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Stability Analysis of Tuberculosis Due to Smoking

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Abstract—In this paper, a mathematical model for the analysis of tuberculosis due to smoking has been developed as a system of non-linear ordinary differential equations. To get cured of this disease a medication is necessary for the suffering individuals. After taking medications some people try to adapt the path of giving up smoking and helps in making society free of smoking. For this, smoking free equilibrium point and smoking existence equilibrium point has been found. Basic reproduction number has been calculated at smoking free equilibrium point which will give us an approximate idea of an individuals who are victim of it in our society. Stability analysis has been carried out at both the equilibrium points. Simulation has been carried out to support the analytical results.

Keywords—Smoking, Tuberculosis, Basic Reproduction Number, Medication, Local stability, Global Stability.

I. INTRODUCTION

From the one set of tobacco, since 6000 B.C. smoking has become a most vital public health hazard all over the world [15]. Smoking is the only legal consumer product that kills you. Tobacco is used to make cigarette. Drug called nicotine is obtained from only the tobacco plant. If this drug is injected into the blood stream in a very minute amount also, it can kill human in less than an hour. It is highly poisonous. Smell and colour of cigarette smoke is because of 'tar'. It is highly sticky as it sticks to clothing, inside part of lungs, skin etc. Tar is too dangerous especially inside lungs. It sticks to the cilia-who are responsible for the cleanliness of lungs, so they are not able to clean out germs and dirt because of it and hence leads to lungs disease. Other than this nicotine and tar are responsible for many diseases like tuberculosis, lung cancer, throat cancer, mouth cancer, heart disease etc. are caused by smoking. Smoking spoils important organs of the body, but the organ which is badly affected are lungs [8].

Lots of people suffers from tuberculosis only because of smoking. Individuals must take care of the following principles for the treatment of disease: treatment must make use of multiple drugs due to which Mycobacterium Tuberculosis are susceptible, medication must be taken regularly, the treatment must be continued until the disease resolves completely. The aim for TB treatment is to reduce the clinical source of TB, prevent complications, prevent the development of latenery recurrences and decrease the transmission of TB, to prevent in disease progression [9].

Taking medication will help person to stop smoking. Medicines like nicotine replacement medicine, nicotine chewing gum, nicotine patch, nicotine spray, nicotine nasal etc. can help individual to quit smoking [7], [11]. Quit to smoke reduces health risks. If a person stops to smoke, their health and body will start to recover. It is very difficult for the individuals who starts to smoke once. Then gradually it becomes a habit. But there are individuals who make tremendous choice of quitting smoking on their own. One must always encourage the smokers to give up from this rigorous habit. A person can only become successful in quitting, if he/she gives a beat for cravings and prompts for it. The reason why the smokers feel tough to quit and seems they need cigarette for days/week/months but when tries to quit they feel sick due to the amount of nicotine in their brain drops. Soon the individuals will quit, faster they will become healthy [10]Awan *et.al.*(2017) [2] has developed a mathematical model entitled "Smoking model with cravings to smoke" in which they have studied the smoking dynamics in a population with the effect of powerful cravings for smoking on temporary quitters.

In this paper, we will analyze the stability for one of the disease caused due to smoking which is tuberculosis using an application of *SEIR* model. Section 2 consists of discussion of mathematical model, notations with its description and parametric values. Stability at both the points i.e. smoking free equilibrium point and smoking existence equilibrium point are calculated in Section 3. Simulation with their interpretation and conclusion are discussed in Section 4 and 5 respectively.

II. MATHEMATICAL MODEL

Here, we formulate a mathematical model for the occurrence of tuberculosis due to smoking using *SEIR* model. The notations along with its description and parametric values are given in below Table1.

Notations	Parameters description	Parametric Values
$S(t)$	Number of smokers at any instant of time t	100
$T_B(t)$	Number of individuals suffering from tuberculosis at any instant of time t	40
$M(t)$	Number of individuals going for medication at any instant of time t	35

$Q(t)$	Number of individuals who quits themselves from smoking at any instant of time t	15
B	New Recruitment	0.15
β	Transmission rate of smokers who gets affected of tuberculosis	0.8
η	Rate at which the individuals who gets affected of tuberculosis again during medication	0.05
γ	Rate at which the individuals goes for medication due to suffering of tuberculosis	0.9
δ	Rate at which the individuals who starts smoking again during medication	0.10
α	Transmission rate of individuals who gives up smoking after taking medication	0.07
θ	Rate of individuals who directly quits smoking on their own at some instant of time without moving into any of the compartments	0.15
μ	Mortality Rate	0.35

Table 1: Notations and its Prametric Values

Let $N(t)$ denotes the sample size of total human population at any instant of time t. Here, $N(t)$ is divided into four compartments $S(t), T_B(t), M(t)$ and $Q(t)$ which are individually described in above Table1. Thus, $N(t)=S(t)+T_B(t)+M(t)+Q(t)$. The transmission diagram for the proposed problem is shown in figure 1.

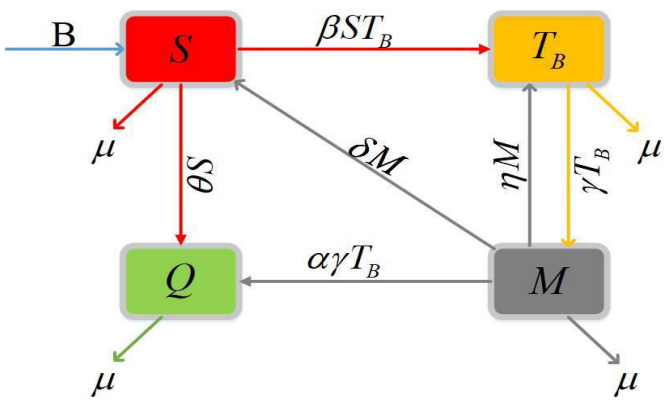

Fig. 1. Motion of Smokers from one Compartment to Other

Persons who smoke (S) for long time get affected of tuberculosis (T_B) at some stage at the rate β. Then he/she has to go for medication as precaution to get cure of this diseases which is described by the rate γ. Individuals who takes medication is not necessary that he will be completely cured from the diseases, it happens sometimes that some symptoms still prevail and a person gets affected of tuberculosis again which is mentioned by the rate η. Also, it happens that a person starts smoking while taking of medication due to his/her habit or once he/ she seems that the disease has started to cure, this is described by the rate δ. Rate α and θ is described for the person who quits smoking after taking medication or directly on his own without suffering from tuberculosis etc. respectively.Now, from the above figure 1 a set of non-linear differential equations has been constructed as a motion of smokers from one compartment to another.

$$\begin{aligned}
\frac{dS}{dt} &= \mathrm{B}-\beta ST_B+\delta M-\theta S-\mu S\\
\frac{dT_B}{dt} &= \beta ST_B+\eta M-(\gamma+\mu)T_B\\
\frac{dM}{dt} &= \gamma(1-\alpha)T_B-(\delta+\eta+\mu)M\\
\frac{dQ}{dt} &= \theta S-\alpha\gamma T_B-\mu Q
\end{aligned} \qquad (1)$$

with $N=S+T_B+M+Q$ and $S>0, T_B\geq 0, M\geq 0, Q\geq 0$

In system of equation (1), $N(t)$ is constant so we assume that $S(t)+T_B(t)+M(t)+Q(t)=1$. Also, as the variable Q does not appear in any of the first three equations from the set of equations (1) we consider the following subsystem of equations

$$\begin{aligned}
\frac{dS}{dt} &= \mathrm{B}-\beta ST_B+\delta M-\theta S-\mu S\\
\frac{dT_B}{dt} &= \beta ST_B+\eta M-(\gamma+\mu)T_B\\
\frac{dM}{dt} &= \gamma(1-\alpha)T_B-(\delta+\eta+\mu)M
\end{aligned} \qquad (2)$$

On adding the above set of equations (2) we get

$$\frac{d}{dt}(S+T_B+M)=\mathrm{B}-\mu(S+T_B+M)-\theta S-\alpha T_B\geq 0$$

This, gives $\limsup_{t\to\infty}(S+T_B+M)\leq \dfrac{\mathrm{B}}{\theta+\mu}$

So, the feasible region for (2) is

$$\Lambda=\left\{(S,T_B,M): S+T_B+M\leq \frac{\mathrm{B}}{\theta+\mu}, S>0, T_B\geq 0, M\geq 0\right\}$$

Thus, smoking free equilibrium of system (2) is $E_0=\left(\dfrac{\mathrm{B}}{\theta+\mu},0,0\right)$.

Now, we are interested in calculating the basic reproduction number which is to be calculated using next generation matrix method [3], [4], [5], [14]. The next generation matrix method is defined as FV^{-1} where F and V both are Jacobian matrices of $\Im$ and v evaluated with respect to the individuals suffering

from tuberculosis (T_B) and the one going for medication (M) at the point E_0.

Let $X = (T_B, M, S)$

$$\therefore \frac{dX}{dt} = \Im(X) - v(X)$$

where $\Im(X)$ denotes the rate of new smokers and $v(X)$ denotes the rate of transfer of smoking which is given as

$$\Im(X) = \begin{bmatrix} \beta S T_B \\ 0 \\ 0 \end{bmatrix} \text{ and } v(X) = \begin{bmatrix} (\gamma+\mu)T_B - \eta M \\ (\eta+\mu)M - \gamma(1-\alpha)T_B + \delta M \\ -\mathrm{B} + \beta S T_B - \delta M + (\theta+\mu)S \end{bmatrix}$$

Now, the derivative of $\Im$ and v at smoking free equilibrium point E_0 gives matrices F and V of order 3×3 defined as

$$F = \left[\frac{\partial \Im_i(E_0)}{\partial X_j}\right] \quad V = \left[\frac{\partial v_i(E_0)}{\partial X_j}\right] \text{ for } i, j = 1,2,3$$

Hence, $F = \begin{bmatrix} \frac{\beta \mathrm{B}}{\theta+\mu} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ and

$$V = \begin{bmatrix} (\gamma+\mu) & -\eta & 0 \\ -\gamma(1-\alpha) & \eta+\mu+\delta & 0 \\ \frac{\beta \mathrm{B}}{\theta+\mu} & -\delta & \theta+\mu \end{bmatrix}$$

where V is non-singular matrix. Thus, the basic reproduction number R_0 which is the spectral radius of matrix FV^{-1} is given as

$$R_0 = \frac{\beta \mathrm{B}(\eta+\mu+\delta)}{(\theta+\mu)(\alpha\eta\gamma + \delta\gamma + \delta\mu + \eta\mu + \gamma\mu + \mu^2)}$$

On equating the set of equations (2) equal to zero, an endemic equilibrium point defined as smoking present equilibrium point (E^*) is obtained which is as follows:

Smoking present equilibrium is $E^* = (S^*, T_B^*, M^*)$ where

$$S^* = \frac{\mathrm{B} + \delta M^*}{\theta + \mu + \beta T_B^*}, \; M^* = \frac{\gamma(1-\alpha)T_B^*}{\eta+\mu+\delta},$$

$$T_B^* = \frac{\mathrm{B}(\eta+\mu+\delta)(R_0-1)}{R_0\left[\mu^2 + \mu(\eta+\gamma+\delta) + \alpha\gamma(\eta+\delta)\right]}$$

III. STABILIY ANALYSIS

In this section, the local and global stability at E_0 and E^* using the linearization method and matrix analysis are to be studied.

A. *Local Stability*

Theorem 3.1.1: (stability of E_0) If $R_0 < 1$ and $R_0 < \frac{\mu}{\theta+\mu}$ then the smoking free equilibrium point E_0 is locally asymptotically stable.

Proof: At point E_0, the Jacobian matrix of the system (2) is

$$J(E_0) = \begin{bmatrix} -(\theta+\mu) & -\frac{\beta \mathrm{B}}{\mu} & \delta \\ 0 & \frac{\beta \mathrm{B}}{\mu} - (\gamma+\mu) & \eta \\ 0 & \gamma(1-\alpha) & -(\delta+\eta+\mu) \end{bmatrix}$$

The characteristic polynomial for the above matrix is

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0$$

where

$a_1 = \frac{1}{\mu}\left[\eta\mu + 3\mu^2 + \delta\mu + \mu\theta + \gamma\mu - \beta \mathrm{B}\right] > 0$ which is obvious.

$$a_2 = \frac{1}{\mu}\begin{pmatrix} \alpha\eta\gamma\mu - \beta \mathrm{B}\delta - \beta \mathrm{B}\eta - 2\mu \mathrm{B}\beta - \mathrm{B}\beta\theta + \delta\gamma\mu \\ +2\delta\mu^2 + \delta\mu\theta + 2\eta\mu^2 + \eta\mu\theta + 2\gamma\mu^2 + \gamma\mu\theta + 3\mu^3 + 2\mu^2\theta \end{pmatrix}$$

$$= \frac{1}{\mu}\begin{bmatrix} (\theta+\mu)(\alpha\eta\gamma + \delta\gamma + 2\delta\mu + \eta\mu + \gamma\mu + \mu^2)\mu + \delta\mu + \eta\mu \\ +\gamma\mu + \mu\theta + 2\mu^2 - \mu \mathrm{B}\beta - \beta \mathrm{B}(\delta+\eta+\mu) \end{bmatrix}$$

$$= \frac{1}{\mu}\begin{bmatrix} (\theta+\mu)(\alpha\eta\gamma + \delta\gamma + 2\delta\mu + \eta\mu + \gamma\mu + \mu^2)\mu \\ +(\delta+\eta+\gamma+\theta+2\mu - \mathrm{B}\beta)\mu - \beta \mathrm{B}(\delta+\eta+\mu) \end{bmatrix}$$

$$= (\theta+\mu)(\alpha\eta\gamma + \delta\gamma + 2\delta\mu + \eta\mu + \gamma\mu + \mu^2)(1-R_0) + (\delta+\eta+\gamma+\theta+2\mu - \mathrm{B}\beta)$$

If $R_0 < 1$ then $a_2 > 0$.

$$a_3 = \frac{1}{\mu}\begin{bmatrix} \alpha\eta\gamma\mu^2 + \alpha\eta\gamma\mu\theta - \mathrm{B}\beta\delta\mu - \mathrm{B}\beta\delta\theta \\ -\mathrm{B}\beta\eta\mu - \mathrm{B}\beta\eta\theta - \mathrm{B}\beta\mu^2 - \mathrm{B}\beta\mu\theta \\ +\delta\gamma\mu^2 + \delta\gamma\mu\theta + \delta\mu^3 + \delta\mu^2\theta + \eta\mu^3 \\ +\eta\mu^2\theta + \gamma\mu^3 + \gamma\mu^2\theta + \mu^4 + \mu^3\theta \end{bmatrix}$$

$$= \frac{1}{\mu}\begin{bmatrix} \mu(\theta+\mu)\{\alpha\eta\gamma + \delta\gamma + \delta\mu + \eta\mu + \gamma\mu + \mu^2\} \\ -\mathrm{B}\beta(\mu+\theta)\{\delta+\eta+\mu\} \end{bmatrix}$$

$$= (\theta+\mu)\{\alpha\eta\gamma + \delta\gamma + \delta\mu + \eta\mu + \gamma\mu + \mu^2\} - \frac{\mathrm{B}\beta(\mu+\theta)\{\delta+\eta+\mu\}}{\mu}$$

$$= (\theta+\mu)\{\alpha\eta\gamma + \delta\gamma + \delta\mu + \eta\mu + \gamma\mu + \mu^2\}\left[1 - \frac{\mathrm{B}\beta(\mu+\theta)\{\delta+\eta+\mu\}}{\mu(\theta+\mu)\{\alpha\eta\gamma + \delta\gamma + \delta\mu + \eta\mu + \gamma\mu + \mu^2\}}\right]$$

$$= (\theta+\mu)\{\alpha\eta\gamma + \delta\gamma + \delta\mu + \eta\mu + \gamma\mu + \mu^2\}\left[1 - \frac{R_0(\theta+\mu)}{\mu}\right]$$

$$= \frac{(\theta+\mu)^2\{\alpha\eta\gamma + \delta\gamma + \delta\mu + \eta\mu + \gamma\mu + \mu^2\}}{\mu}\left[\frac{\mu}{(\theta+\mu)} - R_0\right]$$

If $R_0 < \frac{\mu}{\theta+\mu}$ then $a_3 > 0$.

Also,

$$a_1a_2 - a_3$$

$$= \left[\frac{1}{\mu}\left(\eta\mu + 3\mu^2 + \delta\mu + \mu\theta + \gamma\mu - \beta\mathrm{B}\right)\right]\begin{bmatrix} (\theta+\mu)(\alpha\eta\gamma + \delta\gamma + 2\delta\mu + \eta\mu + \gamma\mu + \mu^2)(1-R_0) \\ +(\delta+\eta+\gamma+\theta+2\mu-\mathrm{B}\beta) \end{bmatrix} - \left[\frac{(\theta+\mu)^2\{\alpha\eta\gamma + \delta\gamma + \delta\mu + \eta\mu + \gamma\mu + \mu^2\}}{\mu}\left[\frac{\mu}{(\theta+\mu)} - R_0\right]\right] > 0$$

$\therefore$ If $R_0 < 1$ and $R_0 < \frac{\mu}{\theta+\mu}$ then $a_1a_2 - a_3 > 0$.

$\therefore$ By Routh Hurwitz criteria [6], if $R_0 < 1$ and $R_0 < \frac{\mu}{\theta+\mu}$ then $a_1 > 0$, $a_3 > 0$ and $a_1a_2 - a_3 > 0$ which proves that E_0 is locally asymptotically stable.

Lemma 3.1.2: (stability of E^*) Let K be a real matrix of order 3×3. If $tr(K)$, $\det(K)$ and $\det(K^{[2]}) < 0$ then all the eigen values of the matrix K have negative real parts.

Proof: On linearizing the set of equations (2) at point $E^* = (S^*, T_B^*, M^*)$ the Jacobian matrix of the system (2) is obtained as follows:

$$J(E^*) = \begin{bmatrix} -(\theta+\mu+\beta T_B^*) & -\beta S^* & \delta \\ \beta T_B^* & -\frac{\eta M^*}{T_B^*} & \eta \\ 0 & \gamma(1-\alpha) & -(\delta+\eta+\mu) \end{bmatrix}$$

$$\therefore trace\left(J\left(E^*\right)\right) = -\beta T_B^* - 2\mu - \theta - \frac{\eta R^*}{T_B^*} - \eta - \delta < 0$$

$$\det\left(J\left(E^*\right)\right) = -\frac{1}{T_B^*}\begin{bmatrix} ST_B^{*2}\beta^2(\delta+\eta+\mu) + M^*T_B^*\beta\eta(\delta+\eta+\mu) \\ -T_B^{*2}\beta\delta\gamma(1-\alpha) - T_B^{*2}\beta\eta\gamma(1-\alpha) \\ -T_B^*\eta\gamma\mu(1-\alpha) - T_B^*\eta\gamma\theta(1-\alpha) \\ +M^*\eta\mu(\delta+\eta+\mu) + M^*\eta\theta(\delta+\eta+\mu) \end{bmatrix}$$

$$= -\frac{\eta M^*(\theta+\mu)(\delta+\eta+\mu)}{T_B^*} + \eta\gamma\mu(1-\alpha) - \eta\gamma\theta(1-\alpha) - \eta M^*\beta(\delta+\eta+\mu) + \eta T_B^*\beta\gamma(1-\alpha) + T_B^*\beta\delta\gamma(1-\alpha) - \beta^2S^*T_B^*(\delta+\eta+\mu)$$

$$= -\beta^2S^*T_B^*(\delta+\eta+\mu) - \eta T_B^*\beta\gamma(1-\alpha) + T_B^*\beta\delta(\delta+\eta+\mu) + \eta T_B^*\beta\gamma(1-\alpha) + \gamma(1-\alpha)\eta(\theta+\mu) - \gamma(1-\alpha)\eta(\theta+\mu)$$

$$= \beta(\delta+\eta+\mu)\left[-\beta S^*T_B^* + \delta M^*\right] < 0$$

Now, the second additive compound matrix of $J\left(E^*\right)$ [1], [13] which is given by $J^{[2]}\left(E^*\right)$ is as follows:

$$J^{[2]}\left(E^*\right) = \begin{bmatrix} -\left(\theta+\mu+\beta T_B^* + \frac{\eta M^*}{T_B^*}\right) & \eta & -\delta \\ \gamma(1-\alpha) & -\left(\theta+\delta+\eta+2\mu+\beta T_B^*\right) & -\beta S^* \\ 0 & \beta T_B^* & -\left(\frac{\eta M^*}{T_B^*} + \delta + \eta + \mu\right) \end{bmatrix}$$

$$\det\left(J^{[2]}\left(E^*\right)\right)=\left(\frac{\eta M^*}{T_B^*}+\delta+\eta+\mu\right)\left[\begin{array}{l}-\left(\theta+\mu+\beta T_B^*+\frac{\eta M^*}{T_B^*}\right)\\ \left(\theta+\delta+\eta+2\mu+\beta T_B^*\right)-\eta\gamma(1-\alpha)\end{array}\right]$$
$$-\beta T_B^*\delta\gamma(1-\alpha)-\beta^2 S^* T_B^*$$
$$\left(\theta+\mu+\beta T_B^*+\frac{\eta M^*}{T_B^*}\right)$$
$$=-\left(\frac{\eta M^*}{T_B^*}+\delta+\eta+\mu\right)$$
$$\left[\begin{array}{l}\left(\theta+\delta+\eta+2\mu\right)\left(\theta+\mu+\beta T_B^*\right)\left(\frac{\eta M^*}{T_B^*}\right)\\ +\left(\theta+\mu+\beta T_B^*\right)\beta T_B^*+\eta\beta M^*\\ +\eta\gamma(1-\alpha)+\beta T_B^*\delta\gamma(1-\alpha)\end{array}\right]$$
$$-\left[\beta^2 S^* T_B^*\left(\theta+\mu+\beta T_B^*+\frac{\eta M^*}{T_B^*}\right)\right]$$
$$<0$$

Hence, E^* is locally asymptotically stable by above lemma.

B. Global Stability

Theorem 3.2.1: (stability of E_0) If $\beta\le\gamma\alpha$ then E_0 is globally asymptotically stable.

Proof: Consider the Lyapunov function

$$L(t)=T_B(t)+M(t)$$
$$\therefore\frac{dL}{dt}=-(\gamma+\mu)T_B+\beta S T_B+\eta M+\gamma(1-\alpha)T_B-(\eta+\mu)M-\delta M$$
$$=-\gamma T_B-\mu T_B+\beta S T_B+\eta M+\gamma T_B-\gamma\alpha T_B-\eta M-\mu M-\delta M$$
$$=-\mu(T_B+M)+T_B(\beta S-\gamma\alpha)-\delta M$$
$$\le 0$$

We have $\frac{dL}{dt}<0$ for $\beta S\le\gamma\alpha$

But we have noted that $S<1$ so $\frac{dL}{dt}<0$ for $\beta\le\gamma\alpha$ and $\frac{dL}{dt}=0$ when $T_B=M=0$.

$\therefore$ By LaSalle's Invariance Principle [12], E_0 isglobally asymptotically stable.

Theorem 3.2.2: (stability of E^*) Consider a piecewise smooth vector field $g(S,T_B,M)=\{g_1(S,T_B,M),g_2(S,T_B,M),g_3(S,T_B,M)\}$ on Λ^* that satisfies the condition $(Curl)\cdot\vec{n}<0$, $g\cdot f=0$ inside Λ^*,where $f=(f_1,f_2,f_3)$ is a Lipschitz continuous field inside Λ^*, $\vec{n}$ is a normal vector to Λ^* and

$$Curl\ g=\left(\frac{\partial g_3}{\partial T_B}-\frac{\partial g_2}{\partial M}\right)\hat{i}-\left(\frac{\partial g_3}{\partial S}-\frac{\partial g_1}{\partial M}\right)\hat{j}+\left(\frac{\partial g_2}{\partial S}-\frac{\partial g_1}{\partial T_B}\right)\hat{k}\ .$$

Then, the system of differential equations $S=f_1,T_B=f_2,M=f_3$ has no homoclinic loops, periodic solutions and oriented phase polygons inside Λ^*.

Proof: Suppose

$$\Lambda^*=\left\{(S,T_B,M):(1+\theta)S+\left(\frac{\mu+\alpha\gamma}{\mu}\right)T_B+M=1,S>0,T_B\ge 0,M\ge 0\right\}$$

Also, it can easily be proved that Λ^* is subset of Λ, Λ^* is positively invariant and endemic equilibrium E^* belongs to Λ^*. Let f_1,f_2 and f_3 represents the right-hand side of equations in set of equations (2) respectively. Using $(1+\theta)S+\left(\frac{\mu+\alpha\gamma}{\mu}\right)T_B+M=1$ to write f_1,f_2 and f_3 in the equivalent forms, we get

$$f_1(S,T_B)=\mathrm{B}-\beta S T_B+\delta M-(\theta+\mu)S$$
$$f_1(S,M)=\mathrm{B}-\beta S\left[\left(1-M-(1+\theta)S\right)\left(\frac{\mu}{\mu+\alpha\gamma}\right)\right]+\delta M-(\theta+\mu)S$$
$$f_2(S,T_B)=\beta S T_B+\eta M-(\gamma+\mu)T_B$$
$$=\beta S T_B-(\gamma+\mu)T_B+\eta\left[1-(1+\theta)S-\left(\frac{\mu+\alpha\gamma}{\mu}\right)T_B\right]$$
$$f_2(T_B,M)=\beta S T_B+\eta M-(\gamma+\mu)T_B$$
$$=\frac{\beta T_B}{1+\theta}\left[1-M-\left(\frac{\mu+\alpha\gamma}{\mu}\right)T_B\right]+\eta M-(\gamma+\mu)T_B$$
$$f_3(S,M)=\gamma T_B(1-\alpha)-(\mu+\eta)M-\delta M$$
$$=\gamma(1-\alpha)\left[\left(1-M-(1+\theta)S\right)\left(\frac{\mu}{\mu+\alpha\gamma}\right)\right]-(\mu+\eta+\delta)M$$
$$f_3(I,R)=\gamma T_B(1-\alpha)-(\mu+\eta+\delta)M$$

Suppose $g=(g_1,g_2,g_3)$ be a vector field such that

$$g_1 = \frac{f_3(S,M)}{SM} - \frac{f_2(S,T_B)}{ST_B}$$
$$= -\frac{\eta}{S} + \frac{\gamma(1-\alpha)\mu}{SM(\mu+\alpha\gamma)} - \frac{\gamma(1-\alpha)\mu}{S(\mu+\alpha\gamma)}$$
$$-\frac{\gamma(1-\alpha)\mu(1+\theta)}{M(\mu+\alpha\gamma)} - \frac{\delta}{S} + \frac{\gamma}{S} - \beta$$
$$-\frac{\eta}{ST_B} - \frac{\eta(1+\theta)}{T_B} + \frac{\eta}{S}\left(\frac{\mu+\alpha\gamma}{\mu}\right)$$

$$g_2 = \frac{f_1(S,T_B)}{ST_B} - \frac{f_3(T_B,M)}{T_BM}$$
$$= \frac{\mathrm{B}}{ST_B} - \beta + \frac{\delta M}{ST_B}$$
$$+\frac{\eta+\mu+\delta}{T_B} - \frac{\gamma(1-\alpha)}{M} + \frac{\delta M}{ST_B}$$

$$g_3 = \frac{f_2(T_B,M)}{T_BM} - \frac{f_1(S,M)}{SM}$$
$$= -\frac{\gamma+\mu}{M} + \frac{\beta}{(1+\theta)M} - \frac{\beta}{(1+\theta)}$$
$$-\frac{\beta T_B}{M(1+\theta)}\left(\frac{\mu+\alpha\gamma}{\mu}\right) + \frac{\eta}{T_B} - \frac{\mathrm{B}}{SM} - \frac{\delta}{S}$$
$$+\frac{\beta}{M}\left(\frac{\mu}{\mu+\alpha\gamma}\right) - \beta\left(\frac{\mu}{\mu+\alpha\gamma}\right)$$
$$-\frac{\beta S\mu}{M}\left(\frac{\mu}{\mu+\alpha\gamma}\right) + \frac{(\theta+\mu)}{M}$$

As the alternate form of f_1, f_2 and f_3 are equivalent

$$g \cdot f = g_1 f_1 + g_2 f_2 + g_3 f_3 = 0$$

Normal vector $\vec{n} = \left(1+\theta, \frac{\mu+\alpha\gamma}{\mu}, 1\right)$

$$Curl\ \underset{g}{\mathrm{r}} = \left[\frac{-\eta}{T_B^2} - \frac{\delta}{ST_B} - \frac{\beta}{M(1+\theta)}\left(\frac{\mu+\alpha\gamma}{\mu}\right)\right]\hat{i}$$
$$-\left[\begin{array}{l}\frac{\mathrm{B}}{S^2M} + \frac{\delta}{S^2} - \frac{\beta(1+\theta)}{M}\left(\frac{\mu}{\mu+\alpha\gamma}\right) \\ -\frac{\gamma(1-\alpha)}{SM^2}\left(\frac{\mu}{\mu+\alpha\gamma}\right) + \frac{\gamma(1-\alpha)(1+\theta)}{M^2}\left(\frac{\mu}{\mu+\alpha\gamma}\right)\end{array}\right]\hat{j}$$
$$+\left[-\frac{\mathrm{B}}{S^2T_B} - \frac{\delta M}{S^2T_B} - \frac{\eta}{ST_B^2} + \frac{\eta(1+\theta)}{T_B^2}\right]\hat{k}$$

$$(Curl\ \underset{g}{\mathrm{r}}) \cdot \underset{n}{\mathrm{r}} = -\frac{\delta(1+\theta)}{ST_B} - \frac{\beta}{M}\left(\frac{\mu+\alpha\gamma}{\mu} - (1+\theta)\right)$$
$$-\frac{\delta(\mu+\alpha\gamma)}{S^2\mu} - \frac{\gamma(1-\alpha)(S(1+\theta)-1)}{SM^2}$$
$$-\frac{\mathrm{B}}{S^2T_B} - \frac{\delta M}{S^2T_B} - \frac{\eta}{ST_B^2} - \frac{2\eta(1+\theta)}{T_B^2}$$
$$< 0$$

So, the system (2) has no homoclinic loops, periodic solutions and oriented phase polygons in the interior of Λ^*.

$\therefore E^*$ is globally asymptotically stable in the interior of Λ^*.

IV. NUMERICAL SIMULATION

In this section, numerical results with their interpretation are discussed which will help us to know the behavior of individuals in each compartment.

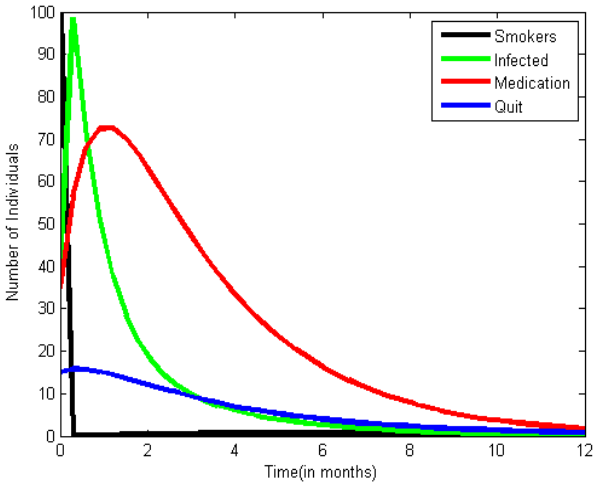

Fig. 2. Motion of Individuals in all Compartments

Figure 2 shows that as the more number of smokers gets affected from tuberculosis simultaneously these large number of infected individuals have to go for medication which is also increasing and after some time both infected and medicated people decreases which shows that they are on the way to quit.

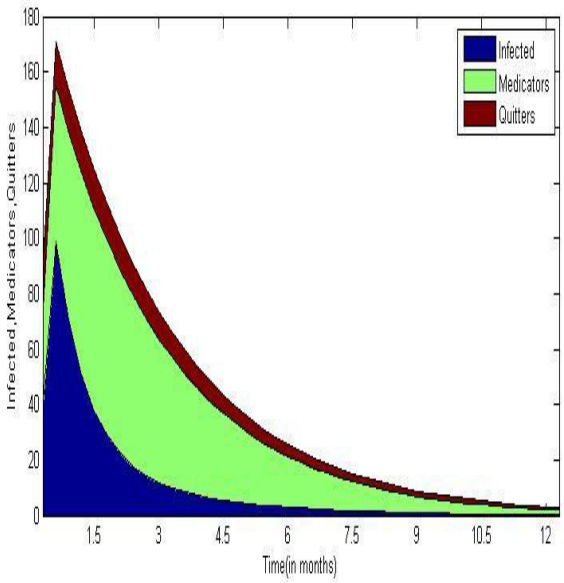

Fig. 3. Motion of Infected, Medication, Quitters in the Compartment

Figure 3 shows that the region acquired by the individuals opting for medication after suffering from tuberculosis is more which is essential to get cure from it whereas there are in all less number of individuals who opts for quitting.

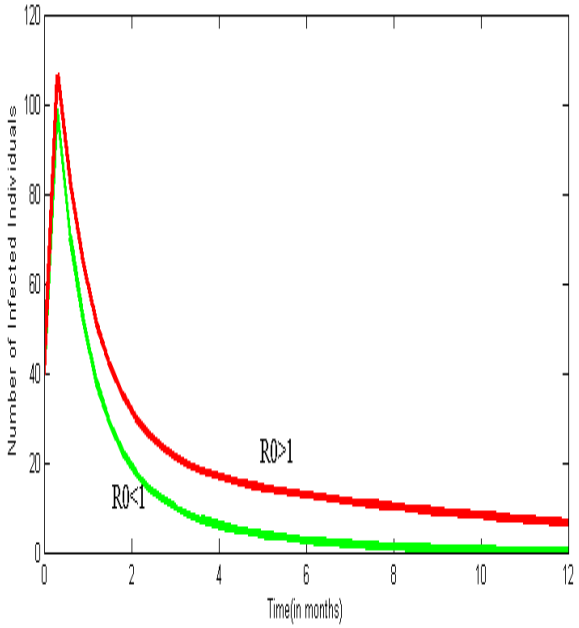

Fig. 4: Motion of Individuals Suffering from Tuberculosis due to Change in R_0

Figure 4 shows that increase in reproduction number increases the infected individuals, equivalently infected individuals spread tuberculosis at a higher rate.

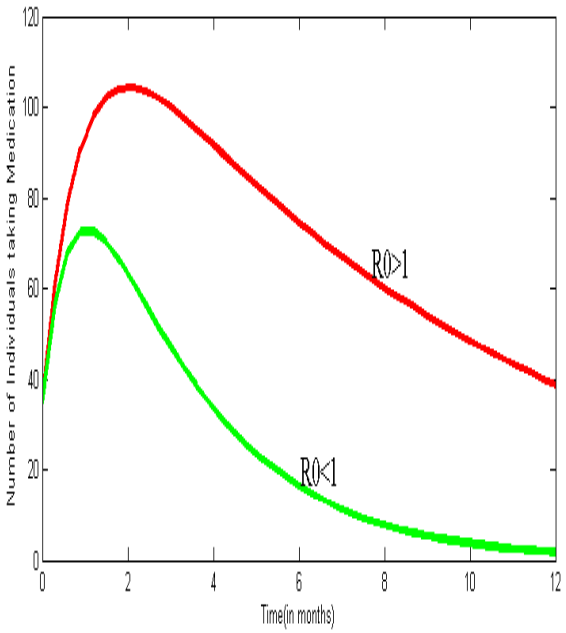

Fig. 5. Motion of Indidviduals Going for Medication for Change in R_0

Figure 5 shows that for both the values of threshold i.e. for threshold less than unity and greater than unity, the number of individuals going for medication increases initially for few time, but it decreases with the passage of time which shows that they have understood the harmfulness of smoking and tries to get rid of it.

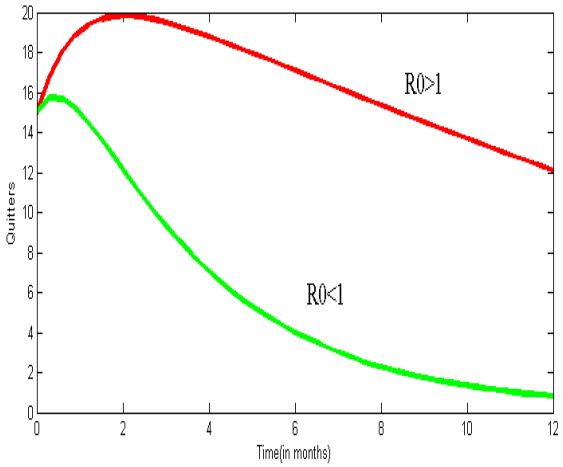

Fig. 6. Motion of Quitters for Change in R_0

Figure 6 shows that for $R_0 > 1$ the number of quitters increases in comparison to $R_0 < 1$ which shows that these increased number of individuals pushes more and more quitters to quit from smoking.

V. CONCLUSION

In this paper, a smoking dynamical system has been constructed to know the various stage faced by a smoker. An individual who smokes continuously will suffer from tuberculosis at some stage in their journey of life. We have tried to analyze that how the smoker can quit themselves from their habit due to their illness using mathematical model. Using the parametric values, 20% of the individuals still persists in the society who suffers from tuberculosis only because of smoking. Medication is the only step to get cured from the disease. Local stability and global stability conditions has also been achieved for smoking free and smoking existence equilibrium points. Analytical results show that it is better to quite themselves from the beginning rather to become victim of the disease and then quit. As this smoking related issues has spread vitally among the young generations, it is advisable to make a ban on the cigarette by the government so as to make the society healthy.

VI. ACKNOWLEDGMENT

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