ISSN No:-2456 –2165

Statistical Analysis of Linear and Non-Linear Smoothing Techniques under the Autoregressive (AR) and Generalized Auotregressive Conditional Heteroscedastic (Garch) Models

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Abstract:-This paper presents a comparison between the moving average and the LULU smoothing techniques for time series analysis under the Autoregressive(AR) and the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) models.. Different methods are being used for smoothing time series data and other smoothing purposes. These methods include moving average, weighted moving average, exponential smoothing, double exponential smoothing, Kernel smoothers, median smoothers, non-linear state space approach and LULU smoothers among others. The comparison between the moving average and the LULU smoothing methods in this paper is performed using monthly inflation data in Ghana from 2008 to 2013 under the AR modeling and the GARCH modeling procedures. The results showed that ARLU (1, 2) model was optimal. This is an indication that the LULU smoothing of order 2 is more accurate in smoothing inflation rates as compared to the moving average smoothing method. It also revealed that the ARLU (1, 2) modelis the best for modeling and forecasting monthly inflation rates in Ghana over the study period. A one year out of sample forecast for the year 2014 by the ARLU (1, 2) model showed that in the short term there would be a consistent increase in the monthly inflation rates in Ghana for the year 2014.

Keywords:-LULU smoothers, Moving average, AR model, GARCH model, Inflation rates, Ghana.

I. INTRODUCTION

Most time series data have high frequency fluctuations which need to be removed in order that reliable modeling and forecasting can be performed. These fluctuations are removed or reduced by smoothing the data. Smoothing is important in many data analysis. It is an operation which removes high frequency fluctuations from a signal. Smoothing methods used in time series analysis can be classified as linear or non-linear. The most common linear smoothing techniques often used in data analysis include moving average, weighted moving average, exponential smoothing and double exponential smoothing among others. Rasmussen (2004) used the exponential smoothing techniques to smooth the data in time series analysis. Also the local regression and Kernel smoothing techniques were employed by Loader (2004) for smoothing data in time series analysis.Usually the smoothing techniques are to remove fluctuations from a time series with the purpose of uncovering patterns in the series, with a minimum of preconceptions and assumptions as to what these patterns should be. In the process of smoothing, the random error is reduced, thus making the variance of the smoothed sequence small relative to the variance of the original sequence (Anderson, 1971).

The median smoothers and the state space approach to smoothing data are some of the non-linear smoothing techniques used for smoothing data. Kitagawa (1991) developed a non-linear state space approach to smoothing time series data. Recently, a class of non-linear smoothers known as LULU smoothers was introduced by (Rohwer, 1989). LULU smoothing is a non-linear mathematical technique for removing impulsive noise from a data sequence such as time series.LULU smoothers are compared in detail to median smoothers by Conradie, det Wet and Jankowitz (2006) and found to be superior in some aspects, particularly in mathematical properties like idempotence and co-idempotence. Idempotence means that there is no “noise” left in the smoothed data whilst co-idempotence means there is no “signal” left in the residual. In this thesis, both the linear smoothing (moving average) and non-linear smoothing (LULU) techniques are used in smoothing the data to compare their accuracies. The smoothing techniques were performed on economic time series data, that is monthly inflation rates in Ghana. Inflation is one economic factor that affects all other levels of the economy and the aim of every country or government is to control inflation rates. Due to the fact that inflation levels affect other levels of the economy especially the business community, it is important to model and forecast or estimate the value of inflation in the future so that such values are incorporated in policy formulation in the country. Several researches have been done in the area of inflation modeling and forecasting in Ghana. One of such researches was done by Suleman and Sarpong (2012).

There exist many models used for time series modeling. A common approach for modelingunivariate time series is the autoregressive (AR) model. Autoregressive models are commonly used in economic data. Autoregressive models including AR, periodic AR and periodic VARhave been used by Osburn and Jeremy (1989) on seasonal U.K. consumption. Also, Franses and Richard (1994) used periodic AR model on several quarterly U.K. macroeconomic data. Time series models like the AR, MA, ARMA, ARIMA and several others assume a constant conditional variance. However, most economic and financial series often exhibit non- constant conditional variance, a condition known as Heteroscedasticity and hence these time series models do not perform well when used to model such time series. Therefore models such as the Autoregressive Conditional Heteroscedastic (ARCH) and by extension the Generalized ARCH (GARCH) and other related models have therefore been used to model such time series. The ARCH model was introduced by Engle (1982) and was later modified by Bollerslev (1986) to the Generalized ARCH (GARCH) model which has been widely used for modeling time-varying volatility and has a much more flexible lag structure compared with the ARCH model. These among other models are used in modeling various time series forecasting.

All over the world researchers have modeled different types of time series data using various statistical models.Bayram and Yuksel (2005) found the GARCH (1, 1) model to be the best for modeling the volatility in the stock market in Turkey. Nandini (2006) concluded that the ARIMA (4, 2, 5) model was best in time series forecasting in India. The financial volatility in China was examined by (Su, 2010). The results showed that the EGARCH model fits the sample data better than the GARCH model in modeling the volatility of Chinese stock returns.

The study conducted by Amos (2010) revealed that the SARIMA (1, 1, 0)×(0, 1, 1)$_{12}$ was the best fitting model from the ARIMA family models while the GARCH (1, 1) was best from the ARCH family of models in modeling inflation data for South Africa. The GARCH (1, 1) model was superior to the SARIMA (1, 1, 0)×(0, 1, 1)$_{12}$ model. Chinwuba and Ibrahim (2013) found ARIMA (2, 2, 3) model to be the most appropriate model for short term forecasting of Nigeria inflation.Olajide et al (2012) sought to forecast inflation rates in Nigeria using Box-Jenkins approach. They identified ARIMA (1, 1, 1) model as the most appropriate for forecasting inflation rates.

Nortey (2002) used the Box-Jenkins approach to model and forecast the catch of fish in Ghana. The study showed that SARIMA (2, 0, 0)×(1, 0. 0)$_{12}$ and SARIMA (1, 0, 0)×(0, 0, 1)$_{12}$ models were appropriate for modeling and forecasting the data on SardinellaAurita and DentexSpp species respectively while ARIMA (1, 0, 1) model was best for modeling and forecasting SardinellaMaderensis and Sparidae species.Appiah and Adetunde (2011) found that ARIMA (1, 1, 1) model was suitable for modeling and forecasting future monthly exchange rates between Ghana Cedi and the U S Dollar.Mbeah-Baiden (2013) modeled inflation in Ghana using the ARCH type models. The EGARCH (2, 1) model was adjudged the most appropriate.

Previous studies have analyzed time series data using various data smoothing methods under different types of time series models. However, not many studies had been done in the area of comparing the linear and non-linear smoothing methods in modeling and forecasting monthly inflation rates in Ghana under the AR and the GARCH models. It is against this background that this study seeks to investigate the accuracy of the linear and non-linear smoothing techniques under the AR and the GARCH models in modeling and forecasting inflation in Ghana.

II. MATERIALS AND METHODS

The paper used sample monthly inflation data from January 2008 to December 2013. The data were obtained from the Ghana Statistical Service (GSS) as published on their official website [www.statsghana.gov.gh.] The data smoothing method was done with the aid of the EXCEL software whilst the modeling processes were conducted using EVIEWS 7.0 and MINITAB 16.0 statistical software. The methods used in this paper are briefly described below:

A. Differencing Process

Most time series data are nonstationary in nature. Stationarity is the foundation of time series analysis.One of the possible remedial measures to make a series X_t stationary is by differencing X_t to remove the trend and/or seasonality.Differencing of time series data can be done once or twice since over differencing also affects specification error of the model (Nortey, 2002). In this paper the differencing was done once as follows;

$$d_t = x_t - x_{t-1}$$

B. Moving Average Smoothing Techniques

Moving averages rank among the most popular techniques for the preprocessing of time series. They are used to filter random "white noise" from the data, to make the time series smoother or even to emphasize certain informational components contained in the time series. According to Velleman&Hoaglin (1981) the most common smoothing technique is the moving average.A moving average is obtained by calculating a series of averages of different subsets of the full data set. A moving average of order n, MA (n) is obtained as follows;

$$M_t = \frac{(x_t + x_{t-1} + \ldots + x_{t-n+1})}{n}$$

OR

$$M_{t-1} = \frac{(x_{t-1} + x_{t-2} + \ldots + x_{t-n})}{n}$$

C. Non-Linear Smoothing Techniques

Non-linear smoothers based on the extreme selectors have been developed as a class of smoothing method with very powerful properties and ideally suitable for application to data having impulsive noise, the type of data that often occur in the engineering and financial fields. Some of their properties make them ideally suitable as a basis for data smoothing. They systematically, measurably and monotonically "peel off" variation until one has a sufficiently smooth result (de Wet &Conradie, 2006). In this paper a class of non-linear smoothers known as LULU smoothers was one of the smoothing techniques used.LULU smoothers is a class of non-linear smoothers introduced by Rohwer (1989), based on extreme selectors within moving windows. They are a class of non-linear smoothers based on compositions of minima and maxima over different window sizes. Let $x = \{\ldots, x_{-3}, x_{-2}, x_{-1}, x_0, x_1, x_2, x_3, \ldots\}$ be a numerical sequence. LULU smoothers are compositions of the following two basic rank selectors operating on x. A forward operator $\vee^n$ is defined as

$$(\vee^n x)_i = Max(x_i, \ldots, x_{i+n})$$

and a backward operator $\wedge^n$ as

$$(\wedge^n x)_i = Min(x_{i-n}, \ldots, x_i)$$

Sequences of consecutive upward and downward impulses of length n will be removed by $\wedge^n \vee^n$ or $\vee^n \wedge^n$. These compositions give rise to half smoothers which are as illustrated below;

$$(U_n x)_i \equiv (\wedge^n \vee^n x)_i = Min\{\max(x_{i-n}, \ldots, x_i), \ldots, \max(x_i, \ldots, x_{i+n})\}$$

OR

$$(L_n x)_i \equiv (\vee^n \wedge^n x)_i = Max\{\min(x_{i-n}, \ldots, x_i), \ldots, \min(x_i, \ldots, x_{i+n})\}$$

D. Autoregressive (AR) Model

The autoregressive (AR) model uses past values of the dependent variable to explain the current value. According to Hamilton (1994) AR model is the most ordinary autoregressive models used in time series analysis. Let $\{\varepsilon_t \mid t \in T\}$ be a white noise process with mean zero and variance σ^2. A process $\{x_t \mid t \in T\}$ is said to be an autoregressive time series of order p (denoted AR (p)) if

$$d_t = \alpha_0 + \alpha_1 x_{t-1} + \alpha_2 x_{t-2} + \ldots. + \alpha_p x_{t-p} + \varepsilon_t$$

$$d_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i x_{t-i} + \varepsilon_t ;$$

where α_0 is a constant, α_i are parameters of the model and ε_t is white noise with zero mean and constant variance σ^2

A model with a combination of autoregressive terms and moving average terms is known as mixed autoregressive moving average model. We use notation ARMA (p, q) to represent these models for our convenience, where p is the order of the autoregressive part and q is the order of the moving average part. The orders of autoregressive and moving average terms in an ARMA model are determined from the pattern of sample autocorrelation and partial autocorrelations. A model for the series X_t can be an AR (p) model or an MA (q) model or a combination of both the AR (p) and the MA (q) models. The latter model is known as an autoregressive moving average of order (p, q), denoted by ARMA (p, q), and is given by

ISSN No:-2456 –2165

$$d_t = \alpha_0 + \alpha_1 x_{t-1} + \ldots + \alpha_p x_{t-p} + \beta_1 \varepsilon_{t-1} + \ldots + \beta_q \varepsilon_{t-q} + \varepsilon_t$$

, where $\alpha_1, \ldots, \alpha_p, \beta_1, \ldots \beta_q$ are model parameters to be estimated, and ε_t is a series of random errors each with zero mean and constant variance σ^2 (Box, 1976).

E. Generalized Autoregressive Conditional Heteroscedastic (GARCH) model

Bollerslev (1986) proposed the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model as an extension of the Autoregressive Conditional Heteroscedastic (ARCH) model much in the same way as the Autoregressive Moving Average (ARMA) model is an extension of the Autoregressive (AR) model. The generalized autoregressive conditional heteroscedasticity (GARCH) model is by far the most popular model for analyzing volatility in financial context. Let $x_t = r_t - u_t$ be the mean corrected return, where r_t is the return of an asset and u_t the conditional mean of x_t .Then the process $\{x_t\}$ is a GARCH (p, q) model if

$$d_t = \sigma_t \varepsilon_t$$

where

$$\sigma_t^2 = \alpha_0 + \alpha_1 x_{t-1}^2 + \alpha_2 x_{t-2}^2 + \ldots + \alpha_p x_{t-p}^2 + \beta_1 \sigma_{t-1}^2 + \beta_2 \sigma_{t-2}^2 + \ldots + \beta_q \sigma_{t-q}^2$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i x_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 ,$$

ε_t is a Gaussian white noise given as $\varepsilon_t \sim iid(0,1)$; $\alpha_i \geq 0$, $i = 0,1,\ldots,p$,

$\beta_j \geq 0$, $j = 1,2,\ldots q$ are parameters of the model and

$\sum_{i=1}^{w} (\alpha_i + \beta_i) < 1$, where $w = \max(p, q)$, $\alpha_i = 0$ for $i > p$ and $\beta_j = 0$ for $j > q$ The constraints on $\alpha_i + \beta_i$ suggests that the unconditional variance of x_t is finite, whilst its conditional variance σ_t^2 evolves over time. Allmodel estimations are performed on the differenced series.

III. RESULTS AND DISCUSSION

It is clear from Figure 1 that both the mean and variance are changing over time. The changing mean and variance over time is an indication of the non-stationarity of the monthly inflation rates. Alsofrom the plots of the ACF and PACF shown by Figure 2a and Figure 2b, it is evident that there exists a correlation in the monthly inflation rates. The ACF plots did not show an exponential decay implying that the inflation series is non-stationary. On this basis we need to transform the data to make it stationary by taking the first difference of the series. The inflation series was therefore differenced once.The ACF plots of the differenced series exhibits an exponential decaying and the PACF plots cutoff to zero after the first lag. This means that there is no significant correlation in the first difference monthly inflation rates indicating that the differenced data is stationary. From the ACF and PACF plots of the first difference data in Figure 3a and Figure 3b, the ACF tails off at lag 2 whilst the PACF spike at lag 1. To confirm the stationarity of data after the first differencing was performed; the Augmented Dickey-Fuller (ADF) test was performed to ascertain the stationarity or otherwise of the first differenced monthly inflation rates. From table 1 the computed ADF test statistic (-3.986613) is smaller than the critical values (-3.527045, -2.903566, -2.589227) at 1%, 5% and 10% respectively. This means that we can reject the null hypothesis that the first differenced inflation series has a unit root implying that the series is stationary at 1%, 5% and 10% significant levels.

Tentative ARMA (p, q) models are fitted using the first differenced inflation data to ascertain the true order(s) of p and q respectively. The most appropriate model was selected based on the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC)significance tests. The criterion is that the smaller the AIC and the BIC values the better the model. Table 2 shows the various suggested models for ARMA after the first differenced inflation data with their respective fit statistics. The ARMA (1, 2) model was the most appropriate. This showed that an AR (1) model is the best model under the moving average smoothing of order 2.In fitting the AR (1) model on the LULU smoothed data the length of the LULU smoothing was used as the order of the LULU much in the same way as the order of the moving average smoothing. Since the order of the moving average as per above was 2, a length of 2 (i. e order 2) of the LULU smoothing was considered. Table 3 gives the ARLU (1, 2) model with its fit statistics. The AR (1) models under the moving average smoothing and the LULU smoothing methods expressed as ARMA (1, 2) and ARLU (1, 2) respectively are then compared in table 4 to determine the accuracy of the smoothing methods. The lesser the AIC and the BIC values the better the model and the more accurate the smoothing method. The results from table 4 showed that the LULU smoothing is more accurate under the AR (1) model.

On the bases of the ARMA (1, 2) model, the tentative GARCH (p, q)) models were built around AR (1) and MA (2). Thus AR (1) implies ARCH (1) and MA (2) means GARCH (2). Table 5 gives the GARCH (p, q) models. GARCH (1, 3) turn out as the best among the GARCH models, this gives a moving average of order 3. Hence the LULU smoothing with a length of 3 (i e order 3) was performed on the data. Thus we fitted GARCH (1, 0) on the LULU smoothed data and named it ARCHLU (1, 3) model since the LULU is of order 3 as seen in table 6. The GARCH (1, 3) under the moving average smoothing and the ARCHLU (1, 3) under the LULU smoothing methods are compared to find out the most appropriate model and the more accurate smoothing method. This is showed in table 7. ARCHLU (1, 3) model has the least AIC and BIC values as compared with the GARCH (1, 3) model. This means that the LULU smoothing method is proven one more time to be more accurate since it produced a good model as compared to the moving average smoothing method. Based on the model output of the ARMA (1, 2), ARLU (1, 2), GARCH (1,3) and ARCHLU (1, 3) models, ARMA (1, 2), ARLU (1, 2) and ARCHLU (1, 3) were selected. Based on the model diagnostics and adequacy checks performed on the ARMA (1, 2), ARLU (1, 2) and ARCHLU (1, 3) models, it was realized that all the three models represent the data adequately. Thus to select the most appropriate model among these models we observed the AIC and the BIC values of these models. Table 8 gives the AIC and BIC values of the aforementioned models. The ARLU (1, 2) stood out as the most appropriate model. The forecasting evaluation and accuracy criteria were also used for the selection of the most appropriate model. The models were evaluated in terms of their forecasting ability of future monthly inflation rates. This is showed in table 9. The results from table 9 again show that the ARLU (1, 2) is the optimal model. This is indeed a confirmation that the LULU smoothing is superior to the moving average method. Figure 4 also confirmed that the LULU smoothers outperformed the moving average smoothing method. Table 10 gives the estimates of the coefficients of the ARLU (1, 2) model.

A one year out of sample forecast for 2014 monthly inflation rates was obtained by ARLU (1, 2) model as showed in table 11. The ARLU (1, 2) model was to forecast inflation values close to the actual monthly inflation rates for January, February, March and April 2014 since these are the only published inflation figures obtained from the official website of Ghana Statistical Service (GSS). Comparing the actual rates for January, February, March and April 2014 Published as 13.80, 14.00, 14.50 and 14.70 respectively to the forecast rates for the same period obtained from the model gives an average absolute percentage error of about 0.87. This is an indication that the ARLU (1, 2) is a good model. The forecast however showed an upward trend implying that Ghana would experience a monotonic increase in the monthly inflation rates for the year 2014 in the short term.

IV. CONCLUSION

In this paper we compared the moving average and the LULU smoothing techniques in smoothing time series data, specifically monthly inflation rates in Ghana from 2008-2013 under the AR and the GARCH models. The obtained results showed that the LULU smoothers always performed better than the moving average smoothers. The AR and the GARCH models were also compared. The results showed that the ARLU (1, 2) model is the most appropriate for modeling and forecasting monthly inflation rates in Ghana for the study period. A one year out of sample forecast of the monthly inflation rates for 2014 from the optimal model provide close figures as compared to the actual inflation figures published by GSS at the time this paper is being prepared. The forecast produced an error margin of about 0.87 implying that the model is a good one.

V. ACKNOWLEDGEMENT

We are highly indebted to the Almighty God for the knowledge and guidance.

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Table 1: Augmented Dickey-Fuller (ADF) Unit Test for the First Difference Monthly Inflation Rates in Ghana (2008-2013)

		t- Statistic	Prob
Augmented Dickey- Fuller test statistic		-3.9866	0.0026
Test Critical values:	1%	-3.5270	
	5%	-2.9036	
	10%	-2.5892	

Table 2: Comparison of Suggested ARMA (p, q) Model with fit Statistics on the first Differenced inflation Data

Model	AIC	BIC
ARMA (1, 2)	1.571	1.699
ARMA (1, 3)	1.593	1.754
ARMA (2, 2)	1.557	1.719
ARMA (2, 3)	1.586	1.780

Table 3: ARLU (1, 2) Model with Fit Statistics

Model	AIC	BIC
ARLU (1, 2)	1.213	1.278

Table 4: Comparison of ARMA (1, 2) and ARLU (1, 2) Models with Fit Statistics

Model	AIC	BIC
ARMA (1, 2)	1.571	1.699
ARLU (1, 2)	**1.213**	**1.278**

Table 5: Comparison of Suggested GARCH (p, q) Models with fit Statistics on the First Differenced Data

Model	AIC	BIC
GARCH (1, 2)	1.729	1.888
GARCH (1, 3)	**1.687**	**1.878**
GARCH (2, 2)	1.763	1.954
GARCH (2, 3)	1.738	1.961

Table 6: ARCHLU (1, 3) Model with fit Statistics

Model	AIC	BIC
ARCHLU (1, 3)	1.277	1.375

Table 7: Comparison of GARCH (1, 3) and ARCHLU (1, 3) Models with Fit Statistics

Model	AIC	BIC
GARCH (1, 3)	1.687	1.878
ARCHLU (1, 3)	**1.277**	**1.375**

Table 8: Selected Models with fit Statistics

Model	AIC	BIC
ARMA (1, 2)	1.571	1.699
ARLU (1, 2)	**1.213**	**1.278**
ARCHLU (1, 3)	1.277	1.375

Table 9: Forecast Performance of Selected Models

Measure	ARMA (1, 2)	ARLU (1, 2)	ARCHLU (1, 3)
Root Mean Square Error (RMSE)	0.61	**0.57**	0.58
Mean Absolute Error (MAE)	0.45	**0.40**	**0.40**
Mean Abs Percent Error (MAPE)	130.15%	**106.63%**	119.16%
Theil's Inequality Coefficient (TIC)	**0.76**	0.83	0.82
Rank	3	**1**	2

Table 10: Estimates of ARLU (1, 2) Model

Variable	Coefficient	Std-Error	t-Statistic	Prob
α_0	0.0534	0.1725	0.3095	0.7579
α_1	0.6924	0.0881	7.8614	0.0000

Table 11: One year out of Sample Forecast of Monthly inflation Rates for 2014 From the ARLU (1, 2) Model

Month	Observed % (2013)	Forecast % (2014)	Forecast error %
Jan	8.80	13.76	4.96
Feb	10.00	14.06	4.06
Mar	10.40	14.30	3.90
Apr	10.60	14.51	3.91
May	10.90	14.71	3.81
Jun	11.20	14.91	3.71
Jul	11.80	15.09	3.29
Aug	11.50	15.28	3.78
Sep	11.90	15.46	3.56
Oct	13.10	15.64	2.54
Nov	13.20	15.81	2.61
Dec	13.50	15.99	2.49

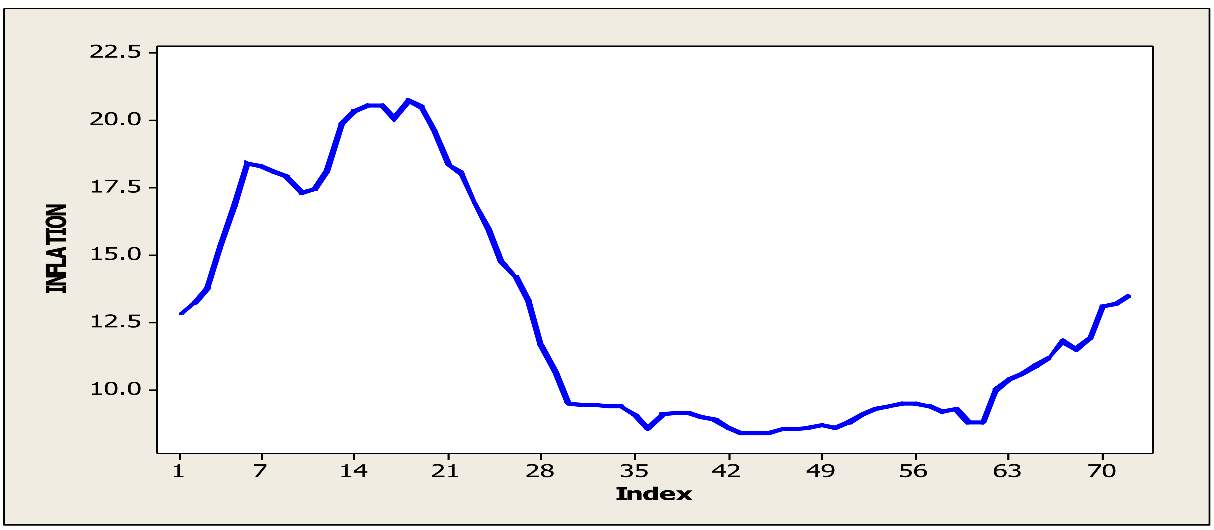

Figure1: Time Series of Monthly inflation Rates in Ghana (2008-2013)

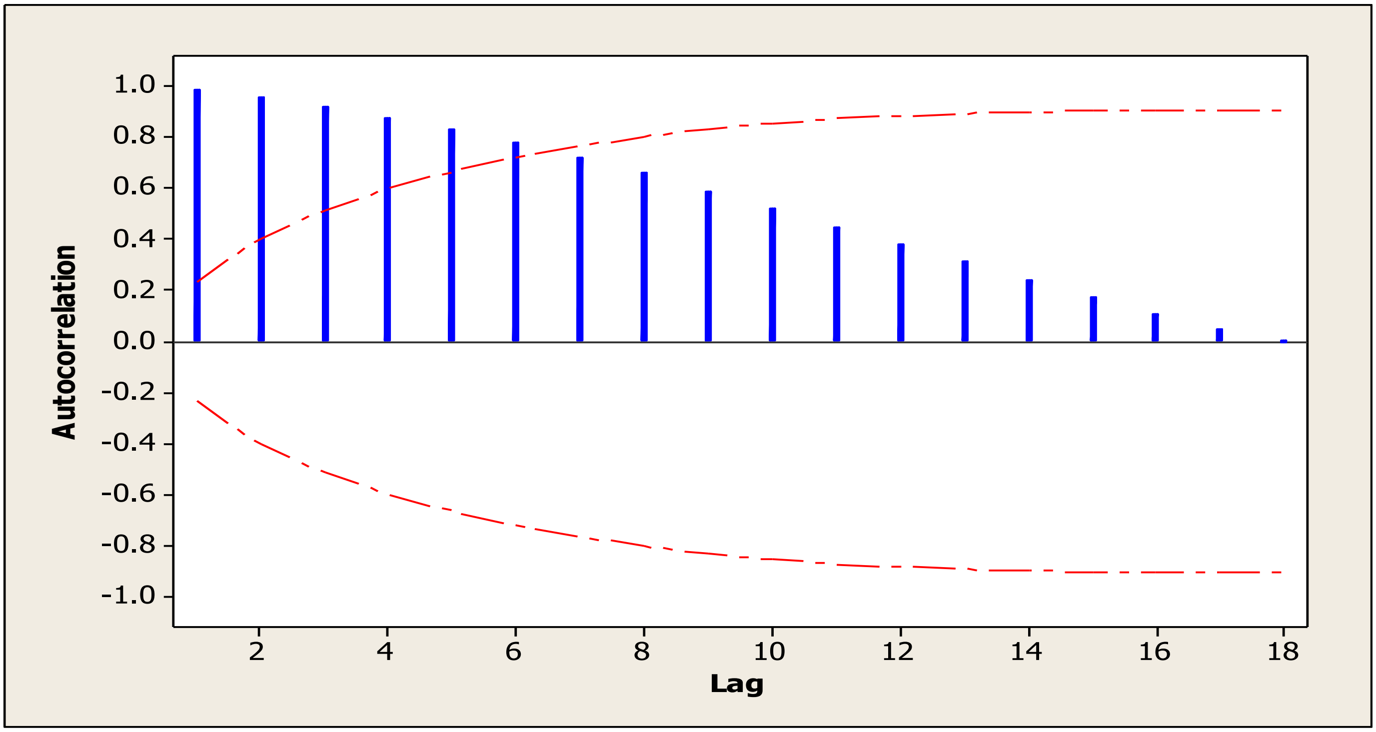

Figure 2a: Autocorrelation Function (ACF) Plots of Monthly inflation Rates in Ghana (2008-2013)

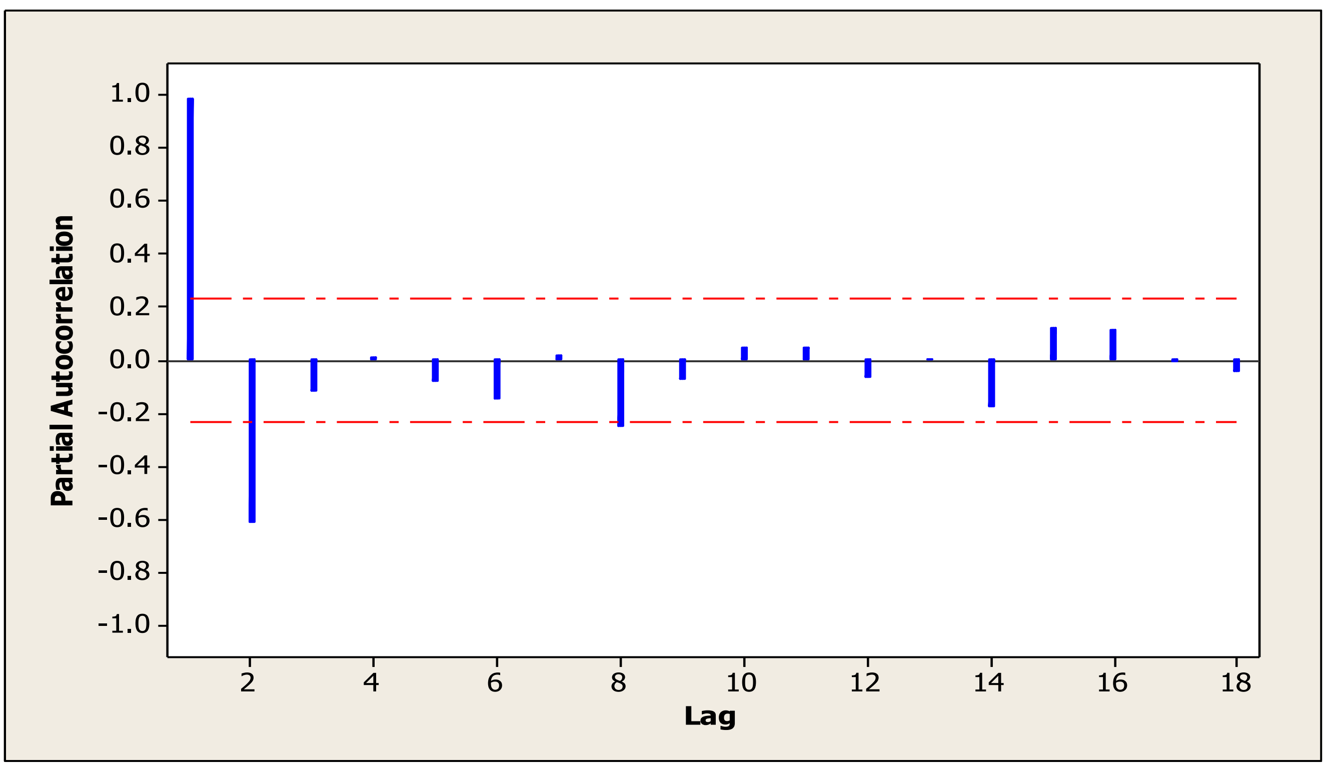

Figure 2b: Partial Autocorrelation Function (PACF) Plots of Monthly inflation Rates in Ghana (2008-2013)

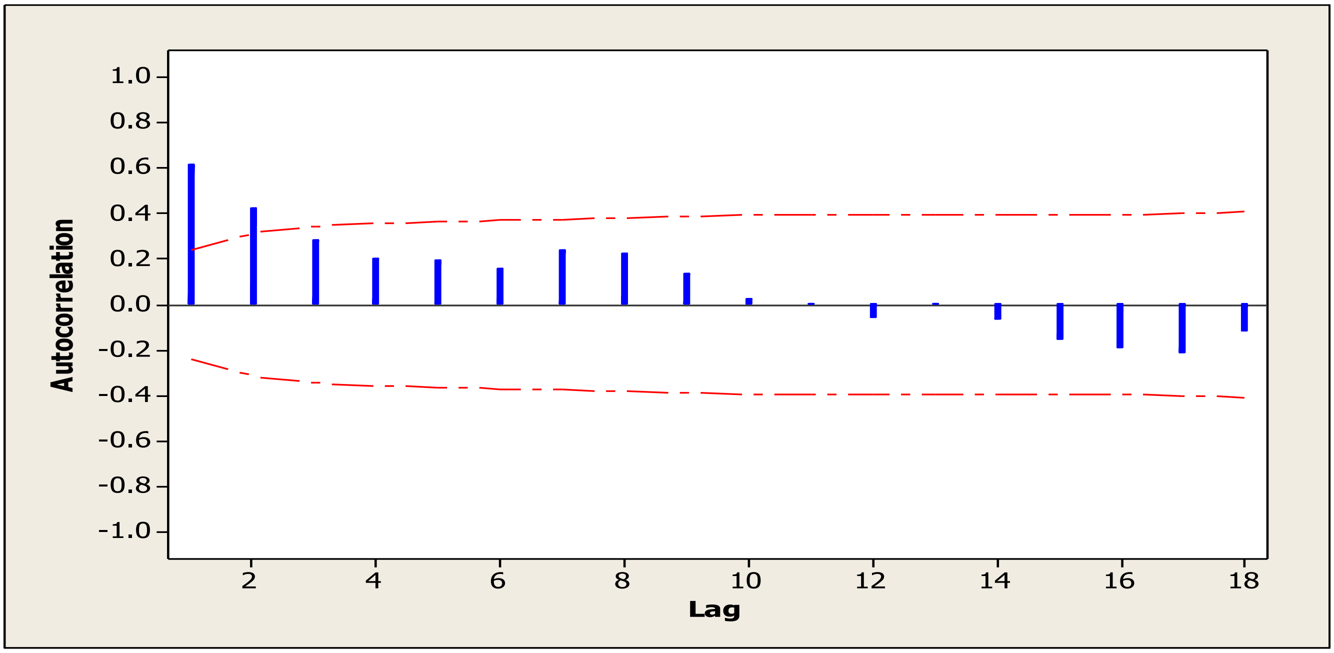

Figure 3a: Autocorrelation Function (ACF) Plots of First Differenced Monthly Inflation Rates in Ghana (2008-2013)

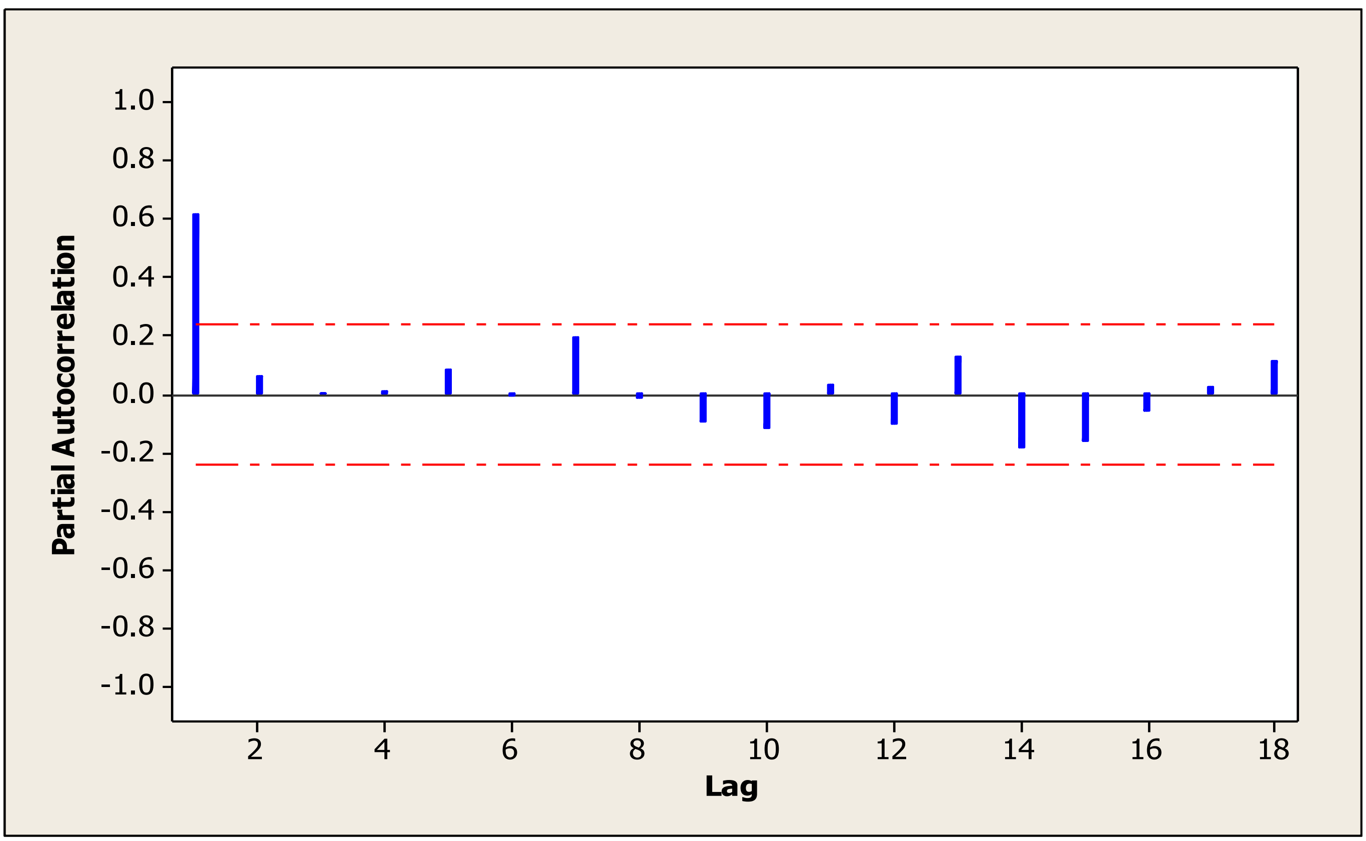

Figure 3b: Partial Autocorrelation Function (PACF) Plots of First Differenced of Monthly inflation Rates in Ghana (2008-2013)

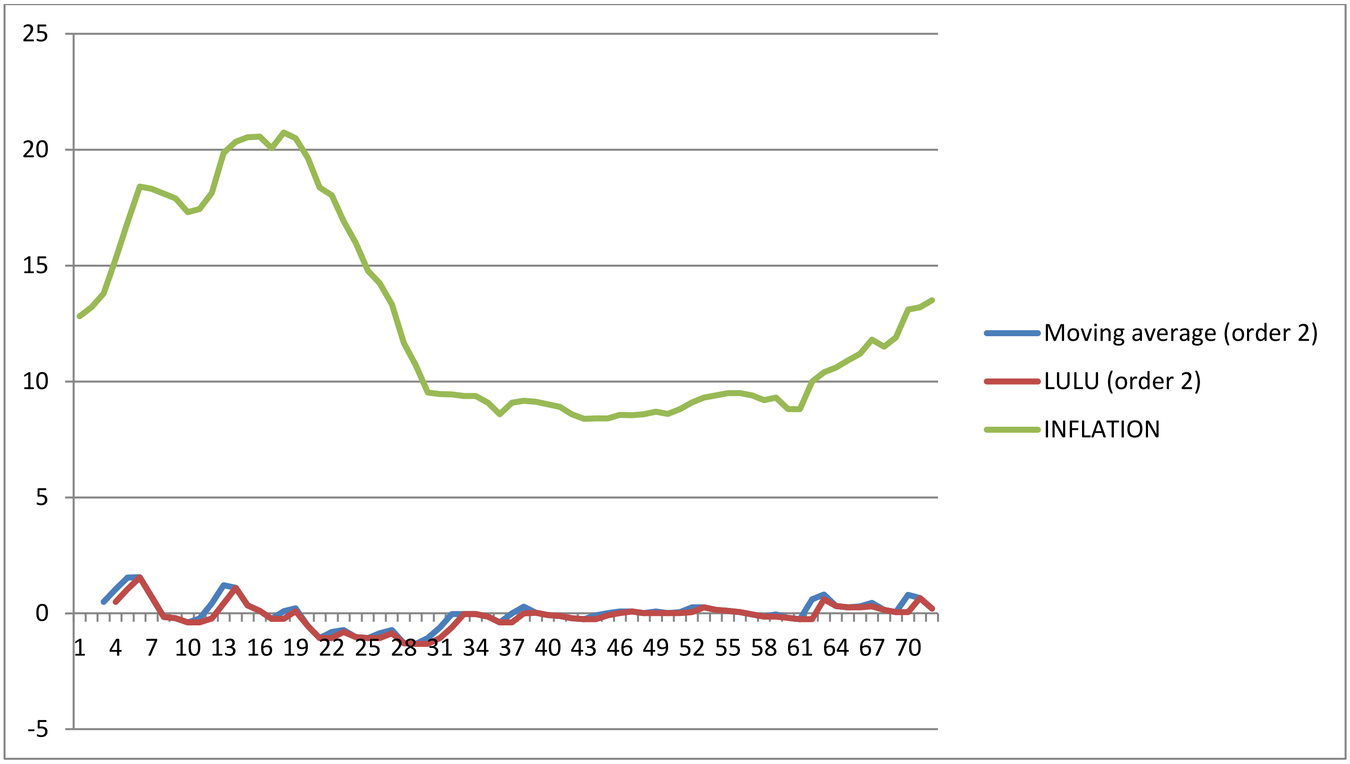

Figure 4: Plots Moving Average Smoothed Data, LULU Smoothed Data and Original Data