}, {x, y, z}, {y, z}}. Let h: (U, τ_R (X)) → (V,$\tau_{R'}$(Y)) be defined as follows h (p) = y, h (q) = x, h (r) = z, h (s) = w. Here {x, y, z} = B, h^{-1} ({x, y, z})

= {p, q, r}. Then h is nano δ-γ-continuous but not nano δ-semi-continuous.

Example 4.6

Let U = {p, q, r, s} with U/R= {{p}, {r}, {q, s}} and X = {p, q}. Then τ_R(X) = { U, ϕ, {q, s}, {p, q, s}, {p}}, τ_{R_δ} (X) = {ϕ, {p}, {q, s}, {p, q, s}}. Let V = {x, y, z, w} with V/R= {{x}, {w}, {y, z}} and Y = {x, y} then τ_R(Y) = {U, ϕ, {x}, {x, y, z}, {y, z}},$\tau_{R'_\delta}$(Y) = {ϕ, {x}, {x, y, z}, {y, z}}. Let f: (U, τ_R (X)) → (V,$\tau_{R'}$(Y)) be a mapping defined as follows f (p) = w, f(q) = x, f (r) = w, f (s) = x. Here {x, y, z} ⊂ B_1, f^{-1}({x, y, z}) = {q}.Then f is nano δ-pre-continuous but not nano δ-α-continuous.

Theorem 4.7

Let (U,τ_R(X)) and (U,$\tau_{R'}$(Y)) be NTS, and let f:(U,τ_R(X)) →(V,$\tau_{R'}$(Y))be a mapping. Then, the following statements are equivalent:

(i) g is N-δ-β continuous.
(ii) The inverse image of every N-closed set B in V is N-δ-β closed in U.
(iii) g(n δ-β cl(B)) ⊆ ncl$_\delta$(g(B)), forevery subset B of U.
(iv) n δ-βcl(g^{-1}(E)) ⊆ g^{-1}(ncl$_\delta$(E)),forevery subset E of
(v) g^{-1}(nint$_\delta$(E)) ⊆ N δ-β int(g^{-1}(E)) , for every subset E of V.

Proof:

(i)⇒ (ii)

Let g be N-δ-βcontinuous and let B be N-closed set in V. That is V − B is N-open in V. Since g is N-δ-β continuous mapping. Then g^{-1} (V − B) is N-δ-β open in U. Then g^{-1}(V − B) = U −g^{-1}(B) which means that, g^{-1}(B) is N-δ-β closed set in U.

(ii) ⇒ (i)

Let B be N-open set in V. Then, g^{-1} (V − B) is N-δ-β closed in U . Then g^{-1} (B) is N-δ-β open in U. Hence U−g^{-1} (B) is a N-δ-β closed in U. Therefore, g is Nδ-βcontinuous mapping.

(i) ⇒ (iii)

Let g be N-δ-β continuous and let B ⊆ U. Since g is N-δ-β continuous and ncl$_\delta$(g (B)) is N-closed in V, g^{-1}(ncl$_\delta$(g(B))) is N-δ-β closed in U. Since g(B) ⊆ ncl$_\delta$(g(B) g^{-1}(g(B)) ⊆ g^{-1}(ncl$_\delta$(g(B))) , then N-δ-βcl(B) ⊆N-δ-βcl[g^{-1}(ncl$_\delta$(g(B)))] = g^{-1} (ncl$_\delta$(g(B))). Thus N-δ-βcl(B) ⊆ g^{-1} (ncl$_\delta$(g(B))). Therefore, g(N-δ-β cl(B))) ⊆ ncl$_\delta$ (g(B)) for every subset B of U.

(iii) ⇒ (i)

Let g(N-δ-β cl(B)) ⊆ ncl$_\delta$(g(B)) for every subset B of U. Let E be N-closed in V, then g(N-δ-β cl(E)) ⊆ ncl$_\delta$(g(g^{-1}(E))) = ncl$_\delta$(E) = E that is g(N-δ-β cl(g^{-1}(E))) ⊆ E. Thus N-δ-β cl (g^{-1}(E)) ⊆ g^{-1}(E),butg^{-1} (E) ⊆N-δ-β cl(g^{-1}(E)). Hence N-δ-βcl(g^{-1}(E)) = g^{-1}(E). Therefore, g^{-1}(E) is N-δ-βclosed in U for every N-closed set E in V. That is g is N-δ-βcontinuous.

(i) ⇒ (iv)

Let g be a N-δ-βcontinuous and let E ⊆V , then ncl$_\delta$(E) is N-closed in V and hence g^{-1}(ncl$_\delta$(E)) is N-δ-β closed in U . Therefore, N-δ-βcl[g^{-1}(ncl$_\delta$(E))] = g^{-1}(ncl$_\delta$(E)). Since E ⊆ N-δ-βcl(E), g^{-1}(E) ⊆ g^{-1}(ncl$_\delta$(E)). Then N-δ-βcl(g^{-1}(E)) ⊆N-δ-βcl(g^{-1}(ncl$_\delta$(E))) = g^{-1}(ncl$_\delta$(E)). Thus N-δ-βcl(g^{-1}(E)) ⊆ g^{-1}(ncl$_\delta$(E)).

(iv) ⇒ (i)

Let N-δ-βcl(g^{-1} (E)) ⊆ g^{-1} (ncl$_\delta$(E)) for every subset E of V. If E be N-closed in V , then ncl$_\delta$(E) = E. By assumption, N-δ-βcl(g^{-1}(E)) ⊆ g^{-1}(ncl$_\delta$(E)) = g^{-1}(E). But g^{-1}(E) ⊆N-δ-βcl (g^{-1}(E)). Therefore, N-δ-βcl(g^{-1}(E)) = g^{-1}(E). That is, g^{-1}(E) is N-δ-β closed in U for every N-closed set E in V. Therefore g is N-δ-βcontinuous.

(i) ⇒ (v)

Let g be a N-δ-βcontinuous and let E ⊆ V, then nint$_\delta$(E) is N-open in V and hence g^{-1} (nint$_\delta$(E)) is N-δ-βopen in U. Therefore N-δ-βint[g^{-1}(nint$_\delta$(E))] = g^{-1}(nint$_\delta$(E)). Also, nint$_\delta$(E) ⊆ E implies that g^{-1}(nint$_\delta$(E)) ⊆ g^{-1}(E). Therefore, N-δ-βint (g^{-1}(nint$_\delta$(E))) ⊆N-δ-βint(g^{-1}(E)). That is, g^{-1}(nint$_\delta$(E)) ⊆N-δ-βint(g^{-1}(E)).

(v)⇒ (i):

Let g^{-1}(nint$_\delta$(E)) ⊆N-δ-βint(g^{-1}(E)) for every E ⊆ V. If E is N-open in V, then nint$_\delta$ (E) = E. By assumption, g^{-1}(nint$_\delta$(E)) ⊆N-δ-βint(g^{-1}(E)). Thus g^{-1}(E) ⊆N-δ-βint(g^{-1}(E)). But N-δ-βint(g^{-1}(E)) ⊆ g^{-1}(E). Therefore, g^{-1}(E) = N-δ-βint(g^{-1}(E)). That is, g^{-1}(E) is N-δ-βopen in U for every N-open set E in V. Therefore, g is N-δ-β continuous.

V. APPLICATION OF NANO TOPOLOGY

In this chapter an example is given for applying nano topology in real life situation.

Example 5.1:

Dengue fever is caused four different viruses and people get the dengue virus from Aedes mosquito. Mosquitoes that spread dengue fever breed in stagnant water in and around our house, generally in confined places like pots, tarpaulins and buckets. There are three types of Dengue fever which are called A, B, C. Type A and B does not make series problems. Its symptoms are fever and drop of platelet counts to very low level. Normal level of platelet counts is 1,50,000 to 4,50,000. If affected people take proper rest and medicines they come out the recover within a week. Papaya leaf and Nilavembu extract are used to treat this types. Most of the people affected in this type recover within 10 days. But type C cause severe problems to the affected persons. Its Symptoms include severe joint and muscle pain, pain behind the eyes, headache, fever, red spots, vomiting, fatigue and temperature. In some cases, symptoms worsen and can become life threatening. Blood vessels often become damaged and leaky. The number of colt-forming cells(platelets) in your bloodstream drops. This can cause a severe form of dengue fever called dengue hemorrhagic fever.

The following table gives data collected from 8 patients affected by Dengue fever:

Symptoms / patients	Red spots	Body pain	Pain behind the eyes	Fatigue	Headache	Vomiting	Temperature	Dengue Fever
V_1	1	0	1	0	0	0	High	1
V_2	1	1	1	0	1	0	Normal	0
V_3	1	0	1	0	0	0	High	0
V_4	1	1	1	0	1	0	Very High	1
V_5	1	1	1	0	1	0	Very High	1
V_6	1	1	1	1	1	1	High	1
V_7	0	1	1	1	1	1	High	0
V_8	1	0	0	0	0	0	High	0

Table 1: Gives Data Collected From 8 Patients Affected By Dengue Fever

In the above table rows represent objects (patients affected by Dengue fever of type C) and columns represent attributes (symptoms of Dengue fever).

In the above table "1" represent "YES" and "0" represent "NO".

Let U = $\{V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8\}$.

Case I:

Let X $=\{V_1, V_4, V_5, V_6\}$ be the set of patients having Dengue fever. Let R be the equivalence relation on U with respect to the set of all the condition attributes.

The set of equivalence Cases corresponding to R is given by $U/I(R)= \{\{V_2\},\{V_1,V_3\}, \{V_4,V_5\},\{V_6\},\{V_7\},\{V_8\}\}$, so the nano topology on U with respect to X is given by τ_R(X)={U,∅,{V_4,V_5, V_6}, {V_1, V_3, V_4,V_5, V_6}{V_1,V_3}. If the attribute "Red Spots" is discarded U/I(R−(R)) = {{V_2},{V_1,V_3}, {V_4,V_5},{V_6,V_7},{V_8}}. Hence $\tau_{R-(R)}$(X) = {U,{V_4,V_5},{V_1, V_3, V_4, V_5, V_6, V_7}, { V_1,V_3, V_6,V_7}, ∅}≠ τ_R(X).

If the attribute "Body Pain" is discarded, then U/I(R−(B)) = {{V_2},{V_1,V_3}, {V_4,V_5}, {V_6},{V_7}, {V_8}} = U/I(R) and hence $\tau_{R-(B)}$(X) = τ_R(X). If the attribute "Pain behind of the eyes" is discarded U/(R−(E)) = {{V_2},{V_4,V_5}, {V_1,V_3, V_8},{V_6},{V_7}}. Hence $\tau_{R-(E)}$(X) = {U,{V_4,V_5,V_6}, {V_1,V_3,V_4, V_5, V_6, V_8}, {V_1,V_3,V_8},∅} ≠ τ_R(X). If the attribute "Fatigue" is discarded U/I(R−(F)) = {{V_2},{V_1,V_3},{V_4,V_5},{V_6},{V_7}, {V_8}}=U/I(R) and hence $\tau_{R-(F)}$(X) = τ_R(X).

If the attribute "Headache" is discarded U/I(R−(H)) = {{V_2},{V_1,V_3}, {V_4,V_5},{V_6},{V_7}, {V_8}} =U/I(R) and hence $\tau_{R-(H)}$(X) = τ_R(X). If the attribute "Vomiting" is discarded U/I(R−(V))={{V_2},{V_1,V_3} {V_4,V_5},{V_6},{V_7}, {V_8}}=U/I(R) and hence $\tau_{R-(V)}$(X) = τ_R(X). If the attribute "Temperature" is discarded U/I(R−(T)) = {{V_2,V_4,V_5}, {{V_1,V_3},{V_6},{V_7},{V_8}}. Therefore $\tau_{R-(T)}$(X) = {U, {V_6}, {V_1,V_2, V_3,V_4, V_5, V_6}, {V_1,V_2, V_3,V_4, V_5}, ∅} ≠ τ_R(X). From Case I, the main Symptoms (R) = {R, E, T}.

Case II

Let X = {V_2,V_3,V_7,V_8} be the set of patients not having Dengue fever. Then U/I(R) = {{V_2},{V_1,V_3}, {V_4,V_5},{V_6},{V_7},{V_8}}, therefore τ_R(X) = {U, {V_2, V_7, V_8},{V_1, V_2, V_3, V_7,V_8}, {V_1, V_3}, ∅}. If the attribute "Red Spots" is discarded, U/I(R−(R)) = {{V_2},{V_4,V_5},{V_1,V_3},{V_6,V_7},{V_8}}, and hence $\tau_{R-(R)}$(X) = {U, {V_2, V_8}, {V_1, V_2, V_3,V_6,V_7, V_8}, { V_1,V_3,V_6, V_7}, ∅} ≠ τ_R(X).

If the attribute "Body pain" is discarded U/I(R−(B)) = {{V_2},{V_1,V_3}, {V_4,V_5},{V_6},{V_7},{V_8}} = U/I(R) and hence $\tau_{R-(B)}$(X) = τ_R(X). If the attribute "Pain behind of the eyes" is discarded U/(R−(E)) = {{V_2},{V_4,V_5},{V_1,V_3,V_8},{V_6},{V_7}}. Hence $\tau_{R-(E)}$(X) = {U,{V_2,V_7},{V_1, V_2,V_3, V_7, V_8}, {V_1,V_3, V_8}, ∅} ≠ τ_R(X). If the attribute "Fatigue" is discarded U/I(R−(F)) = {{V_2},{V_1,V_3}, {V_4,V_5},{V_6},{V_7},{V_8}}, =U/I(R) and hence $\tau_{R-(F)}$(X) = τ_R(X).

If the attribute "Headache" is discarded U/I(R−(H)) = {{V_2},{V_1,V_3}, {V_4,V_5},{V_6},{V_7},{V_8}} =U/I(R) and hence $\tau_{R-(H)}$(X) = τ_R(X). If the attributes "Vomiting" is discarded U/I(R−(V)) = {{V_2},{V_1,V_3}, {V_4,V_5},{V_6},{V_7},{V_8}} =U/I(R) and hence $\tau_{R-(V)}$(X) = τ_R(X). If the attribute "Temperature" is discarded U/I(R−(T)) = {{V_2,V_4,V_5},{{V_1,V_3},{V_6},{V_7},{V_8}}. Therefore $\tau_{R-(T)}$(X) = {U, {V_7,V_8}, {V_1,V_2, V_3,V_4, V_5, V_7,V_8}, {V_1,V_2, V_3,V_5, V_5}, ∅} ≠ τ_R(X). From Case II the main Symptoms (R) = {R, E, T}.

VI. OBSERVATION

The investigation from above two cases are "Red spots", "Pain behind of the Eyes" and "Temperature" are main symptoms of Dengue fever.

VII. CONCLUSION

δ-open sets in δ-nano topological space were introduced and relation between them were studied. δ-nano continuity was introduced and several properties of these types of near nano continuous were discussed. An application for nano topology in real life situation also discussed.

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