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Four Node Tandem Communication Network Model with Feedback Having Homogeneous Poisson Arrivals and Dynamic Bandwidth Allocation

B. Kalaiselvi[1] ,V. Thangarathinam[2]
[1]Associate Professor in Mathematics, Sri G.V.G Visalakshi College for Women, Udumalpet.
[2]Research Scholar in Mathematics, Sri G.V.G Visalakshi College for Women, Udumalpet.

ABSTRACT

Tandem queues are widely used in mathematical modelling of tandem processes describing the operation of Manufacturing systems, supply chains, Computer and telecommunication networks. In many of the communication systems the arrivals are time dependent and can be characterized by a non homogenous Poisson process. In this paper we developed and analyzed the four node tandem communication network model with feedback to each and every node assuming that the arrivals are follows homogeneous Poisson process. The transmission time required by each packet at each node is dependent on the content of the buffer connected to it. The transient behaviour of the network model is analyzed by deriving system performance like exploitation, throughput, mean number of packets, mean delay. The sensitive analysis of the reveals that homogeneous Poisson arrivals improve the quality of service.

Keywords:-Tandem Network, Feedback, Homogeneous Poisson Process, Performance Measures, Sensitive Analysis.

1. INTRODUCTION

Tandem queuing systems are the simple queuing networks consisting of finite number of nodes in series. Such systems can be used for modelling real-life networks having a linear topology as well as for validation of general decomposition algorithms in network. In general, in communication networks the models are analyzed under steady state behaviour, due to its simplicity. But in many communication networks, the steady state measures of system performance simply make sense when the particular needs to know how the system up to some specified time[1]. The behaviour of the system could be understood more effectively with the help of time dependent analysis. The laboratory experimentation is time consuming and expensive, hence it is desirable to develop communication network models and their analysis under transient conditions.

In communication networks the exploitation of the resources is one of the major considerations. In designing the communication networks two aspects are to be considered. They are congestion and packet scheduling. These two aspects are needed in order to utilize resources more effectively and efficiently. Due to the unpredicted nature of the transmission lines, congestion occurs in communication systems. In order to analyze the communication network efficiently, one has to consider the analogy between communication networks and waiting line models. Generally the analysis in a communication system is mainly concerned with the problem of allocation and distribution of data or voice packetization, statistical multiplexing, bit dropping, flow control, link assignment delay and routing etc. For efficient exploitation of the resource, mathematical modelling provides the basic frame work in communication networks.

The communication networks are modelled as interconnected queues by viewing the message as the customer, communication buffer as waiting line and all activities necessary for transmission of the message as service. This representation is the most natural with respect to the actual operation of such systems. This leads a communication network to view as a tandem or serial queuing network.

Some algorithms have been developed with different protocols and allocation strategies for optimal exploitation of bandwidth [2][3][4]. These strategies are developed based on arrival process of the packets through bit dropping and flow control techniques. It is needed to utilize the bandwidth maximum possible by developing strategies of transmission control based on buffer size. One such strategy is adjusted instantaneously depending upon the control of the buffer. P. Suresh Varma et al have developed some communication network models using dynamic bandwidth allocation. They considered that the arrivals of packets to the buffer are single [5]. But, in store-and-forward communication the messages are packetized and transmitted. When a message is packetized, the number of packets of that message is random having bulk in size. Hence, considering single packet arrival to the initial node may not accurately evaluate the performance of the communication network. M.V. Rama Sundari et al have developed three node communication network model with feedback for first two nodes [6]. In this paper we developed and analyzed a four node feedback tandem queue model assuming that arrivals of packets are homogeneous Poisson and by using difference- differential equations the performance is analyzed by deriving the joint probability function of the number of packets in each buffer. The performance measures like average number of packets in the buffer and in the network, the average waiting time of packets in the buffer and in the network etc., of the developed network model are derived explicitly. Sensitivity analysis of this model is carried with respect to other parameters. This model is useful for evaluating communication networks.

2. FOUR NODE TANDEM COMMUNICATION NEWTWORK MODEL WITH DBA AND HP ARRIVALS WITH FEEDBACK

An open queuing model of tandem communication network having four nodes is taken. Each node has a buffer and a transmitter. In tandem A_1, A_2, A_3, A_4 are four buffers andB_1, B_2, B_3, B_4 are transmitters linked in it. At the first nodethe arrival of packets follows homogeneous Poisson processes together with a mean arrival rate as a function of t and is in the form of $\omega(t) = \omega$. Then over the transmitters that the packets are transmitters and in the transmitter the mean service rate is linearly dependent on the content of the buffer linked to it. Suppose that the packet following reaching transmitted over first transmitter may connect the second buffer which is in sequence linked to B_2 otherwise may be returned back buffer linked to B_1 and the packets reaching transmitted may connect the third buffer which is in sequencelinked to B_3otherwise may be returned back buffer linked to B_2 and the packets reaching transmitted over third transmitter may connect the fourth buffer which is in sequencelinked to B_4 otherwise may be returned back to B_3 for retransmission together with certain probabilities . The packets delivered from third node and arrived at the fourth node may be transmitted out of the network or returned back toA_4 for retransmission.

In this system First In First Out (FIFO) procedure is follows by the buffers of the nodes for transmitting the packets over transmitters. Afterwards reaching transmitted from the first transmitter the packets are forwarded to A_2 for forward transmission having probability $(1-\varepsilon)$ Or returned back to the A_1having probabilityε.The packets are forwarded to A_3 from the second transmitters having probability $(1-\tau)$ otherwise returned back to the A_2 having probability τ.The packets are forwarded to A_4having probability $(1-\varphi)$ otherwise returned back to the A_3 having probabilityφ. The packets arrived at from the fourth transmitter are forwarded in the network with the probability $(1-\vartheta)$ or returned back with ϑ. The completion of service in all the transmitters follows Poisson process having the parameters $\alpha_1, \beta_2, \gamma_3$ and δ_4 for the first, second, third and fourth transmitters. Before transmission, the transmission rate of each packet is adjusted depending on the content of the buffer linked to the transmitter. At time t letn_1, n_2, n_3 and n_4 be the number of packets in $A_1, A_2, A_3$and A_4 and $P_{n_1}(t)$ be the probability that there are n_1 packets inA_1, $P_{n_2}(t)$ be the probability that there are n_2 packets in A_2 ,$P_{n_3}(t)$be the probability that there are n_3 packets inA_3, and $P_{n_4}(t)$ be the probability that there are n_4 packets in A_4.For this model, the difference differential equations are given below:

$$\frac{\partial P_{n_1 n_2 n_3 n_4}(t)}{\partial t} = -\Big(\omega(t)+n_1\alpha_1(1-\varepsilon)+n_2\beta_2(1-\tau)+n_3\gamma_3(1-\varphi) \quad +n_4\delta_4(1-\vartheta)\Big) P_{n_1, n_2, n_3 n_4}(t)+\omega(t)P_{n_1-1, n_2, n_3, n_4}(t)\beta_2(1-\tau)$$

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$$P_{n_1,n_{2+1},n_{3-1},n_4}(t)+(n_2+1)+(n_3+1)\gamma_3(1-\varphi)P_{n_1,n_2,n_{3+1},n_{4-1}}(t)+(n_4+1)\delta_4(1-\vartheta)P_{n_1,n_2,n_3,n_{4+1}}(t)$$

$$\frac{\partial P_{n_1,n_2,n_3,0}(t)}{\partial t} = -\left(\omega(t)+n_1\alpha_1(1-\varepsilon)+n_2\beta_2(1-\tau)+n_3\gamma_3(1-\varphi)\right)P_{n_1,n_2,n_3,0}(t) +\omega(t)P_{n_{1-1},n_2,n_3,0}(t)+ (n_1+1)\alpha_1(1-\varepsilon)P_{n_{1+1},n_{2-1},n_3,0}(t)+(n_2+1)\beta_2(1-\tau)P_{n_1,n_{2+1},n_{3-1},0}(t)+\delta_4(1-\vartheta)P_{n_1,n_2,n_3,1}(t)$$

$$\frac{\partial P_{0,n_2,n_3,n_4}(t)}{\partial t}=-\left(\omega(t)+n_2\beta_2(1-\tau)+n_3\gamma_3(1-\varphi)+n_4\delta_4\right)P_{0,n_2,n_{,3},n_4}(t) +\omega(t)P_{n_{1-1},n_2,n_3,n_4}(t)+\alpha_1(1-\varepsilon)P_{1,n_{2-1},n_3,n_4}(t) +(n_2+1)\beta_2(1-\tau)P_{0,n_{2+1},n_{3-1},n_4}(t)+(n_3+1)\gamma_3(1-\varphi)P_{0,n_2,n_{3+1},n_{4-1}}(t)+(n_4+1)\delta_4P_{0,n_2,n_3,n_{4+1}}(t)$$

$$\frac{\partial P_{n_1,n_2,n_3,0}(t)}{\partial t} = -\left(\omega(t)+n_1\alpha_1(1-\varepsilon)+n_2\beta_2(1-\tau)+n_3\gamma_3(1-\varphi)\right)P_{n_1,n_2,n_3,0}(t)+\omega(t)P_{n_{1-1},n_2,n_3,0}(t)+(n_1+1) \alpha_1(1-\varepsilon)P_{n_{1+1},n_{2-1},n_3,0}(t)+(n_2+1)\beta_2(1-\tau)P_{n_1,n_{2+1},n_{3-1},0}(t)+\delta_4(1-\vartheta)P_{n_1,n_2,n_3,1}(t)$$

$$\frac{\partial P_{0,0,n_3,n_4}(t)}{\partial t} = -\left(\omega(t)+n_3\gamma_3(1-\varphi)+n_4\delta_4(1-\vartheta)\right)P_{0,0,n_3,n_4}(t)+\beta_2(1-\tau)P_{0,1,n_{3-1},n_4}(t)+(n_3+1)\gamma_3(1-\varphi)P_{0,0,n_{3+1},n_{4-1}}(t) +(n_4+1)\delta_4(1-\vartheta)P_{0,0,n_3,n_{4+1}}(t)$$

$$\frac{\partial P_{n_1,n_2,0,0}(t)}{\partial t}=-\left(\omega(t)+n_1\alpha_1(1-\varepsilon)+n_2\beta_2(1-\tau)\right)P_{n_1,n_2,0,0}(t)+\omega(t)P_{n_{1-1},n_2,0,0}(t)+(n_1+1)\alpha_1(1-\varepsilon)P_{n_{1+1},,n_{2-1},0,0}(t) +\delta_4(1-\vartheta)P_{n_1,n_2,0,1}(t)$$

$$\frac{\partial P_{n_1 0,n_3,0}(t)}{\partial t} =-\left(\omega(t)+n_1\alpha_1(1-\varepsilon)+n_3\gamma_3(1-\varphi)\right)P_{n_1,0,n,_3 0}(t)+\omega(t)P_{n_{1-1},0,n_3,0}(t)+\beta_2(1-\tau)P_{n_1,,1,n_{3-1},0}(t) +\delta_4(1-\vartheta)P_{n_1,0,n_3,1}(t)$$

$$\frac{\partial P_{0,n_2,0,n_4}(t)}{\partial t} = -\left(\omega(t)+n_2\beta_2(1-\tau)+n_4\delta_4(1-\vartheta)\right)P_{0,n_2,0,n_4}(t)+\alpha_1(1-\varepsilon)P_{1,n_{2-1},0,n_4}(t)+\gamma_3(1-\varphi)P_{0,n_2,1,n_{4-1}}(t) +(n_4+1)\delta_4(1-\vartheta)P_{0,n_2,0,n_{4+1}}(t)$$

$$\frac{\partial P_{0,0,0,n_4}(t)}{\partial t} =-\left(\omega(t)+n_4\delta_4(1-\vartheta)\right)P_{0,0,0,n_4}(t)+\gamma_3(1-\varphi)P_{0,0,1,n_{4-1}}(t)+(n_4+1)\delta_4(1-\vartheta)P_{0,0,0,n_{4+1}}(t)$$

$$\frac{\partial P_{n_1,0,0,0}(t)}{\partial t} = -\left(\omega(t)+n_1\alpha_1(1-\varepsilon)\right)P_{n_1,0,0,0}(t)+\omega(t)P_{n_{1-1},n_2,n_3,n_4}(t)+\delta_4(1-\vartheta)P_{n_1,0,0,1}(t)$$

$$\frac{\partial P_{0,n_2,0,0}(t)}{\partial t} = -\left(\omega(t)+n_2\beta_2(1-\tau)\right)P_{0,n_2,0,0}(t)+\alpha_1(1-\varepsilon)P_{1,n_{2-1},0,0}(t)\delta_4(1-\vartheta)P_{0,n_2,0,1}(t)$$

$$\frac{\partial P_{0,0,n_3,0}(t)}{\partial t} = -\left(\omega(t)+n_3\gamma_3(1-\varphi)\right)P_{0,0,n_3,0}(t)+\beta_2(1-\tau)P_{0,1,n_{3-1},0}(t)\delta_4(1-\vartheta)P_{0,0,n_3,1}(t)$$

$$\frac{\partial P_{0,0,0,0}(t)}{\partial t} = -\left(\omega(t)\right)P_{0,0,0,,0}(t)+\delta_4\left(1-\vartheta\right)P_{0,0,0,1}(t) \qquad (2.1)$$

Consider the joint probability generating function $P(B_1, B_2, B_3, B_4 ; t)$of $P_{n_1n_2n_3n_4}(t)$. Multiply the above equations with $B_1^{n_1}B_2^{n_2}B_3^{n_3}B_4^{n_4}$ and summing over all n_1, n_2, n_3, n_4, it becomes

$$\frac{\partial P(B_1, B_2, B_3, B_4 ;t)}{\partial t} = \sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty}\sum_{n_3=1}^{\infty}\sum_{n_4=1}^{\infty} \begin{bmatrix} -\left(\omega(t)+n_1\alpha_1(1-\varepsilon)+n_2\beta_2(1-\tau)+n_3\gamma_3(1-\varphi)+\delta_4(1-\vartheta)\right)P_{n_1,n_2,n_3n_4}(t) \\ +\omega(t)P_{n_1-1,n_2,n_3,n_4}(t)+(n_1+1)\alpha_1(1-\varepsilon)P_{n_1+1,n_2-1,n_3,n_4}(t)+(n_2+1)\beta_2(1-\tau) \\ P_{n_1,n_2+1,n_3-1,n_4}(t)+(n_3+1)\gamma_3(1-\varphi)P_{n_1,n_2,n_3+1,n_4-1}(t)+(n_4+1)\delta_4 \\ (1-\vartheta)P_{n_1,n_2,n_3,n_4+1}(t) \end{bmatrix}$$

$$B_1^{n_1}B_2^{n_2}B_3^{n_3}B_4^{n_4}+\ldots+-(\omega)P_{0,0,0,0}(t)+\delta_4(1-\vartheta)P_{0,0,0,1}(t) \quad (2.2)$$

After simplifying (2.2), it gives

$$\frac{\partial(B_1,B_2,B_3,B_4 :t)}{\partial t} = \omega(t)P(B_1-1)+\alpha_1(1-\varepsilon)\frac{\partial P}{\partial B_1}(B_2-B_1)+\beta_2(1-\tau)\frac{\partial P}{\partial B_2}(B_3-B_2)$$

$$+\gamma_3(1-\varphi)\frac{\partial P}{\partial B_2}(B_4-B_3)+\delta_4(1-\vartheta)\frac{\partial P}{\partial B_4}(1-B_4) \quad (2.3)$$

Using Lagrangian's method for solving (2.3), the auxiliary equation is

$$\frac{dt}{1} = \frac{dB_1}{\alpha_1(1-\varepsilon)(B_1-B_2)} = \frac{dB_2}{\beta_2(1-\tau)(B_2-B_3)} = \frac{dB_3}{\gamma_3(1-\varphi)(B_3-B_4)} = \frac{dB_4}{\delta_4(1-\vartheta)(B_4-1)} = \frac{dP}{\omega(t)P(B_1-1)} \quad (2.4)$$

$\omega(t)$is given and the mean arrival rate $\omega(t) = \omega$, where $\omega > 0$.To solve the equations in (2.4) the functional form of ω is required.

Solving first and fifth terms in equation (2.4),

$$u = (B_4 - 1)e^{-\delta_4(1-\vartheta)t}$$

(2.5)

Solving first and fourth terms in equation (2.4),

$$v=(B_3-1)e^{-\gamma_3(1-\varphi)t}+\frac{(B_4-1)\gamma_3(1-\varphi)e^{-\gamma_3(1-\varphi)t}}{\left(\delta_4(1-\vartheta)-\gamma_3(1-\varphi)\right)} \quad (2.6)$$

Solving first and third terms in equation (2.4),

$$w=(B_2-1)e^{-\beta_2(1-\tau)t}+\frac{(B_3-1)\beta_2(1-\tau)e^{-\beta_2(1-\tau)t}}{\left(\gamma_3(1-\varphi)-\beta_2(1-\tau)\right)}+\frac{(B_4-1)\beta_2(1-\tau)\gamma_3(1-\varphi)e^{-\beta_2(1-\tau)t}}{\left(\delta_4(1-\vartheta)-\beta_2(1-\tau)\right)\left(\gamma_3(1-\varphi)-\beta_2(1-\tau)\right)} \quad (2.7)$$

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Solving first and fourth terms in equation (2.4),

$$x = (B_1-1)e^{-\alpha_1(1-\varepsilon)t} + \frac{(B_2-1)\alpha_1(1-\varepsilon)e^{-\alpha_1(1-\varepsilon)t}}{\left(\beta_2(1-\tau)-\alpha_1(1-\varepsilon)\right)} + \frac{(B_3-1)\beta_2(1-\tau)\alpha_1(1-\varepsilon)e^{-\alpha_1(1-\varepsilon)t}}{\left(\gamma_3(1-\varphi)-\alpha_1(1-\varepsilon)\right)\left(\beta_2(1-\tau)-\alpha_1(1-\varepsilon)\right)} + \frac{(B_4-1)\gamma_3(1-\varphi)\beta_2(1-\tau)\alpha_1(1-\varepsilon)e^{-\alpha_1(1-\varepsilon)t}}{\left(\delta_4(1-\vartheta)-\alpha_1(1-\varepsilon)\right)\left(\beta_2(1-\tau)-\alpha_1(1-\varepsilon)\right)} \quad (2.8)$$

Solving first and fourth terms in equation (2.4),

$$y = P\exp\left\{-\left[\frac{B_1-1(\omega)}{\alpha_1(1-\varepsilon)} + \frac{(B_2-1)(\omega)}{\beta_2(1-\tau)} + \frac{(B_3-1)(\omega)}{\gamma_3(1-\varphi)} + \frac{(B_4-1)(\omega)}{\delta_4(1-\vartheta)}\right]\right\} \quad (2.9)$$

Here u, v, w, x and y are arbitrary constants.

At time t, from the general solution of (2.4), the probability generating function of n_1, n_2, n_3 in A_1, A_2, A_3 are found. Let it be $P(B_1, B_2, B_3, B_4; t)$.That is,

$$P(B_1,B_2,B_3,B_4;t) = \exp\left\{\begin{aligned}
&\frac{B_1-1(\omega)}{\alpha_1(1-\varepsilon)} + \frac{B_2-1(\omega)}{\beta_2(1-\tau)} + \frac{B_2-1(\omega)}{\left(\beta_2(1-\tau)-\alpha_1(1-\varepsilon)\right)}\begin{pmatrix} e^{-\beta_2(1-\tau)t} - \\ e^{-\alpha_1(1-\varepsilon)} \end{pmatrix} + \frac{B_3-1(\omega)}{\gamma_3(1-\varphi)}\left(1-e^{-\gamma_3(1-\varphi)t}\right) + \frac{B_3-1(\omega)}{\left(\gamma_3(1-\varphi)-\beta_2(1-\tau)\right)} \\
&\begin{pmatrix} e^{-\gamma_3(1-\varphi)t} - \\ e^{-\beta_2(1-\tau)t} \end{pmatrix}(B_3-1)(\beta_2(1-\tau)\left(\left(\begin{aligned} &\frac{e^{-\alpha_1(1-\varepsilon)t}}{\left(\alpha_1(1-\varepsilon)-\gamma_3(1-\varphi)\right)\left(\beta_2(1-\tau)-\alpha_1(1-\varepsilon)\right)} + \\ &\frac{e^{-\beta_2(1-\tau)t}}{\left(\beta_2(1-\tau)-\alpha_1(1-\varepsilon)\right)\left(\gamma_3(1-\varphi)-\beta_2(1-\tau)\right)} + \\ &\frac{e^{-\gamma_3(1-\varphi)t}}{\left(\gamma_3(1-\varphi)-\left(\beta_2(1-\tau)\right)\right)\left(\gamma_3(1-\varphi)\alpha_1(1-\varepsilon)\right)} \end{aligned}\right)(\omega)\right) \\
&+ \frac{B_4-1(\omega)}{\delta_4(1-\vartheta)}\left(1-e^{-\delta_4(1-\vartheta)t}\right) + \frac{B_4-1(\omega)}{\left(\delta_4(1-\vartheta)-\gamma_3(1-\varphi)\right)}\begin{pmatrix} e^{-\delta_4(1-\vartheta)t} - \\ e^{-\gamma_3(1-\varphi)t} \end{pmatrix} + (B_4-1)\gamma_3(1-\varphi) \\
&\left(\begin{aligned} &\frac{e^{-\alpha_1(1-\varepsilon)t}}{\left(\alpha_1(1-\varepsilon)-\delta_4(1-\vartheta)\right)\left(\beta_2(1-\tau)-\alpha_1(1-\varepsilon)\right)} + \frac{e^{-\beta_2(1-\tau)t}}{\left(\beta_2(1-\tau)-\alpha_1(1-\varepsilon)\right)\left(\gamma_3(1-\varphi)-\beta_2(1-\tau)\right)} + \\ &\frac{e^{-\gamma_3(1-\varphi)t}}{\left(\gamma_3(1-\varphi)-\beta_2(1-\tau)\right)\left(\gamma_3(1-\varphi)-\delta_4(1-\vartheta)\right)} + \frac{e^{-\delta_4(1-\vartheta)t}}{\left(\gamma_3(1-\varphi)-\delta_4(1-\vartheta)\right)\left(\delta_4(1-\vartheta)-\alpha_1(1-\varepsilon)\right)} \end{aligned}\right)(\omega)
\end{aligned}\right\}$$

(2.10)

3. PERFORMANCE MEASURES OF THE NETWORK MODEL

The performance measures of the network following the given transient conditions are discussed in the following section. The probability $P_{0000}(t)$ that the network is empty is found as follows:

$$P_{0000}(t) = \exp\left\{\begin{array}{l} \frac{-1(\omega)}{\alpha_1(1-\varepsilon)}\left(1-e^{-\alpha_1(1-\varepsilon)t}\right) - \frac{1(\omega)}{\beta_2(1-\tau)} - \frac{1(\omega)}{\left(\beta_2(1-\tau)-\alpha_1(1-\varepsilon)\right)}\left(e^{-\beta_2(1-\tau)t} - e^{-\alpha_1(1-\varepsilon)}\right) - \frac{1(\omega)}{\gamma_3(1-\varphi)}\left(1-e^{-\gamma_3(1-\varphi)t}\right) - \frac{1(\omega)}{\left(\gamma_3(1-\varphi)-\beta_2(1-\tau)\right)} \\ \left(e^{-\gamma_3(1-\varphi)t} - e^{-\beta_2(1-\tau)t}\right) + (-1)(\beta_2(1-\tau)\left(\left(\begin{array}{c} \frac{e^{-\alpha_1(1-\varepsilon)t}}{\left(\alpha_1(1-\varepsilon)-\gamma_3(1-\varphi)\right)\left(\beta_2(1-\tau)-\alpha_1(1-\varepsilon)\right)} + \\ \frac{e^{-\beta_2(1-\tau)t}}{\left(\beta_2(1-\tau)-\alpha_1(1-\varepsilon)\right)\left(\gamma_3(1-\varphi)-\beta_2(1-\tau)\right)} + \\ \frac{e^{-\gamma_3(1-\varphi)t}}{\left(\gamma_3(1-\varphi)-\left(\beta_2(1-\tau)\right)\right)\left(\gamma_3(1-\varphi)\alpha_1(1-\varepsilon)\right)} \end{array}\right)(\omega)\right) - \frac{1(\omega)}{\delta_4(1-\vartheta)}\left(1-e^{-\delta_4(1-\vartheta)t}\right) \\ - \frac{1(\omega)}{\left(\delta_4(1-\vartheta)-\gamma_3(1-\varphi)\right)}\left(e^{-\delta_4(1-\vartheta)t} - e^{-\gamma_3(1-\varphi)t}\right) + (-1)\gamma_3(1-\varphi)\left(\begin{array}{c} \frac{e^{-\alpha_1(1-\varepsilon)t}}{\left(\alpha_1(1-\varepsilon)-\delta_4(1-\vartheta)\right)\left(\beta_2(1-\tau)-\alpha_1(1-\varepsilon)\right)} + \\ \frac{e^{-\beta_2(1-\tau)t}}{\left(\beta_2(1-\tau)-\alpha_1(1-\varepsilon)\right)\left(\gamma_3(1-\varphi)-\beta_2(1-\tau)\right)} + \\ \frac{e^{-\gamma_3(1-\varphi)t}}{\left(\gamma_3(1-\varphi)-\beta_2(1-\tau)\right)\left(\gamma_3(1-\varphi)-\delta_4(1-\vartheta)\right)} + \\ \frac{e^{-\delta_4(1-\vartheta)t}}{\left(\gamma_3(1-\varphi)-\delta_4(1-\vartheta)\right)\left(\delta_4(1-\vartheta)-\alpha_1(1-\varepsilon)\right)} \end{array}\right)(\omega) \end{array}\right\} \tag{3.1}$$

Put $B_2, B_3, B_4 = 1$ in (3.1) in the first buffer ,the probability generating function of the number of packets in B_1 is

$$P(B_1;t) = \exp\left\{\frac{(B_1-1)(\omega)}{\alpha_1(1-\varepsilon)}\left(1-e^{-\alpha_1(1-\varepsilon)t}\right)\right\} \tag{3.2}$$

Since $B_1 = 0$, equation (3.2) becomes,

$$P_0(t) = \exp\left\{\frac{-1(\omega)}{\alpha_1(1-\varepsilon)}\left(1-e^{-\alpha_1(1-\varepsilon)t}\right)\right\} \tag{3.3}$$

Put $B_1, B_3, B_4 = 1$ in (3.1) in the second the probability generating function of the number of packets in B_2 is

$$P(B_2;t) = \exp\left\{\frac{B_2-1(\omega)}{\beta_2(1-\tau)}\left(1-e^{-\beta_2(1-\tau)t}\right) + \frac{B_2-1(\omega)}{\left(\beta_2(1-\tau)-\alpha_1(1-\varepsilon)\right)}\left(e^{-\beta_2(1-\tau)t}-e^{-\alpha_1(1-\varepsilon)t}\right)\right\} \tag{3.4}$$

Since $B_2 = 0$, equation (3.4) becomes

$$P_0(t) = \exp\left\{\frac{-1(\omega)}{\beta_2(1-\tau)}\left(1-e^{\beta_2(1-\tau)}\right)-\frac{1(\omega)}{\left(\beta_2(1-\tau)-\alpha_1(1-\varepsilon)\right)}\left(e^{-\beta_2(1-\tau)t}-e^{-\alpha_1(1-\varepsilon)t}\right)\right\} \quad (3.5)$$

Put $B_1, B_2, B_4 = 1$ in (3.1) in the third buffer, the probability generating function of the number of packets in B_3 is

$$P(B_3;t) = \exp\left\{\begin{array}{c}\frac{B_3-1(\omega)}{\gamma_3(1-\varphi)}\left(1-e^{-\gamma_3(1-\varphi)t}\right)++\frac{B_3-1(\omega)}{\gamma_3(1-\varphi)-\beta_2(1-\tau)}\left(e^{-\gamma_3(1-\varphi)t}-e^{-\beta_2(1-\tau)t}\right)+(B_3-1)\left(\beta_2(1-\tau)\right)\\ \left(\begin{array}{c}\frac{e^{-\alpha_1(1-\varepsilon)t}}{\left(\alpha_1(1-\varepsilon)-\gamma_3(1-\varphi)\right)\left(\beta_2(1-\tau)-\alpha_1(1-\varepsilon)\right)}+\frac{e^{-\beta_2(1-\tau)t}}{\left(\beta_2(1-\tau)-\alpha_1(1-\varepsilon)\right)\left(\gamma_3(1-\varphi)-\beta_2(1-\tau)\right)}\\ +\frac{e^{-\gamma_3(1-\varphi)t}}{\left(\beta_2(1-\tau)-\gamma_3(1-\varphi)\right)\left(\gamma_3(1-\varphi)-\alpha_1(1-\varepsilon)\right)}\end{array}\right)(\omega)\end{array}\right\} \quad (3.6)$$

Since $B_3 = 0$, equation (3.6) becomes

$$P_0(t) = \exp\left\{\begin{array}{c}\frac{-1(\omega)}{\gamma_3(1-\varphi)}\left(1-e^{-\gamma_3(1-\varphi)t}\right)-\frac{1(\omega)}{\gamma_3(1-\varphi)-\beta_2(1-\tau)}\left(e^{-\gamma_3(1-\varphi)t}-e^{-\beta_2(1-\tau)t}\right)-1\left(\beta_2(1-\tau)\right)\\ \left(\begin{array}{c}\frac{e^{-\alpha_1(1-\varepsilon)t}}{\left(\alpha_1(1-\varepsilon)-\gamma_3(1-\varphi)\right)\left(\beta_2(1-\tau)-\alpha_1(1-\varepsilon)\right)}+\frac{e^{-\beta_2(1-\tau)t}}{\left(\beta_2(1-\tau)-\alpha_1(1-\varepsilon)\right)\left(\gamma_3(1-\varphi)-\beta_2(1-\tau)\right)}\\ +\frac{e^{-\gamma_3(1-\varphi)t}}{\left(\beta_2(1-\tau)-\gamma_3(1-\varphi)\right)\left(\gamma_3(1-\varphi)-\alpha_1(1-\varepsilon)\right)}\end{array}\right)(\omega)\end{array}\right\} \quad (3.7)$$

Put $B_1, B_2, B_3 = 1$ in (3.1) in the fourth buffer, the probability generating function of the number of packets in B_4 is

$$P(B_4;t) = \exp\left\{\begin{array}{c}\frac{B_4-1(\omega)}{\delta_4(1-\vartheta)}\left(1-e^{-\delta_4(1-\vartheta)t}\right)+\frac{B_4-1(\omega)}{\left(\delta_4(1-\vartheta)-\gamma_3(1-\varphi)\right)}\left(\begin{array}{c}e^{-\delta_4(1-\vartheta)t}\\ -e^{-\gamma_3(1-\varphi)t}\end{array}\right)+(B_4-1)\gamma_3(1-\varphi)\\ \left(\begin{array}{c}\frac{e^{-\alpha_1(1-\varepsilon)t}}{\left(\alpha_1(1-\varepsilon)-\delta_4(1-\vartheta)\right)\left(\beta_2(1-\tau)-\alpha_1(1-\varepsilon)\right)}+\frac{e^{-\beta_2(1-\tau)t}}{\left(\beta_2(1-\tau)-\alpha_1(1-\varepsilon)\right)\left(\gamma_3(1-\varphi)-\beta_2(1-\tau)\right)}+\\ \frac{e^{-\gamma_3(1-\varphi)t}}{\left(\beta_2(1-\tau)-\gamma_3(1-\varphi)\right)\left(\gamma_3(1-\varphi)-\delta_4(1-\vartheta)\right)}+\frac{e^{-\delta_4(1-\vartheta)t}}{\left(\gamma_3(1-\varphi)-\delta_4(1-\vartheta)\right)\left(\delta_4(1-\vartheta)-\alpha_1(1-\varepsilon)\right)}\end{array}\right)(\omega)\end{array}\right\} \quad (3.8)$$

Since $B_4 = 0$, equation (3.8) becomes

$$P_0(t) = \exp\left\{\begin{matrix} \frac{-1(\omega)}{\delta_4(1-\vartheta)}\left(1\text{-}e^{-\delta_4(1-\vartheta)t}\right) - \frac{1(\omega)}{\left(\delta_4(1-\vartheta)\text{-}\gamma_3(1\text{-}\varphi)\right)}\begin{pmatrix} e^{-\delta_4(1-\vartheta)t} \\ -e^{-\gamma_3(1\text{-}\varphi)t}\end{pmatrix}(-1)\gamma_3(1\text{-}\varphi) \\ \left(\begin{matrix}\frac{e^{-\alpha_1(1\text{-}\varepsilon)t}}{\left(\alpha_1(1\text{-}\varepsilon)\text{-}\delta_4(1-\vartheta)\right)\left(\beta_2(1\text{-}\tau)\text{-}\alpha_1(1\text{-}\varepsilon)\right)}+\frac{e^{-\beta_2(1\text{-}\tau)t}}{\left(\beta_2(1\text{-}\tau)\text{-}\alpha_1(1\text{-}\varepsilon)\right)\left(\gamma_3(1\text{-}\varphi)\text{-}\beta_2(1\text{-}\tau)\right)}+ \\ \frac{e^{-\gamma_3(1\text{-}\varphi)t}}{\left(\beta_2(1\text{-}\tau)\text{-}\gamma_3(1\text{-}\varphi)\right)\left(\gamma_3(1\text{-}\varphi)\text{-}\delta_4(1-\vartheta)\right)}+\frac{e^{-\delta_4(1-\vartheta)t}}{\left(\gamma_3(1\text{-}\varphi)\text{-}\delta_4(1-\vartheta)\right)\left(\delta_4(1-\vartheta)\text{-}\alpha_1(1\text{-}\varepsilon)\right)}\end{matrix}\right)(\omega)\end{matrix}\right\} \quad (3.9)$$

In A_1, the mean number of packets is

$$M_1(t) = \frac{\partial P(B_1;t)}{\partial B_1} = \frac{1(\omega)}{\alpha_1(1\text{-}\varepsilon)}\left(1\text{-}e^{-\alpha_1(1\text{-}\varepsilon)t}\right) \quad (3.10)$$

In B_1, the exploitation is

$$E_1(t) = 1\text{-}P_0(t) = 1\text{-}\exp\left\{\frac{-1(\omega)}{\alpha_1(1\text{-}\varepsilon)}\left(1\text{-}e^{-\alpha_1(1\text{-}\varepsilon)t}\right)\right\} \quad (3.11)$$

In A_1, the variance of the number of packets is

$$V_1(t) = \frac{1(\omega)}{\alpha_1(1\text{-}\varepsilon)}\left(1\text{-}e^{-\alpha_1(1\text{-}\varepsilon)t}\right) \quad (3.12)$$

In B_1, the throughput is

$$Th_1 = \alpha_1\left(1\text{-}P_0(t)\right) = \alpha_1\left(1+\exp\left\{\frac{1(\omega)}{\alpha_1(1\text{-}\varepsilon)}\left(1\text{-}e^{-\alpha_1(1\text{-}\varepsilon)t}\right)\right\}\right) \quad (3.13)$$

In A_1 , average waiting time is

$$W_1(t) = \frac{M_1(t)}{\alpha_1\left(1\text{-}P_0(t)\right)} \quad (3.14)$$

In $A_{2,}$ the mean number of packets is

$$M_2(t) = \frac{\partial P(B_2;t)}{\partial B_2} = \frac{1(\omega)}{\beta_2(1\text{-}\tau)}\left(1\text{-}e^{-\beta_2(1\text{-}\tau)}\right) + \frac{1(\omega)}{\left(\beta_2(1\text{-}\tau)\text{-}\alpha_1(1\text{-}\varepsilon)\right)}\left(e^{-\beta_2(1\text{-}\tau)t}\text{-}e^{-\alpha_1(1\text{-}\varepsilon)t}\right) \quad (3.15)$$

In B_2, the exploitation is

$$E_2(t) = 1\text{-}P_0(t) = 1\text{-}\exp\left\{\frac{-1(\omega)}{\beta_2(1\text{-}\tau)}\left(1\text{-}e^{-\beta_2(1\text{-}\tau)}\right) - \frac{1(\omega)}{\left(\beta_2(1\text{-}\tau)\text{-}\alpha_1(1\text{-}\varepsilon)\right)}\begin{pmatrix} e^{-\beta_2(1\text{-}\tau)t} \\ -e^{-\alpha_1(1\text{-}\varepsilon)t}\end{pmatrix}\right\} \quad (3.16)$$

In A_2, variance of the number of packets is

$$V_2(t) = \left\{\frac{1(\omega)}{\beta_2(1\text{-}\tau)}\left(1\text{-}e^{-\beta_2(1\text{-}\tau)}\right) + + \frac{1(\omega)}{\left(\beta_2(1\text{-}\tau)\text{-}\alpha_1(1\text{-}\varepsilon)\right)}\left(e^{-\beta_2(1\text{-}\tau)t}\text{-}e^{-\alpha_1(1\text{-}\varepsilon)t}\right)\right\} \quad (3.17)$$

In B_2, the throughput is

$$Th_2 = \beta_2\left(1\text{-}P_0(t)\right) = \beta_2\left(1\text{- } \exp\left\{\frac{1(\omega)}{\beta_2(1\text{-}\tau)}\left(1\text{-}e^{\text{-}\beta_2(1\text{-}\tau)}\right)+\frac{1(\omega)}{\left(\beta_2(1\text{-}\tau)\text{-}\alpha_1(1\text{-}\varepsilon)\right)}\left(e^{\text{-}\beta_2(1\text{-}\tau)t}\text{-}e^{\text{-}\alpha_1(1\text{-}\varepsilon)t}\right)\right\}\right) \quad (3.18)$$

In A_2 , average waiting time is

$$W_2(t) = \frac{M_2(t)}{\beta_2(1\text{-}P_0)} \quad (3.19)$$

In A_3, the mean number of packets is

$$M_3(t) = \frac{\partial P(B_3;t)}{\partial B_3} = \left\{\begin{matrix}\frac{1(\omega)}{\gamma_3(1\text{-}\varphi)}\left(1\text{-}e^{\text{-}\gamma_3(1\text{-}\varphi)t}\right)+\frac{1(\omega)}{\gamma_3(1\text{-}\varphi)\text{-}\beta_2(1\text{-}\tau)}\begin{pmatrix}e^{\text{-}\gamma_3(1\text{-}\varphi)t}\\ \text{-}e^{\text{-}\beta_2(1\text{-}\tau)t}\end{pmatrix}+\left(\beta_2(1\text{-}\tau)\right)\\ \begin{pmatrix}\frac{e^{-\alpha_1(1\text{-}\varepsilon)t}}{\left(\alpha_1(1\text{-}\varepsilon)\text{-}\gamma_3(1\text{-}\varphi)\right)\left(\beta_2(1\text{-}\tau)\text{-}\alpha_1(1\text{-}\varepsilon)\right)}+\frac{e^{-\beta_2(1\text{-}\tau)t}}{\left(\beta_2(1\text{-}\tau)\text{-}\alpha_1(1\text{-}\varepsilon)\right)\left(\gamma_3(1\text{-}\varphi)\text{-}\beta_2(1\text{-}\tau)\right)}+\\ \frac{e^{\text{-}\gamma_3(1\text{-}\varphi)t}}{\left(\beta_2(1\text{-}\tau)\text{-}\gamma_3(1\text{-}\varphi)\right)\left(\gamma_3(1\text{-}\varphi)\text{-}\alpha_1(1\text{-}\varepsilon)\right)}\end{pmatrix}(\omega)\end{matrix}\right\} \quad (3.20)$$

In $B_{3,}$ the exploitation is

$$E_3(t) = 1\text{- } \exp\left\{\begin{matrix}\frac{\text{-}1(\omega)}{\gamma_3(1\text{-}\varphi)}\left(1\text{-}e^{-\gamma_3(1\text{-}\varphi)t}\right)\text{-}\frac{1(\omega)}{\gamma_3(1\text{-}\varphi)\text{-}\beta_2(1\text{-}\tau)}\begin{pmatrix}e^{\text{-}\gamma_3(1\text{-}\varphi)t}\\ \text{-}e^{-\beta_2(1\text{-}\tau)t}\end{pmatrix}-1\left(\beta_2(1\text{-}\tau)\right)\\ \begin{pmatrix}\frac{e^{-\alpha_1(1\text{-}\varepsilon)t}}{\left(\alpha_1(1\text{-}\varepsilon)\text{-}\gamma_3(1\text{-}\varphi)\right)\left(\beta_2(1\text{-}\tau)\text{-}\alpha_1(1\text{-}\varepsilon)\right)}+\frac{e^{-\beta_2(1\text{-}\tau)t}}{\left(\beta_2(1\text{-}\tau)\text{-}\alpha_1(1\text{-}\varepsilon)\right)\left(\gamma_3(1\text{-}\varphi)\text{-}\beta_2(1\text{-}\tau)\right)}+\\ \frac{e^{\text{-}\gamma_3(1\text{-}\varphi)t}}{\left(\beta_2(1\text{-}\tau)\text{-}\gamma_3(1\text{-}\varphi)\right)\left(\gamma_3(1\text{-}\varphi)\text{-}\alpha_1(1\text{-}\varepsilon)\right)}\end{pmatrix}(\omega)\end{matrix}\right\} \quad (3.21)$$

In A_3, the variance of the number of packets is

$$V_3(t) = \left\{\begin{matrix}\frac{1(\omega)}{\gamma_3(1\text{-}\varphi)}\left(1\text{-}e^{\text{-}\gamma_3(1\text{-}\varphi)t}\right)+\frac{1(\omega)}{\gamma_3(1\text{-}\varphi)\text{-}\beta_2(1\text{-}\tau)}\begin{pmatrix}e^{\text{-}\gamma_3(1\text{-}\varphi)t}\\ \text{-}e^{\text{-}\beta_2(1\text{-}\tau)t}\end{pmatrix}+\left(\beta_2(1\text{-}\tau)\right)\\ \left(\frac{e^{-\alpha_1(1\text{-}\varepsilon)t}}{\left(\alpha_1(1\text{-}\varepsilon)\text{-}\gamma_3(1\text{-}\varphi)\right)\left(\beta_2(1\text{-}\tau)\text{-}\alpha_1(1\text{-}\varepsilon)\right)}+\frac{e^{-\beta_2(1\text{-}\tau)t}}{\left(\beta_2(1\text{-}\tau)\text{-}\alpha_1(1\text{-}\varepsilon)\right)\left(\gamma_3(1\text{-}\varphi)\text{-}\beta_2(1\text{-}\tau)\right)}+\frac{e^{-\gamma_3(1\text{-}\varphi)t}}{\left(\beta_2(1\text{-}\tau)\text{-}\gamma_3(1\text{-}\varphi)\right)\left(\gamma_3(1\text{-}\varphi)\text{-}\alpha_1(1\text{-}\varepsilon)\right)}\right)(\omega)\end{matrix}\right\}$$

(3.22)

In B_3, the throughput is

$$Th_3 = \gamma_3\left(1\text{-}P_0\right) = 1+ \exp\left\{\begin{matrix}\frac{1(\omega)}{\gamma_3(1\text{-}\varphi)}\left(1\text{-}e^{\text{-}\gamma_3(1\text{-}\varphi)t}\right)\text{-}\frac{1(\omega)}{\gamma_3(1\text{-}\varphi)\text{-}\beta_2(1\text{-}\tau)}\left(e^{-\gamma_3(1\text{-}\varphi)t}\text{-}e^{-\beta_2(1\text{-}\tau)t}\right)+\left(\beta_2(1\text{-}\tau)\right)\\ \begin{pmatrix}\frac{e^{\text{-}\alpha_1(1\text{-}\varepsilon)t}}{\left(\alpha_1(1\text{-}\varepsilon)\text{-}\gamma_3(1\text{-}\varphi)\right)\left(\beta_2(1\text{-}\tau)\text{-}\alpha_1(1\text{-}\varepsilon)\right)}+\frac{e^{-\beta_2(1\text{-}\tau)t}}{\left(\beta_2(1\text{-}\tau)\text{-}\alpha_1(1\text{-}\varepsilon)\right)\left(\gamma_3(1\text{-}\varphi)\text{-}\beta_2(1\text{-}\tau)\right)}+\\ \frac{e^{\text{-}\gamma_3(1\text{-}\varphi)t}}{\left(\beta_2(1\text{-}\tau)\text{-}\gamma_3(1\text{-}\varphi)\right)\left(\gamma_3(1\text{-}\varphi)\text{-}\alpha_1(1\text{-}\varepsilon)\right)}\end{pmatrix}(\omega)\end{matrix}\right\} \quad (3.23)$$

In A_3, average waiting time is

$$W_3(t) = \frac{M_3(t)}{\gamma_3(1\text{-}P_0)}$$

In A_4, the mean number of packets is

$$M_4(t) = \frac{\partial P(B_4;t)}{\partial B_4} = \left\{ \begin{array}{c} \frac{1(\omega)}{\delta_4(1-\vartheta)}\left(1\text{-}e^{-\delta_4(1-\vartheta)t}\right) + \frac{1(\omega)}{\left(\delta_4(1-\vartheta)\text{-}\gamma_3(1\text{-}\varphi)\right)}\left(e^{-\delta_4(1-\vartheta)t}\text{-}e^{-\gamma_3(1\text{-}\varphi)t}\right) + \gamma_3(1\text{-}\varphi) \\ \left(\begin{array}{c} \frac{e^{-\alpha_1(1\text{-}\varepsilon)t}}{(\alpha_1(1\text{-}\varepsilon)\text{-}\delta_4)\left(\beta_2(1\text{-}\tau)\text{-}\alpha_1(1\text{-}\varepsilon)\right)} + \frac{e^{-\beta_2(1\text{-}\tau)t}}{\left(\beta_2(1\text{-}\tau)\text{-}\alpha_1(1\text{-}\varepsilon)\right)\left(\gamma_3(1\text{-}\varphi)\text{-}\beta_2(1\text{-}\tau)\right)} + \\ \frac{e^{-\gamma_3(1\text{-}\varphi)t}}{\left(\beta_2(1\text{-}\tau)\text{-}\gamma_3(1\text{-}\varphi)\right)\left(\gamma_3(1\text{-}\varphi)\text{-}\delta_4(1-\vartheta)\right)} + \frac{e^{-\delta_4(1-\vartheta)t}}{\left(\gamma_3(1\text{-}\varphi)\text{-}\delta_4(1-\vartheta)\right)\left(\delta_4(1-\vartheta)\text{-}\alpha_1(1\text{-}\varepsilon)\right)} \end{array} \right)(\omega) \end{array} \right\}$$

In B_4, the exploitation is

$$E_4(t) = 1\text{-}P_0 = 1\text{-}\exp\left\{ \begin{array}{c} \frac{-1(\omega)}{\delta_4(1-\vartheta)}\left(1\text{-}e^{-\delta_4(1-\vartheta)t}\right)\text{-}\frac{1(\omega)}{\left(\delta_4(1-\vartheta)\text{-}\gamma_3(1\text{-}\varphi)\right)}\left(e^{-\delta_4(1-\vartheta)t}\text{-}e^{-\gamma_3(1\text{-}\varphi)t}\right)(-1)\gamma_3(1\text{-}\varphi) \\ \left(\begin{array}{c} \frac{e^{-\alpha_1(1\text{-}\varepsilon)t}}{(\alpha_1(1\text{-}\varepsilon)\text{-}\delta_4)\left(\beta_2(1\text{-}\tau)\text{-}\alpha_1(1\text{-}\varepsilon)\right)} + \frac{e^{-\beta_2(1\text{-}\tau)t}}{\left(\beta_2(1\text{-}\tau)\text{-}\alpha_1(1\text{-}\varepsilon)\right)\left(\gamma_3(1\text{-}\varphi)\text{-}\beta_2(1\text{-}\tau)\right)} + \\ \frac{e^{-\gamma_3(1\text{-}\varphi)t}}{\left(\beta_2(1\text{-}\tau)\text{-}\gamma_3(1\text{-}\varphi)\right)\left(\gamma_3(1\text{-}\varphi)\text{-}\delta_4(1-\vartheta)\right)} + \frac{e^{-\delta_4(1-\vartheta)t}}{\left(\gamma_3(1\text{-}\varphi)\text{-}\delta_4(1-\vartheta)\right)\left(\delta_{4(1-\vartheta)}\text{-}\alpha_1(1\text{-}\varepsilon)\right)} \end{array} \right)(\omega) \end{array} \right\} \quad (3.26)$$

In A_4, variance of the number of packets is

$$V_4(t) = \left\{ \begin{array}{c} \frac{1(\omega)}{\delta_4(1-\vartheta)}\left(1\text{-}e^{-\delta_4(1-\vartheta)t}\right) + \frac{1(\omega)}{\left(\delta_4(1-\vartheta)\text{-}\gamma_3(1\text{-}\varphi)\right)}\left(e^{-\delta_4(1-\vartheta)t}\text{-}e^{-\gamma_3(1\text{-}\varphi)t}\right) \\ +\gamma_3(1\text{-}\varphi)\left(\begin{array}{c} \frac{e^{-\alpha_1(1\text{-}\varepsilon)t}}{\left(\alpha_1(1\text{-}\varepsilon)\text{-}\delta_4(1-\vartheta)\right)\left(\beta_2(1\text{-}\tau)\text{-}\alpha_1(1\text{-}\varepsilon)\right)} + \frac{e^{-\beta_2(1\text{-}\tau)t}}{\left(\beta_2(1\text{-}\tau)\text{-}\alpha_1(1\text{-}\varepsilon)\right)\left(\gamma_3(1\text{-}\varphi)\text{-}\beta_2(1\text{-}\tau)\right)} + \\ \frac{e^{-\gamma_3(1\text{-}\varphi)t}}{\left(\beta_2(1\text{-}\tau)\text{-}\gamma_3(1\text{-}\varphi)\right)\left(\gamma_3(1\text{-}\varphi)\text{-}\delta_4\right)} + \frac{e^{-\delta_4(1-\vartheta)t}}{\left(\gamma_3(1\text{-}\varphi)\text{-}\delta_4(1-\vartheta)\right)\left(\delta_4(1-\vartheta)\text{-}\alpha_1(1\text{-}\varepsilon)\right)} \end{array} \right)(\omega) \end{array} \right\} \quad (3.27)$$

In B_4, the throughput is

$$Th_4 = \delta_4\left(1\text{-}P_0\right) = \delta_4\left(1+\exp\left\{ \begin{array}{c} \frac{1(\omega)}{\delta_4(1-\vartheta)}\left(1\text{-}e^{-\delta_4(1-\vartheta)t}\right) + \frac{1(\omega)}{\left(\delta_4(1-\vartheta)\text{-}\gamma_3(1\text{-}\varphi)\right)}\left(e^{-\delta_4(1-\vartheta)t}\text{-}e^{-\gamma_3(1\text{-}\varphi)t}\right)(\text{-}1)\gamma_3(1\text{-}\varphi) \\ \left(\begin{array}{c} \frac{e^{-\alpha_1(1\text{-}\varepsilon)t}}{\left(\alpha_1(1\text{-}\varepsilon)\text{-}\delta_4(1-\vartheta)\right)\left(\beta_2(1\text{-}\tau)\text{-}\alpha_1(1\text{-}\varepsilon)\right)} + \frac{e^{-\beta_2(1\text{-}\tau)t}}{\left(\beta_2(1\text{-}\tau)\text{-}\alpha_1(1\text{-}\varepsilon)\right)\left(\gamma_3(1\text{-}\varphi)\text{-}\beta_2(1\text{-}\tau)\right)} + \\ \frac{e^{-\gamma_3(1\text{-}\varphi)t}}{\left(\beta_2(1\text{-}\tau)\text{-}\gamma_3(1\text{-}\varphi)\right)\left(\gamma_3(1\text{-}\varphi)\text{-}\delta_4(1-\vartheta)\right)} + \frac{e^{-\delta_4(1-\vartheta)t}}{\left(\gamma_3(1\text{-}\varphi)\text{-}\delta_4(1-\vartheta)\right)\left(\delta_4(1-\vartheta)\text{-}\alpha_1(1\text{-}\varepsilon)\right)} \end{array} \right)(\omega) \end{array} \right\} \right)$$

(3.28)

In A_4, the average waiting time is given by

$$W_4(t) = \frac{M_4(t)}{\delta_4(1-P_0)}$$

At time t, the mean number of packets in the total network is

$$M(t) = M_1(t)+M_2(t)+M_3(t)+M_4(t) \quad (3.30)$$

In the network, changeability of the number of packets is

$$V(t) = V_1(t)+V_2(t)+V_3(t)+V_4(t) \quad (3.31)$$

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4. PERFORMANCE EVALUATION OF THE NETWORK MODEL

The performance of the network model is given with numerical example. Various values of the parameters are given for bandwidth allocation and arrival of packets. The deviation of the packet arrival (ω) is from 2x10^4packets/sec to 7x10^4 packets/sec, the difference of probabilities parameters ($\epsilon, \tau, \varphi, \vartheta$) is from 0.1 to 0.9, the difference of transmission rate for first transmitter (α_1) is from 5x10^4packets/sec to 9x10^4packets/sec, the difference of second transmitter (β_2) is from 15x10^4packets/sec to 19x10^4 packets/sec, the difference of third transmitter (γ_3) is from 25x10^4packets/sec to 29x10^4 packets/sec , the difference of fourth transmitter (δ_4) is from 35x10^4packets/sec to 39x10^4 packets/sec. By the application of the Dynamic Bandwidth Allocation strategy for three transmitters, the transmission rate of each packet based on the number of packets in the buffer linked to equivalent transmitter.

The exploitation of the transmitters and throughput of the transmitters are calculated by using equations (4.2.11),(4.2.13),(4.2.16),(4.2.18),(4.2.21),(4.2.23),(4.2.26) and (4.2.28) for various values of the parameters t,$\omega, \epsilon, \tau, \varphi, \vartheta, \alpha_1, \beta_2, \gamma_3, \delta_4$. The results and relationship between exploitation of the transmitters and throughput of the transmitters are given below:

Table 4.1: Values of Exploitation and Throughput of the Network with DBA and Homogeneous Poisson Arrivals.

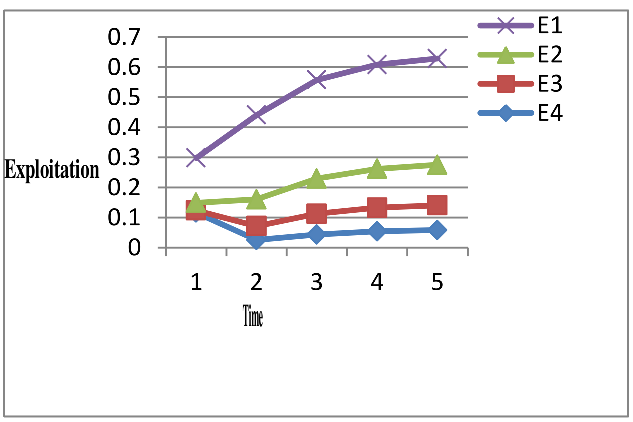
t	ω	ε	τ	φ	ϑ	α_1	β_2	γ_3	δ_4	$E_1(t)$	$E_2(t)$	$E_3(t)$	$E_4(t)$	$Th_1(t)$	$Th_2(t)$	$Th_3(t)$	$Th_4(t)$
0.1	2	0.1	0.1	0.1	0.1	5	15	25	35	0.1488	0.0253	0.0079	0.1161	0.7438	0.3799	0.1969	4.064
0.3	2	0.1	0.1	0.1	0.1	5	15	25	35	0.2805	0.0877	0.0465	0.0252	1.4026	1.3161	1.1616	0.883
0.5	2	0.1	0.1	0.1	0.1	5	15	25	35	0.3281	0.1173	0.0689	0.0437	1.6404	1.7671	1.7239	1.528
0.7	2	0.1	0.1	0.1	0.1	5	15	25	35	0.3465	0.1295	0.0785	0.0541	1.7325	1.9418	1.9626	1.893
0.9	2	0.1	0.1	0.1	0.1	5	15	25	35	0.3538	0.1344	0.0824	0.0585	1.7692	2.0153	2.0598	2.047
0.5	**3**	0.1	0.1	0.1	0.1	5	15	25	35	0.4492	0.1707	0.1016	0.0648	2.2461	2.5612	2.5409	2.266
0.5	**4**	0.1	0.1	0.1	0.1	5	15	25	35	0.5485	0.2209	0.1332	0.0854	2.7426	3.3136	3.3291	2.989
0.5	**5**	0.1	0.1	0.1	0.1	5	15	25	35	0.6299	0.2680	0.1636	0.1056	3.1495	4.0206	4.0896	3.695
0.5	**6**	0.1	0.1	0.1	0.1	5	15	25	35	0.6966	0.3123	0.1929	0.1253	3.4832	4.6849	4.8235	4.386
0.5	**7**	0.1	0.1	0.1	0.1	5	15	25	35	0.7513	0.3539	0.2213	0.1446	3.7566	5.3089	5.5316	5.061

0.5	2	**0.1**	0.1	0.1	0.1	5	15	25	35	0.3281	0.1173	0.0689	0.0437	1.6404	1.7601	1.7239	1.528
0.5	2	**0.3**	0.1	0.1	0.1	5	15	25	35	0.3763	0.1073	0.0623	0.0356	1.8816	1.6088	1.5566	1.243
0.5	2	**0.5**	0.1	0.1	0.1	5	15	25	35	0.4349	0.0916	0.0524	0.0233	2.1746	1.3744	1.3091	0.8168
0.5	2	**0.7**	0.1	0.1	0.1	5	15	25	35	0.5052	0.0671	0.0376	0.0049	2.5258	1.0063	0.9387	0.1724
0.5	2	**0.9**	0.1	0.1	0.1	5	15	25	35	0.5872	0.0279	0.0153	0.0023	2.9360	0.4191	0.3815	-0.815
0.5	2	0.1	**0.1**	0.1	0.1	5	15	25	35	0.3281	0.1173	0.0689	0.0437	1.6404	1.7601	1.7239	1.528
0.5	2	0.1	**0.3**	0.1	0.1	5	15	25	35	0.3281	0.1445	0.0667	0.0368	1.6404	2.1678	1.6680	1.288
0.5	2	0.1	**0.5**	0.1	0.1	5	15	25	35	0.3281	0.1860	0.0623	0.0281	1.6404	2.7902	1.5573	0.9819
0.5	2	0.1	**0.7**	0.1	0.1	5	15	25	35	0.3281	0.2620	0.0495	1.000	1.6404	3.9304	1.2384	35.00
0.5	2	0.1	**0.9**	0.1	0.1	5	15	25	35	0.3281	0.3680	0.0269	-0.240	1.6404	5.5200	0.6724	-8.411
0.5	2	0.1	0.1	**0.1**	0.1	5	15	25	35	0.3281	0.1173	0.0689	0.0437	1.6404	1.7601	1.7239	1.528
0.5	2	0.1	0.1	**0.3**	0.1	5	15	25	35	0.3281	0.1173	0.0863	0.0483	1.6404	1.7601	2.1571	1.691
0.5	2	0.1	0.1	**0.5**	0.1	5	15	25	35	0.3281	0.1173	0.1145	0.0456	1.6404	1.7601	2.8635	1.595
0.5	2	0.1	0.1	**0.7**	0.1	5	15	25	35	0.3281	0.1173	0.1666	0.0509	1.6404	1.7601	4.1649	1.782
0.5	2	0.1	0.1	**0.9**	0.1	5	15	25	35	0.3281	0.1173	0.2775	0.0363	1.6404	1.7601	6.9382	1.272
0.5	2	0.1	0.1	0.1	**0.1**	5	15	25	35	0.3281	0.1173	0.0689	0.0437	1.6404	1.7601	1.7239	1.528
0.5	2	0.1	0.1	0.1	**0.3**	5	15	25	35	0.3281	0.1173	0.0689	0.0544	1.6404	1.7601	1.7239	1.905
0.5	2	0.1	0.1	0.1	**0.5**	5	15	25	35	0.3281	0.1173	0.0689	0.0717	1.6404	1.7601	1.7239	2.51
0.5	2	0.1	0.1	0.1	**0.7**	5	15	25	35	0.3281	0.1173	0.0689	0.0994	1.6404	1.7601	1.7239	3.479
0.5	2	0.1	0.1	0.1	**0.9**	5	15	25	35	0.3281	0.1173	0.0689	0.4344	1.6404	1.7601	1.7239	15.2
0.5	2	0.1	0.1	0.1	0.1	**5**	15	25	35	0.3281	0.1173	0.0689	0.0437	1.7527	1.7601	1.7239	1.528
0.5	2	0.1	0.1	0.1	0.1	**6**	15	25	35	0.2921	0.1234	0.0732	0.0487	1.8342	1.8505	1.8287	1.704
0.5	2	0.1	0.1	0.1	0.1	**7**	15	25	35	0.2620	0.1275	0.0761	0.0523	1.8941	1.9126	1.9035	1.829
0.5	2	0.1	0.1	0.1	0.1	**8**	15	25	35	0.2368	0.1304	0.0783	0.0548	1.9388	1.9554	1.9574	1.919
0.5	2	0.1	0.1	0.1	0.1	**9**	15	25	35	0.2154	0.1323	0.0799	0.0567	1.6404	1.9851	1.9965	1.985
0.5	2	0.1	0.1	0.1	0.1	5	**15**	25	35	0.3281	0.1173	0.0689	0.0437	1.6404	1.7601	1.7239	1.528
0.5	2	0.1	0.1	0.1	0.1	5	**16**	25	35	0.3281	0.1109	0.0694	0.0451	1.6404	1.7758	1.7353	1.579
0.5	2	0.1	0.1	0.1	0.1	5	**17**	25	35	0.3281	0.1053	0.0698	0.0464	1.6404	1.7895	1.7449	1.624
0.5	2	0.1	0.1	0.1	0.1	5	**18**	25	35	0.3281	0.1000	0.0701	0.0475	1.6404	1.8015	1.7532	1.663

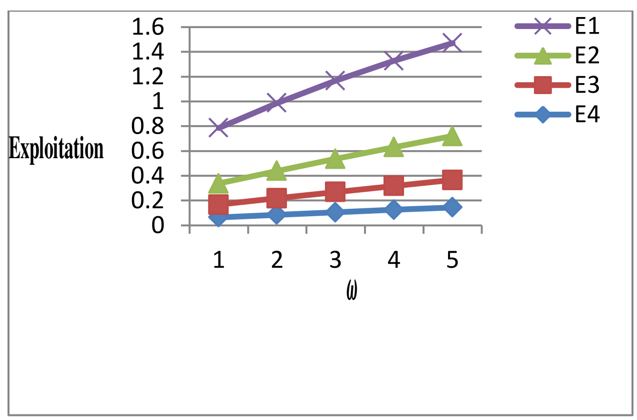
0.5	2	0.1	0.1	0.1	0.1	5	**19**	25	35	0.3281	0.0954	0.0704	0.0485	1.6404	1.8122	1.7604	1.697
0.5	2	0.1	0.1	0.1	0.1	5	15	**25**	35	0.3281	0.1173	0.0689	0.0437	1.6404	1.7601	1.7239	1.528
0.5	2	0.1	0.1	0.1	0.1	5	15	**26**	35	0.3281	0.1173	0.0665	0.0429	1.6404	1.7601	1.7301	1.5
0.5	2	0.1	0.1	0.1	0.1	5	15	**27**	35	0.3281	0.1173	0.0643	0.0421	1.6404	1.7601	1.7357	1.473
0.5	2	0.1	0.1	0.1	0.1	5	15	**28**	35	0.3281	0.1173	0.0622	0.0413	1.6404	1.7601	1.7409	1.446
0.5	2	0.1	0.1	0.1	0.1	5	15	**29**	35	0.3281	0.1173	0.0602	0.0406	1.6404	1.7601	1.7456	1.419
C.5	2	0.1	0.1	0.1	0.1	5	15	25	**35**	0.3281	0.1173	0.0689	0.0437	1.6404	1.7601	1.7239	1.528
0.5	2	0.1	0.1	0.1	0.1	5	15	25	**36**	0.3281	0.1173	0.0689	0.0426	1.6404	1.7601	1.7239	1.532
0.5	2	0.1	0.1	0.1	0.1	5	15	25	**37**	0.3281	0.1173	0.0689	0.0415	1.6404	1.7601	1.7239	1.537
0.5	2	0.1	0.1	0.1	0.1	5	15	25	**38**	0.3281	0.1173	0.0689	0.0405	1.6404	1.7601	1.7239	1.541
0.5	2	0.1	0.1	0.1	0.1	5	15	25	**39**	0.3281	0.1173	0.0689	0.0396	1.6404	1.7601	1.7239	1.545

From table 4.1 it is observed that the exploitation of the transmitters increases when the time(t), ω increases and the other parameters are constants, the exploitation of first transmitter increases and the exploitation of the second, third, and fourth transmitters decreases when ϵ increases from 0.1 to 0.9, the exploitation of the first transmitter is stable, exploitation of the second transmitter increases and the exploitation of the third , fourth transmitters increases when τ varies from 0.1 to 0.9, the exploitation of the first, second transmitter is stable and the exploitation of the third, fourth transmitters increases when φ varies from 0.1 to 0.9, the exploitation of the first, second, third transmitter is stable and the exploitation of the, fourth transmitters increases when ϑ varies from 0.1 to 0.9, the exploitation of the first transmitter decreases ant the exploitation of second, third and fourth transmitters increases when the transmission rate of the first transmitter (α_1) increases from 5 to 9 and the remaining parameters are stable, the exploitation of the first transmitter is stable , the exploitation of the second transmitter increases and the exploitation of the third, fourth transmitter decreases when the transmission rate of the second transmitter (β_2) increases from 15 to 19 and the remaining parameters are fixed, the exploitation of first , second transmitters are stable and the exploitation of third transmitter increases and fourth transmitters decreases when the transmission rate of the third transmitter(γ_3) increases from 25 to 29 and the remaining parameters are fixed, the exploitation of the first, second, third transmitters are stable and the exploitation of the fourth transmitter increases when the transmission rate of the fourth transmitter(δ_4)differs from 35 to 39 and the remaining parameters are fixed. t and ω, increases and the other parameters are constant, the throughput of the first, transmitters increases and the throughput of the second, third , fourth transmitters increases when εvaries from 0.1 to 0.9, the throughput of the first transmitter is stable , throughput of the second transmitter increases, throughput of the third and fourth transmitters decreases when τdiffers from 0.1 to 0.9, the throughput of the first, second transmitters is stable, throughput of the fourth transmitters increases when φ differs from 0.1 to 0.9, the throughput of the first, second transmitters is stable, throughput of the third and fourth transmitters increases when ϑ varies from 0.1 to 0.9, the throughput of the first second third and fourth transmitters increases, when the transmission rate of the first transmitter (α_1) increases. the throughput of the first transmitter is stable and the throughput of the second third and fourth transmitters increases when the transmission rate of the second transmitter (β_2) increases. the throughput of the first, second transmitters is stable and throughput of the third and fourth transmitters increases when the transmission rate of the third transmitter (γ_3) increases. the throughput of the first, second and third transmitters is stable and the throughput of the fourth transmitter increases when the transmission rate of the fourth transmitter (δ_4) increases. The following graphs shows the relationship between the exploitation, throughput of the transmitters and various parameters:

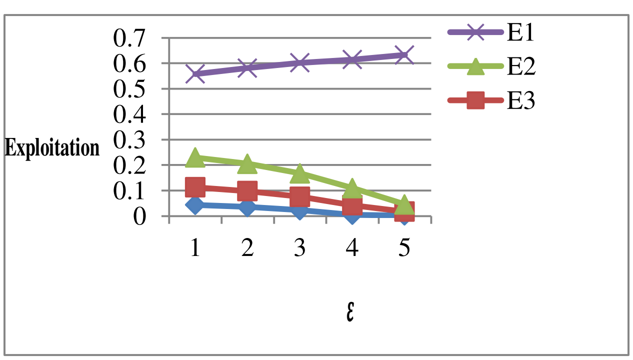
Time vs Exploitation



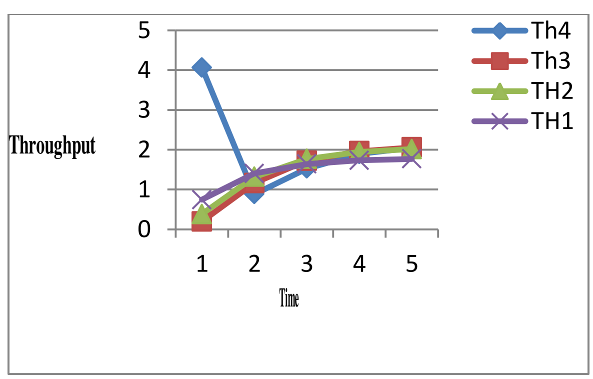
ω vs Exploitation



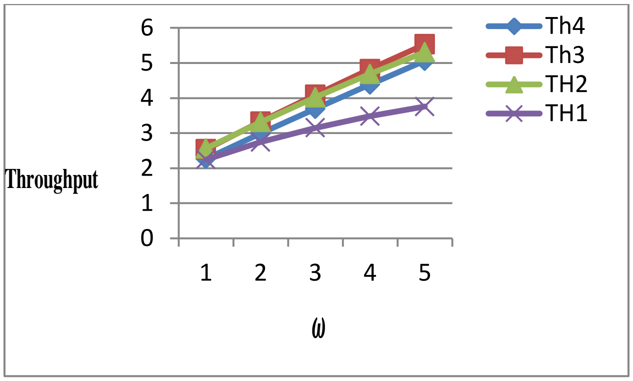
ε vs Exploitation



Time vs Throughput



ω vs Throughput **ε vs Throughput**



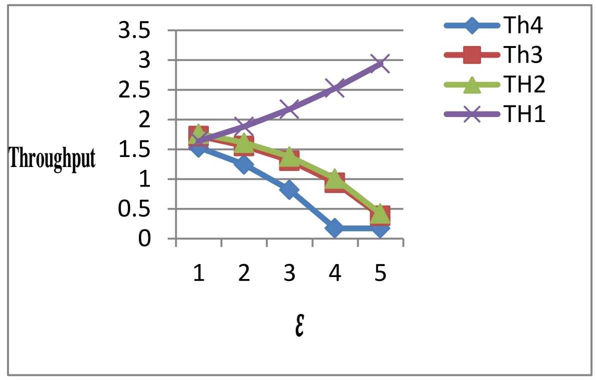


Fig 4.1: The Relationship Between Exploitation and Throughput and Other Parameters

Mean number of packets in the four buffers and mean delay in transmission of the four transmitters are calculated for different values of the parameters by using equations (4.2.10), (4.2.14), (4.2.15), (4.2.19), (4.2.20), (4.2.24), (4.2.25) and (4.2.29) which are given in the following table. The graphs showing relationship between parameters and performance measures are shown in figure 4.2.

Table 4.2: Values for Mean Number of Packets and Mean Delay of the Network Model With DBA and Homogeneous Arrivals.

t	ω	ε	τ	φ	ϑ	α_1	β_2	γ_3	δ_4	$M_1(t)$	$M_2(t)$	$M_3(t)$	$M_4(t)$	$W_1(t)$	$W_2(t)$	$W_3(t)$	$W_4(t)$
0.1	2	0.1	0.1	0.1	0.1	5	15	25	35	0.1611	0.0257	0.0079	0.1234	0.2165	0.0675	0.0402	0.0304
0.3	2	0.1	0.1	0.1	0.1	5	15	25	35	0.3292	0.0918	0.0476	0.0255	0.2347	0.0698	0.0409	0.0289
0.5	2	0.1	0.1	0.1	0.1	5	15	25	35	0.3976	0.1248	0.0715	0.0446	0.2424	0.0709	0.0415	0.0292
0.	2	0.	0.	0.	0.	5	15	2	3	0.425	0.138	0.081	0.055	0.245	0.071	0.041	0.029

7		1	1	1	1			5	5	4	6	8	6	6	4	7	4
0.9	2	0.1	0.1	0.1	0.1	5	15	25	35	0.4367	0.1443	0.0859	0.0603	0.2468	0.0716	0.0417	0.0294
0.5	**3**	0.1	0.1	0.1	0.1	5	15	25	35	0.5964	0.1872	0.1072	0.0669	0.2655	0.0731	0.0422	0.0295
0.5	**4**	0.1	0.1	0.1	0.1	5	15	25	35	0.7952	0.2496	0.1429	0.0893	0.2899	0.0753	0.0429	0.0299
0.5	**5**	0.1	0.1	0.1	0.1	5	15	25	35	0.9940	0.3120	0.1786	0.1116	0.3156	0.0776	0.0437	0.0302
0.5	**6**	0.1	0.1	0.1	0.1	5	15	25	35	1.1928	0.3744	0.2144	0.1339	0.3425	0.0799	0.0444	0.0305
0.5	**7**	0.1	0.1	0.1	0.1	5	15	25	35	1.3916	0.4368	0.2501	0.1562	0.3704	0.0823	0.0452	0.0309
0.5	2	**0.1**	0.1	0.1	0.1	5	15	25	35	0.3976	0.1248	0.0715	0.0446	0.2424	0.0709	0.0415	0.0292
0.5	2	**0.3**	0.1	0.1	0.1	5	15	25	35	0.4721	0.1135	0.0643	0.0362	0.2509	0.0705	0.0413	0.0.291
0.5	2	**0.5**	0.1	0.1	0.1	5	15	25	35	0.5708	0.0961	0.0538	0.0236	0.2625	0.0699	0.0411	0.0289
0.5	2	**0.7**	0.1	0.1	0.1	5	15	25	35	0.7035	0.0694	0.0383	0.0049	0.2785	0.0690	0.0408	0.0286
0.5	2	**0.9**	0.1	0.1	0.1	5	15	25	35	0.8848	0.0283	0.0154	-0.023	0.3014	0.0676	0.0403	0.0282
0.5	2	0.1	**0.1**	0.1	0.1	5	15	25	35	0.3976	0.1248	0.0714	0.0446	0.2424	0.0709	0.0415	0.0292

0.5	2	0.1	**0.3**	0.1	0.1	5	15	25	35	0.3976	0.1561	0.0691	0.0375	0.2424	0.0720	0.0414	0.0292
0.5	2	0.1	**0.5**	0.1	0.1	5	15	25	35	0.3976	0.2058	0.0643	0.0285	0.2424	0.0738	0.0413	0.0289
0.5	2	0.1	**0.7**	0.1	0.1	5	15	25	35	0.3976	0.3039	0.0508	0.0284	0.2424	0.0773	0.0410	0.0287
0.5	2	0.1	**0.9**	0.1	0.1	5	15	25	35	0.3976	0.4589	0.0273	0.0283	0.2424	0.0831	0.0406	0.0256
0.5	2	0.1	0.1	**0.1**	0.1	5	15	25	35	0.3976	0.1248	0.0715	0.0446	0.2424	0.0709	0.0415	0.0292
0.5	2	0.1	0.1	**0.3**	0.1	5	15	25	35	0.3976	0.1248	0.0902	0.0495	0.2424	0.0709	0.0418	0.0293
0.5	2	0.1	0.1	**0.5**	0.1	5	15	25	35	0.3976	0.1248	0.1216	0.0467	0.2424	0.0709	0.0425	0.0292
0.5	2	0.1	0.1	**0.7**	0.1	5	15	25	35	0.3976	0.1248	0.1822	0.0523	0.2424	0.0709	0.0438	0.0293
0.5	2	0.1	0.1	**0.9**	0.1	5	15	25	35	0.3976	0.1248	0.3251	0.3701	0.2424	0.0709	0.0469	0.0291
0.5	2	0.1	0.1	0.1	**0.1**	5	15	25	35	0.3976	0.1248	0.0715	0.0446	0.2424	0.0709	0.0415	0.0292
0.5	2	0.1	0.1	0.1	**0.3**	5	15	25	35	0.3976	0.1248	0.0715	0.0559	0.2424	0.0709	0.0415	0.0294
0.5	2	0.1	0.1	0.1	**0.5**	5	15	25	35	0.3976	0.1248	0.0715	0.0744	0.2424	0.0709	0.0415	0.0297
0.	2	0.	0.	0.	**0.**	5	15	2	3	0.397	0.124	0.071	0.104	0.242	0.070	0.041	0.030

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5		1	1	1	**7**			5	5	6	8	5	7	4	9	5	1
0.5	2	0.1	0.1	0.1	**0.9**	5	15	25	35	0.3976	0.1248	0.0715	0.5699	0.2424	0.0709	0.0415	0.0375
0.5	2	0.1	0.1	0.1	0.1	**5**	15	25	35	0.3976	0.1248	0.0715	0.0446	0.2424	0.0709	0.0415	0.0292
0.5	2	0.1	0.1	0.1	0.1	**6**	15	25	35	0.3976	0.1248	0.0759	0.0499	0.1971	0.0712	0.0415	0.0293
0.5	2	0.1	0.1	0.1	0.1	**7**	15	25	35	0.3976	0.1248	0.0792	0.0537	0.1657	0.0713	0.0416	0.0294
0.5	2	0.1	0.1	0.1	0.1	**8**	15	25	35	0.3976	0.1248	0.0815	0.0564	0.1427	0.0714	0.0417	0.0294
0.5	2	0.1	0.1	0.1	0.1	**9**	15	25	35	0.3976	0.1248	0.0832	0.0584	0.1251	0.0715	0.0417	0.0294
0.5	2	0.1	0.1	0.1	0.1	5	**15**	25	35	0.3976	0.1248	0.0715	0.4463	0.2424	0.0709	0.0415	0.0292
0.5	2	0.1	0.1	0.1	0.1	5	**16**	25	35	0.3976	0.1248	0.0719	0.0462	0.2424	0.0663	0.0415	0.0292
0.5	2	0.1	0.1	0.1	0.1	5	**17**	25	35	0.3976	0.1248	0.0724	0.0475	0.2424	0.0622	0.0415	0.0293
0.5	2	0.1	0.1	0.1	0.1	5	**18**	25	35	0.3976	0.1248	0.0727	0.0487	0.2424	0.0585	0.0415	0.0293
0.5	2	0.1	0.1	0.1	0.1	5	**19**	25	35	0.3976	0.1248	0.0730	0.0497	0.2424	0.0553	0.0415	0.0293
0.5	2	0.1	0.1	0.1	0.1	5	15	**25**	35	0.3976	0.1248	0.0715	0.0446	0.2424	0.0709	0.0415	0.0292

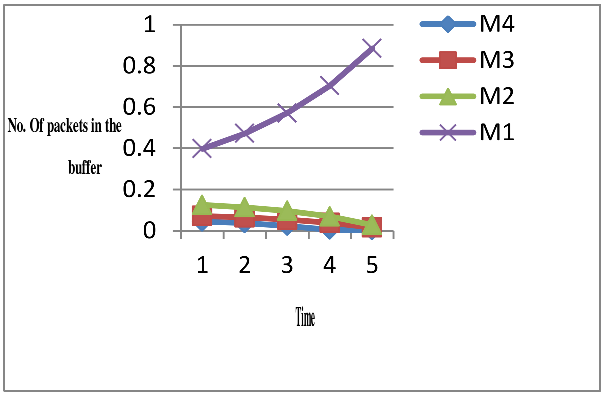
0.5	2	0.1	0.1	0.1	0.1	5	15	**26**	35	0.3976	0.1248	0.0689	0.0438	0.2424	0.0709	0.0398	0.0292
0.5	2	0.1	0.1	0.1	0.1	5	15	**27**	35	0.3976	0.1248	0.0664	0.043	0.2424	0.0709	0.0383	0.0292
0.5	2	0.1	0.1	0.1	0.1	5	15	**28**	35	0.3976	0.1248	0.0642	0.0422	0.2424	0.0709	0.0369	0.0292
0.5	2	0.1	0.1	0.1	0.1	5	15	**29**	35	0.3976	0.1248	0.0621	0.0414	0.2424	0.0709	0.0356	0.0292
0.5	2	0.1	0.1	0.1	0.1	5	15	25	**35**	0.3976	0.1248	0.0715	0.0446	0.2424	0.0709	0.0415	0.0292
0.5	2	0.1	0.1	0.1	0.1	5	15	25	**36**	0.3976	0.1248	0.0715	0.0435	0.2424	0.0709	0.0415	0.0284
0.5	2	0.1	0.1	0.1	0.1	5	15	25	**37**	0.3976	0.1248	0.0715	0.0424	0.2424	0.0709	0.0415	0.0276
0.5	2	0.1	0.1	0.1	0.1	5	15	25	**38**	0.3976	0.1248	0.0715	0.0414	0.2424	0.0709	0.0415	0.0269
0.5	2	0.1	0.1	0.1	0.1	5	15	25	**39**	0.3976	0.1248	0.0715	0.0404	0.2424	0.0709	0.0415	0.0262

It is observed from table 4.2 that as the mean number of packets in the four buffers and in the network increases when t varies from 0.1 to 0.9 and other parameters are constants. The mean number of packets in the first, second, third and fourth buffers and in the network increases when ω increases. The mean number of packets in the first buffer increases and decreases in the second, third and fourth buffer when ε varies from 0.1 to 0.9. $\tau,$ varies from 0.1 to 0.9 the mean number of packets in the first buffer remains constant and increases in the second buffer due to packets arrived directly from the first transmitter. The mean number of packets in the first, second buffers remains constant and in the third, fourth buffers increases when φ varies from 0.1 to 0.9. The mean number of packets in the first, second, third buffers remains constant and in the fourth buffer increases when ϑ varies from 0.1 to 0.9. The mean number of packets in the first buffer decreases, in the second, third and fourth buffers decreases and in the network decreases, when the transmission rate of the first transmitter (α_1) varies from $5x10^4$ to $9x10^4$. The mean number of packets in the first buffer remains constant, decreases in the in the second buffer and increases in the third ,fourth buffers, when the transmission rate of the second transmitter (β_2) varies from $15x10^4$ to1 $9x10^4$. The mean number of packets in the first buffer, second buffers remains constant, decreases in the in the third buffer and increases in the ,fourth buffer, when the transmission rate of the third transmitter (γ_3) varies from $25x10^4$ to 2 $9x10^4$.The mean number of packets in the first, second, third buffers remains constant, increases in the fourth buffer when the transmission rate of the fourth transmitter (δ_4) varies from $35x10^4$ to $39x10^4$..

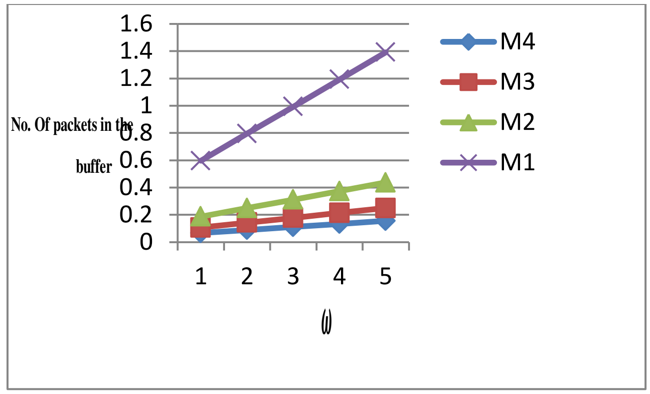
From the table 4.2, it is also observed that with time(t) and $\omega$the mean delay in the four buffers increases for constants values of other parameters. The mean delay in the first buffer increases and decreases in the second, third and fourth buffers when ε varies from 0.1 to 0.9. The mean delay in the first buffer remains constant ,increases in the second buffer and decreases in the third, fourth buffers when τ varies from 0.1 to 0.9. The mean delay in the first, second buffers remains constant, increases in the third buffer and decreases in the fourth buffers when φ varies from 0.1 to 0.9. The mean delay in the first, second, third buffers remains constant ,decreases in the fourth buffer when ϑ varies from 0.1 to 0.9. When the transmission rate of the first transmitter (α_1) varies, the mean delay of the first buffer decreases, second, third and fourth buffers increases. When the transmission rate of the second transmitter (β_2) varies, the mean delay of the first buffer remains constant, decreases in the second buffer, third and fourth buffers increases. When the transmission rate of the third transmitter (γ_3) varies, the mean delay of the first , second buffer remains constant, decreases in the third buffer and in the fourth buffers increases. When the transmission rate of the fourth transmitter (δ_4) varies, the mean delay of the first , second,third buffer remains constant, decreases in the fourth buffer.

From the above analysis, it is observed that the dynamic bandwidth allocation strategy has a significant influence on all performance measures of the network. We also observed that the performance measures are highly sensitive towards smaller values of time. Hence, by dynamic bandwidth allocation and transient conditions it is optimal and also observed that the congestion in buffers and delays in transmission can be reduced to a minimum level by adopting dynamic bandwidth allocation.

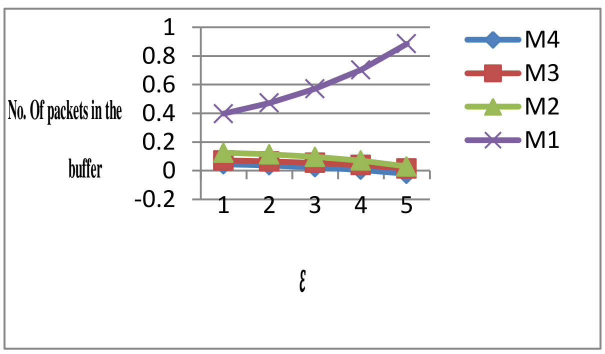
Time vs No. of packets in the buffer



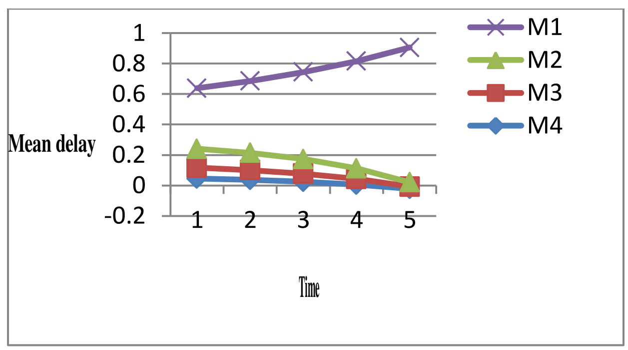
ω **vs No. of packets in the buffer**



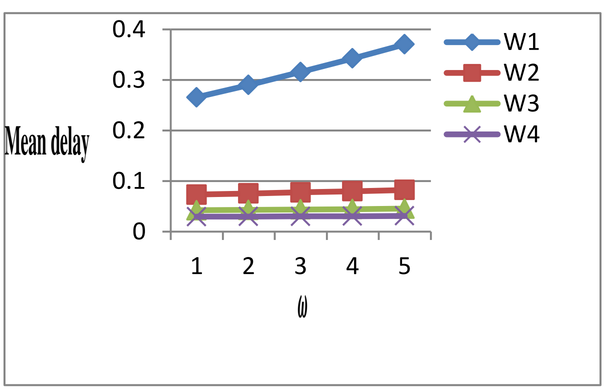
ε **vs No. of packets in the buffer**



Time vs Mean delay



ω **vs Mean delay**



ε **vs Mean delay**

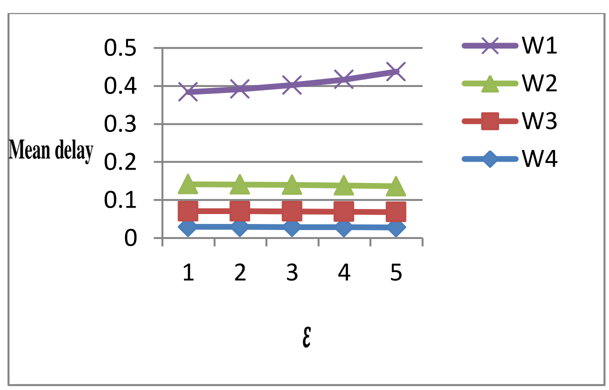


Fig 4.2: The Relationship Between Mean No. of Packets, Mean Delay and Various Parameters

5. SENSITIVITY ANALYSIS

Sensitivity analysis of the proposed network model with respect to the changes in the parameters t, $\omega, \varepsilon, \tau, \varphi, \vartheta$ on the mean number of packets, exploitation of the transmitters, mean and throughput of the four transmitters is presented in this section. The values considered for the sensitivity analysis are, t = 0.5 sec, ω = 2x10^4 packets/sec, α_1= 5x10^4 packets/sec, β_2=15x10^4 packets/sec, γ_3 = 25x10^4 packets/sec, δ_4= 35x10^4 packets/sec, = 0.1, τ = 0.1, φ = 0.1 and ϑ = 0.1. The mean number of packets, exploitation of the transmitters, mean delay and throughput of the transmitters are computed with variation of -15%, -10%, -5%, 0%, 5%, 10%, 15% on the model and are presented in the table 4.5.1. The performance measures are highly affected by the changes in the values of time (t), arrival and probability constants $(\varepsilon, \tau, \varphi, \vartheta)$.

When the time (t) increases from -15% to 15% the average number of packets in the four buffers increase along with the exploitation, throughput of the transmitters, and average delay in buffers. As the arrival parameter (ω) increases from -15% to 15% the average number of packets in the four buffers increase along with the exploitation, throughput of the transmitters, and average delay in buffers. As the probability parameter of the ε increases from -15% to 15% the average number of packets in the first buffers increase along with the exploitation, throughput of the transmitters, and average delay in buffers. But average number of packets in the second, third and fourth buffer decreases along with the exploitation, throughput of the transmitters, and average delay in buffers. Similarly, when the probability parameter of the second buffer τ increases from -15% to 15% the average number of packets, exploitation, throughput, and the average delay in the first buffer remains constant, when as the average number of packets in the second buffer increase along with the exploitation, throughput of the transmitters, and average delay in buffers and average number of packets in the third and fourth buffer decreases along with the exploitation, throughput of the transmitters, and average delay in buffers. When the probability parameter φ increases from -15% to 15% the average number of packets, exploitation, throughput, and the average delay in the first, second buffer remains constant where as the average number of packets in the third buffer increase along with the exploitation, throughput of the transmitters, and average delay in buffers and average number of packets in the fourth buffer decreases along with the exploitation, throughput of the transmitters, and average delay in buffers. When the probability parameter ϑ increases from -15% to 15% the average number of packets, exploitation, throughput, and the average delay in the first, second, third buffer remains constant where as the average number of packets in the fourth buffer increase along with the exploitation.

From the above analysis it is observed that the dynamic bandwidth allocation strategy has an important influence on all performance measures of the network. It is also observed that these performance measures are also sensitive towards the probability parameters $(\varepsilon, \tau, \varphi, \vartheta)$, which causes feedback of packets to the first, second and third transmitters.

Table 5.1:Sensitivity Analysis of the Model

Parameter	**Performance Measure**	**% change in parameter**						
		-15	**-10**	**-5**	**0**	**5**	**10**	**15**
t=0.5	$M_1(t)$	0.3788	0.3858	0.3920	0.3976	0.4025	0.4070	0.4110
	$M_2(t)$	0.1156	0.1190	0.1221	0.1248	0.1273	0.1295	0.1315
	$M_3(t)$	0.0646	0.0671	0.0694	0.0715	0.0733	0.0749	0.0764
	$M_4(t)$	0.0380	0.0403	0.0426	0.0446	0.0465	0.0482	0.0498
	$E_1(t)$	0.3153	0.3201	0.3243	0.3281	0.3314	0.3344	0.3370
	$E_2(t)$	0.1091	0.1122	0.1149	0.1173	0.1195	0.1215	0.1232
	$E_3(t)$	0.0626	0.0649	0.0671	0.0690	0.0707	0.0722	0.0736
	$E_4(t)$	0.0372	0.0395	0.0417	0.0436	0.0455	0.0471	0.0486
	$Th_1(t)$	1.5766	1.6004	1.6216	1.6403	1.6571	1.6719	1.6851
	$Th_2(t)$	1.6370	1.6827	1.7235	1.7601	1.7927	1.8218	1.8479
	$Th_3(t)$	1.5646	1.6235	1.6764	1.7240	1.7666	1.8047	1.8389
	$Th_4(t)$	1.3034	1.3838	1.4586	1.5277	1.5908	1.6483	1.7004
	$W_1(t)$	0.2357	0.2379	0.2401	0.2424	0.2446	0.2469	0.2492
	$W_2(t)$	0.0703	0.0705	0.0707	0.0709	0.0711	0.0713	0.0716
	$W_3(t)$	0.0413	0.0414	0.0414	0.0414	0.0415	0.0415	0.0415
	$W_4(t)$	0.0291	0.0292	0.0292	0.0292	0.0292	0.0293	0.0293

$\omega = 2$	$M_1(t)$	0.03380	0.3578	0.3777	0.3976	0.4175	0.4374	0.4572
	$M_2(t)$	0.1061	0.1123	0.1186	0.1248	0.1310	0.1373	0.,1435
	$M_3(t)$	0.0607	0.0643	0.0679	0.0715	0.0750	0.0786	0.0822
	$M_4(t)$	0.0379	0.0402	0.0424	0.0446	0.0469	0.0491	0.0513
	$E_1(t)$	0.2868	0.3008	0.3146	0.3281	0.3413	0.3543	0.3670
	$E_2(t)$	0.1007	0.1063	0.1118	0.1173	0.1228	0.1283	0.1337
	$E_3(t)$	0.0589	0.0623	0.0656	0.0690	0.0723	0.0756	0.0789
	$E_4(t)$	0.0372	0.0394	0.0415	0.0436	0.0458	0.0479	0.0500
	$Th_1(t)$	1.4339	1.5041	1.5729	1.6403	1.7065	1.7713	1.8349
	$Th_2(t)$	1.5099	1.5938	1.6772	1.7601	1.8424	1.9243	2.0056
	$Th_3(t)$	1.4732	1.5571	1.6407	1.7240	1.8070	1.8897	1.9721
	$Th_4(t)$	1.3028	1.3779	1.4529	1.5277	1.6023	1.6767	1.7510
	$W_1(t)$	0.2357	0.2379	0.2401	0.2424	0.2446	0.2469	0.2492
	$W_2(t)$	0.0703	0.0705	0.0707	0.0709	0.0711	0.0713	0.0716
	$W_3(t)$	0.0412	0.0413	0.0414	0.0414	0.0415	0.0416	0.0417
	$W_4(t)$	0.0291	0.0291	0.0292	0.0292	0.0292	0.0293	0.0293
	$M_1(t)$	0.3928	0.3944	0.3960	0.3976	0.3992	0.4009	0.4025
	$M_2(t)$	0.1255	0.1253	0.1250	0.1248	0.1246	0.1244	0.1241
	$M_3(t)$	0.0719	0.0717	0.0716	0.0714	0.0713	0.0711	0.0710
	$M_4(t)$	0.0451	0.0450	0.0448	0.0446	0.0445	0.0443	0.0441
	$E_1(t)$	0.3248	0.3259	0.3270	0.3281	0.3292	0.3303	0.3314

	$E_2(t)$	0.1179	0.1177	0.1175	0.1173	0.1171	0.1169	0.1167
	$E_3(t)$	0.0694	0.0692	0.0691	0.0690	0.0688	0.0687	0.0685
$\varepsilon = 0.1$	$\square_4(\square)$	0.0441	0.0440	0.0438	0.0436	0.0435	0.0433	0.0431
	$\square h_1(\square)$	1.6241	1.6295	1.6349	1.6403	1.6458	1.6513	1.6568
	$\square h_2(\square)$	1.7690	1.7661	1.7631	1.7601	1.7570	1.7540	1.7508
	$\square h_3(\square)$	1.7341	1.7308	1.7274	1.7240	1.7205	1.7170	1.7135
	$\square h_4(\square)$	1.5448	1.5392	1.5334	1.5277	1.5218	1.5159	1.5100
	$\square_1(\square)$	0.2418	0.2420	0.2422	0.2424	0.2426	0.2428	0.2429
	$\square_2(\square)$	0.0709	0.0709	0.0709	0.0709	0.0709	0.0709	0.0709
	$\square_3(\square)$	0.0415	0.0415	0.0414	0.0414	0.0414	0.0414	0.0414
	$\square_4(\square)$	0.0292	0.0292	0.0292	0.0292	0.0292	0.0292	0.0292
	$\square_1(\square)$	0.3976	0.3976	0.3976	0.3976	0.3976	0.3976	0.3976
	$\square_2(\square)$	0.1229	0.1236	0.1242	0.1248	0.1245	0.1261	0.1267
	$\square_3(\square)$	0.0716	0.0715	0.0715	0.0715	0.0714	0.0714	0.0713
	$\square_4(\square)$	0.0450	0.0449	0.0448	0.0446	0.0445	0.0443	0.0442
	$\square_1(\square)$	0.3281	0.3281	0.3281	0.3281	0.3281	0.3281	0.3281
	$\square_2(\square)$	0.1157	0.1162	0.1168	0.1173	0.1179	0.1185	0.1190
	$\square_3(\square)$	0.0691	0.0690	0.0690	0.0690	0.0689	0.0689	0.0688
$\square = 0.1$	$\square_4(\square)$	0.0440	0.0439	0.0438	0.0436	0.0435	0.0434	0.0432
	$\square h_1(\square)$	1.6403	1.6403	1.6403	1.6403	1.6403	1.6403	1.6403
	$\square h_2(\square)$	1.7353	1.7435	1.7517	1.7601	1.7685	1.7770	1.7855

	□h_3(□)	1.7270	1.7260	1.7250	1.7240	1.7229	1.7219	1.7208
	□h_4(□)	1.5413	1.5368	1.5323	1.5277	1.5230	1.5183	1.5135
	□$_1$(□)	0.2424	0.2424	0.2424	0.2424	0.2424	0.2424	0.2424
	□$_2$(□)	0.0708	0.0709	0.0709	0.0709	0.0709	0.0710	0.0710
	$W_3(t)$	0.0414	0.0414	0.0414	0.0414	0.0414	0.0414	0.0414
	$W_4(t)$	0.0292	0.0292	0.0292	0.0292	0.0292	0.0292	0.0292
	$M_1(t)$	0.3976	0.3976	0.3976	0.3976	0.3976	0.3976	0.3976
	$M_2(t)$	0.1248	0.1248	0.1248	0.1248	0.1248	0.1248	0.1248
	$M_3(t)$	0.0703	0.0707	0.0711	0.0715	0.0718	0.0722	0.0726
	$M_4(t)$	0.0443	0.0444	0.0445	0.0446	0.0447	0.0449	0.0450
□ = 0.1	$E_1(t)$	0.3281	0.3281	0.3281	0.3281	0.3281	0.3281	0.3281
	$E_2(t)$	0.1173	0.1173	0.1173	0.1173	0.1173	0.1173	0.1173
	$E_3(t)$	0.0679	0.0683	0.0686	0.0690	0.0693	0.0697	0.0700
	$E_4(t)$	0.0433	0.0434	0.0435	0.0436	0.0438	0.0439	0.0440
	$Th_1(t)$	1.6403	1.6403	1.6403	1.6403	1.6403	1.6403	1.6403
	$Th_2(t)$	1.7601	1.7601	1.7601	1.7601	1.7601	1.7601	1.7601
	$Th_3(t)$	1.6983	1.7068	1.7153	1.7240	1.7327	1.7415	1.7505
	$Th_4(t)$	1.5162	1.5200	1.5238	1.5277	1.5315	1.5353	1.5391
	$W_1(t)$	0.2424	0.2424	0.2424	0.2424	0.2424	0.2424	0.2424
	$W_2(t)$	0.0709	0.0709	0.0709	0.0709	0.0709	0.0709	0.0709
	$W_3(t)$	0.0414	0.0414	0.0414	0.0414	0.0415	0.0415	0.0415

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	$W_4(t)$	0.0292	0.0292	0.0292	0.0292	0.0292	0.0292	0.0292
$\vartheta = 0.1$	$M_1(t)$	0.3976	0.3976	0.3976	0.3976	0.3976	0.3976	0.3976
	$M_2(t)$	0.1248	0.1248	0.1248	0.1248	0.1248	0.1248	0.1248
	$M_3(t)$	0.0715	0.0715	0.0715	0.0715	0.0715	0.0715	0.0715
	$M_4(t)$	0.0440	0.0442	0.0444	0.0446	0.0449	0.0451	0.0453
	$E_1(t)$	0.3281	0.3281	0.3281	0.3281	0.3281	0.3281	0.3281
	$E_2(t)$	0.1173	0.1173	0.1173	0.1173	0.1173	0.1173	0.1173
	$E_3(t)$	0.0690	0.0690	0.0690	0.0690	0.0690	0.0690	0.0690
	$E_4(t)$	0.0430	0.0432	0.0434	0.0436	0.0439	0.0441	0.0443
	$Th_1(t)$	1.6403	1.6403	1.6403	1.6403	1.6403	1.6403	1.6403
	$Th_2(t)$	1.7601	1.7601	1.7601	1.7601	1.7601	1.7601	1.7601
	$Th_3(t)$	1.7240	1.7240	1.7240	1.7240	1.7240	1.7240	1.7240
	$Th_4(t)$	1.5053	1.5727	1.5201	1.5277	1.5353	1.5430	1.5507
	$W_1(t)$	0.2424	0.2424	0.2424	0.2424	0.2424	0.2424	0.2424
	$W_2(t)$	0.0709	0.2424	0.2424	0.2424	0.2424	0.2424	0.2424
	$W_3(t)$	0.0414	0.2424	0.2424	0.2424	0.2424	0.2424	0.2424
	$W_4(t)$	0.0292	0.0292	0.0292	0.0292	0.0292	0.0292	0.0292

6. CONCLUSION

This paper deals with the development and analysis of communication network model with dynamic bandwidth allocation having feedback. The dynamic bandwidth allocation is adapted by instantaneous adjustment of packet service time by utilizing idle bandwidth in the transmitter. A numerical study reveals the that this communication model is capable of predicting the performance measures of the network like mean delays, average content of the buffers, throughput of the transmitters etc, explicitly. It is observed that the feedback probability parameters $(\varepsilon, \tau, \varphi, \vartheta)$ have significant influence on the overall performance of the network. The sensitivity analysis of the network reveals that the dynamic bandwidth allocation strategy can reduce the congestion in buffers and meandelay in transmission. The communication network model is much useful in analyzing the performance of several communication network at Tele and Satellite communications, Computer communications, ATM scheduling, Bandwidth allocation etc.

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