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# Solving Emden Fowler Type Equations by Adjusted Adomian Decomposition Strategy

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**Abstract**

In this work we show an efficient numerical calculation for comprehending Emden Fowler Type Equations of various request, we proposed another administrator for taking care of solitary Boundary value Problems (BVPs). A few illustrative cases are given to show the adequacy of the present strategy.

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## Chapter 1:- Introduction

We consider the Emden - Fowler Equations of the type

$$y' + \frac{n}{x}y + f(x,y) = 0 \qquad , \qquad (1)$$

wheref(x) and g(x) are functions of x and y, respectively where n >1 is called the shape factor. These issues by and large emerge every now and again in numerous ranges of science and building, for instance, fluid mechanics, quantum mechanics, idea control, chemical reactor hypothesis, optimal design, response dispersion handle, geophysics, and so forth.

The Adomian Decomposition Method (ADM) Adomian, proposed by Adomian toward the start of 1980s, has gotten huge consideration in the previous two decades. Adomian declares that the decay technique gives an efficient and computationally helpful strategy for producing estimated arrangement to the wide class of conditions. It has been utilized by Adomian and numerous different creators to research a huge assortment of scientific and physical issues including (standard or incomplete) differential, indispensable, integro-differential, arithmetical and frameworks of such conditions. The ADM was utilized by Adomian and Rach to examine non-direct multidimensional BVPs. Wazwaz, made a slight however capable modification for ADM to accelerate the union of the arrangement. An efficient path for utilizing the ADM legitimately for particular BVPs was presented and a system managing the solitary BVPs by and large was exhibited by Inc, Ergut and Cherruault.

We aim in this work to establish various kinds of Emden-Fowler type equations of various order. Our approach depends mainly on using different orders of different operators involved in the Emden-Fowler sense given in (2). Our next goal of this work is to apply Adomian Decomposition Strategy to handle the derived Emden Fowler type equations of various kind. Several numerical examples, with boundary conditions of each model will be examined to handle the singularity point that exist in each model.

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## Chapter 2:- Building Emden - Fowler Kinds Equations

It is interesting to note that the Emden - Fowler equation (1) was derived by using the equation

$$x^{-n}\frac{d}{dx}x^n(y)+f(x,y)=0 \quad , \tag{2}$$

To derive the Emden-Fowler kind equations of different order, we use the sense of (2) and set

$$x^{-n}\frac{d^m}{dx^m}x^n(y)+f(x,y)=0 \tag{3}$$

where $n$ is a real number. To determine such different order equations we set $m$ to different values i.e

### 2.1 First Kind for $m = 1$

Substituting for $m = 1$ in equation (3) gives

$$y'+\frac{n}{x}y+f(x,y)=0 \tag{4}$$

### 2.2 Second Kind for $m = 2$

Substituting for $m = 2$ in equation (3) gives

$$y''+\frac{2n}{x}y'+\frac{n(n-1)}{x^2}y+f(x,y)=0 \quad , \tag{5}$$

### 2.3 Third Kind for $m = 3$

Substituting for $m = 3$ in equation (3) gives

$$y'''+\frac{3n}{x}y''+\frac{3n(n-1)}{x^2}+\frac{n(n-1)(n-2)y}{x^3}+f(x,y)=0 \tag{6}$$

### 2.4 Fourth Kind for $m = 4$

Substituting for $m = 4$ in equation (3) gives

$$\begin{aligned} y^{(iv)} &+ \frac{4n}{x}y'''+\frac{6n(n-1)}{x^2}y''+\frac{4n(n-1)(n-2)}{x^3}y' \\ &+\frac{n(n-1)(n-2)(n-3)}{x^4}y+f(x,y)=0, \end{aligned} \tag{7}$$

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$$y^m + \frac{nm}{x} y^{(m-1)} + \dots \frac{n!}{(n-r)!} \cdot \frac{m!}{(m-r)!r!} x^{-r} y^{(m-r)} + f(x,y) = 0$$

## 2.5 Sort Equations of Higher Order

In perspective of the determined Emden Fowler sort conditions, we are currently ready to exhibit the summed up Emden Fowler sort conditions for higher requests for each $m$ and $n$.

### 2.5.1 Theorem If $m \in$ N then

$$x^{-n} \frac{d^m}{dx^m} x^n y = \sum_{r=0}^{m} \frac{n!}{(n-r)!} \binom{m}{r} x^{-r} y^{(m-r)}$$

Proof: We will prove this with mathematical induction.
If m = 1, this statement is

$$x^{-n} \frac{d}{dx} x^n y = \sum_{r=0}^{1} \frac{n!}{(n-r)} \binom{1,}{0} x^{-r} y^{(1-r)} \tag{8}$$

Since the left-hand side is $y' + \frac{n}{x} y$, and the right-hand side is $y' + \frac{n}{x} y$, the equation

$$x^{-n} \frac{d}{dx} x^n y = \sum_{r=0}^{1} \frac{n!}{(n-r)} \binom{1}{0} x^{-r} y^{(1-r)}$$

holds, as both sides are $y' + \frac{n}{x} y$.
We must now prove $S_k \Rightarrow S_{k+1}$ for any $k \geq 1$. That is, we must show that if

$$x^{-n} \frac{d^k}{dx^k} x^n y = \sum_{r=0}^{k} \frac{n!}{(n-r)!} \binom{k}{r} x^{-r} y^{(k-r)},$$

then

$$x^{-n} \frac{d^{k+1}}{dx^{k+1}} x^n y = \sum_{r=0}^{k+1} \frac{n!}{(n-r)!} \binom{k+1}{r} x^{-r} y^{(k+1-r)}, \tag{9}$$

We use direct proof. Suppose

$$x^{-n} \frac{d^k}{dx^k} x^n y = \sum_{r=0}^{k} \frac{n!}{(n-r)!} \binom{k}{r} x^{-r} y^{(k-r)},$$

then

$$x^{-n}\frac{d^{k+1}}{dx^{k+1}}x^n y = x^{-n}\frac{d^k}{dx^k}\left(x^n y' + nx^{n-1}y\right)$$

$$= x^{-n}\frac{d^k}{dx^k}x^n y' + x^{-n}\frac{d^k}{dx^k}nx^{n-1}y$$

$$= y^{k+1} + \sum_{r=1}^{k}\frac{n!}{(n-r)!}\binom{k}{r}x^{-r}y^{(k+1-r)} + \sum_{r=0}^{k}\frac{n!}{(n-1-r)!}\binom{k}{r}x^{-r-1}y^{(k-r)}$$

$$= y^{k+1} + \sum_{r=1}^{k}\frac{n!}{(n-r)!}[\binom{k}{r} + \binom{k}{r-1}]x^{-r}y^{(k+1-r)}$$

$$= y^{k+1} + \sum_{r=1}^{k}\frac{n!}{(n-r)!}\binom{k+1}{r}x^{-r}y^{(k+1-r)}$$

$$= \sum_{r=0}^{k+1}\frac{n!}{(n-r)!}\binom{k+1}{r}x^{-r}y^{(k+1-r)}$$

Therefore

$$x^{-n}\frac{d^m}{dx^m}x^n y = \sum_{r=0}^{m}\frac{n!}{(n-r)!}\binom{m}{r}x^{-r}y^{(m-r)} =$$

It follows by induction that

for each $n \in$ N $$x^{-n}\frac{d^m}{dx^m}x^n y = \sum_{r=0}^{m}\frac{n!}{(n-r)!}\binom{m}{r},x^{-r}y^{(m-r)} \tag{10}$$

2.5.2 Corollary

If $m \in \aleph$ and $n \leq 0$ then

$$x^{-n}\frac{d^m}{dx^m}x^n y = \sum_{r=0}^{m}\frac{(-1)^r\Gamma(r-n)}{\Gamma(-n)}\binom{m}{r}x^{-r}y^{(m-r)}$$

3 Examination of the Method and Numerical Examples

Problem 1. Consider the nonlinear Emden Fowler type equation

$y(0) = 0$ $$y' + \frac{3}{x}y = e^x(x - x^2e^x + 4) + y^2 , \tag{11}$$

with exact solution = $xe^x$

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Obtained by substituting $n = 3$ in (10) and by setting $g(y) = y^2$
First Eq. (11) can be written as

$$Ly = e^x\left(x - x^2e^x + 4\right) + y^2 \quad (12)$$

where

$$L(.) = x^{-1}\frac{d}{dx}x(.)$$

and Operating

$$L^{-1} = x^3\int_0^x x^{-3}(.)dx,$$

on both sides of (12), and using the initial conditions at $x = 0$, yields

$$L^{-1}(Ly) = L^{-1}\left(e^x(x - x^2e^x + 4) + y^2\right)$$

$$L^{-1}(Ly) = L^{-1}\left(e^x(x - x^2e^x + 4)\right) + L^{-1}\left(y^2\right)$$

$$y(x) = L^{-1}\left(e^x(x - x^2e^x + 4)\right) + L^{-1}\left(y^2\right) \quad (13)$$

Substituting the decomposition series $\sum_{n=0}^{\infty} y_n(x)$ for $y(x)$ into (13) gives

$$\sum_{n=0}^{\infty} y_n(x) = x + x^2 + \frac{1}{3}x^3 - \frac{5}{42}x^4 - \frac{5}{24}x^5 - \frac{151}{1080}x^6 - \frac{79}{1200}x^7 - \ldots + L^{-1}\left(y^2\right), \quad (14)$$

Identifying the Zeroth component $y_0(x)$ by all terms that are not included under the inverse operator $L^{-1}$ and following the above discussion leads to the recursive relation

$$y_o = x + x^2 + \frac{1}{3}x^3 - \frac{5}{42}x^4 - \frac{5}{24}x^5 - \frac{151}{1080}x^6 - \frac{79}{1200}x^7 - \ldots$$

$$y_{n+1} = L^{-1}\left(A_n\right), n \geq 0 \quad (15)$$

The Adomian polynomials for the exponential nonlinearity $y_0^2$ are given as

$$A_0 = y_0^2$$
$$A_1 = 2y_0y1$$
$$A_2 = y_1^2 + 2y_0y_2$$
$$A3 = 2y1y2 + 2y0y3$$
$$A_4 = y_2^2 + 2y_1y3 + 2y_0y_4$$
$$A5 = 2y2y3 + 2y1y4 + 2y0y5 \quad (16)$$

Using (15), the first several calculated solution components are

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$$y_o(x) = x + x^2 + \frac{1}{3}x^3 - \frac{5}{42}x^4 - \frac{5}{24}x^5 - \frac{151}{1080}x^6 - \frac{79}{1200}x^7 - \dots$$

$$y_1(x) = \frac{1}{6}x^3 + \frac{2}{7}x^4 + \frac{5}{24}x^5 + \frac{1}{21}x^6 - \frac{137}{2520}x^7 - \frac{733}{10395}x^8 - \frac{141829}{3175200}x^9 - \frac{69799}{4054050}x^{10} - \frac{299711}{139708800}x^{11} + \dots$$

$$y_2(x) = \frac{1}{24}x^5 + \frac{19}{189}x^6 + \frac{277}{2520}x^7 + \frac{167}{2772}x^8 - \frac{71}{70560}x^9 - \frac{27023}{810810}x^{10} - \frac{16296691}{488980800}x^{11} - \frac{42623953}{2270268000}x^{12} + \dots$$

$$y_3(x) = \frac{290600117634571}{154057117294080000}x^{17} + \frac{291730707235321}{139701795000768000}x^{18} + \frac{4393421676399972707}{3380783439018585600000}x^{19} + \text{———}$$

| X | Exact | Adm | Absolute Error |
|---|---|---|---|
| 0 | 0.00000000 | 0.00000000 | 0.00000000 |
| 0.1 | 0.11051709 | 0.11051709 | 0.00000000 |
| 0.2 | 0.24428055 | 0.24428054 | 0.00000001 |
| 0.3 | 0.40495764 | 0.40495733 | 0.00000031 |
| 0.4 | 0.59672988 | 0.59672539 | 0.00000449 |
| 0.5 | 0.82436064 | 0.82431871 | 0.00004193 |
| 0.6 | 1.09327128 | 1.09298338 | 0.00028790 |
| 0.7 | 1.40962690 | 1.40806466 | 0.00156224 |
| 0.8 | 1.78043274 | 1.77340788 | 0.00702486 |
| 0.9 | 2.21364280 | 2.18661668 | 0.02702612 |
| 1 | 2.71828183 | 2.62745582 | 0.09082601 |

Table 1. Comparison of numerical errors

and so on. This gives the approximation of $y(x)$ in a series form by

$$y(x) = x+x^2+\frac{1}{2}x^3+\frac{1}{6}x^4+\frac{1}{24}x^5+\frac{1}{120}x^6+\frac{1}{1200}x^7-\frac{31}{10800}x^9-\frac{183241}{16216200}x^{10}-\frac{20829449}{977961600}x^{11}+\dots \quad (17)$$

**Problem 2**. We now consider the nonlinear Emden Fowler type equation

$$y(0)=1, y'(0)=0, y''(0)=0' \quad (18)$$

$$y'' + \frac{2}{x}y' - 2(2x^2+3)y = 0$$

**with exact solution =** $e^{x^2}$ obtained by substituting $n$ = 1 in (10) and by setting $f(x)g(y) = -2(2x^2+3)y$
First Eq. (18) can be written as

$$Ly = 2(2x^2+3)y \quad (19)$$

Operating

$$L^{-1} = x^1 \int_0^x \int_0^x x^{-1}(.)dxdx,$$

on both sides of (19), and using the initial conditions at $x$ = 0, yields

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$$L^{-1}(Ly) = L^{-1}\Big(2(2x^2+3)y\Big)$$

$$y(x) = y(0) + L^{-1}\Big(2(2x^2+3)y\Big) \tag{20}$$

Substituting the decomposition series $\sum_{n=0}^{\infty} y_n(x)$ for $y(x)$ into (20) gives

$$\sum_{n=0}^{\infty} y_n(x) = 1 + L^{-1}\left(2(2x^2+3)y\right) , \tag{21}$$

Identifying the Zeroth component $y_0(x)$ by all terms that are not included under the inverse operator $L^{-1}$ and following the above discussion leads to the recursive relation

$$y_0 = 1$$
$$y_{n+1} = L^{-1}\left(2\left(2x^2+3\right)y_n\right) \quad n \geq 0 \tag{22}$$

Using (22), the first several calculated solution components are

$$\begin{aligned} y_0(x) &= 1 \\ y_1(x) &= x^2 + \frac{1}{5}x^4 \\ y_2(x) &= \frac{3}{10}x^4 + \frac{8}{105}x^6 + \frac{1}{180}x^8 \\ y_3(x) &= \frac{3}{70}x^6 + \frac{37}{2520}x^8 + \frac{13}{7700}x^{10} + \frac{1}{14040}x^{12} \end{aligned} \tag{23}$$

and so on. This gives the approximation of $y(x)$ in a series form by

$$y(x) = 1 + x^2 + \frac{1}{2}x^4 + \frac{5}{42}x^6 + \frac{17}{840}x^8 + \frac{13}{7700}x^{10} \ldots \tag{24}$$

| X | Exact | Madm | Error |
|---|---|---|---|
| 0.0 | 1.01005017 | 1.01005012 | 5E-08 |
| 0.1 | 1.04081077 | 1.04080767 | 3.1E-06 |
| 0.2 | 1.09417428 | 1.09413812 | 3.616E-05 |
| 0.3 | 1.17351087 | 1.17330106 | 0.00020981 |
| 0.4 | 1.28402542 | 1.28319084 | 0.00083458 |
| 0.5 | 1.43332942 | 1.43070457 | 0.00262485 |
| 0.6 | 1.63231622 | 1.6252712 | 0.00704502 |
| 0.7 | 1.89648088 | 1.87958918 | 0.0168917 |
| 0.8 | 2.24790799 | 2.21063742 | 0.03727057 |
| 0.9 | 2.71828183 | 2.64104525 | 0.07723658 |
| 1.0 | 1 | 1.02380901 | 0.02380901 |

Table 2:Comparison of numerical errors

Problem 3. We now consider the nonlinear Emden Fowler type equation

$$y''' + \frac{6}{x}y'' + \frac{6}{x^2}y' - 6(10 + 2x^3 + x^6)e^{-3y} = 0 \tag{25}$$

$$y(0) = 0, y'(0) = 0, y'''(0) = 0$$

with exact solution: $\ln(1 + x^3)$ obtained by substituting $n = 2$ in (10) and by setting $f(x)g(y) = 6(10 + 2x^3 + x^6)e^{-3y}$

First Eq. (25) can be written as
Operating

$$Ly = 6(10 + 2x^3 + x^6)e^{-3y}$$
$$L^{-1} = x^2 \int_0^x \int_0^x \int_0^x x^{-2}(.)dxdxdx, \tag{26}$$

on both sides of (26), and using the initial conditions at $x = 0$, yields

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$$L^{-1}(Ly) = L^{-1}\left(6(10+2x^3+x^6)e^{-3y}\right)$$

$$y(x) = y(0) + L^{-1}\left(6(10+2x^3+x^6)e^{-3y}\right) \tag{27}$$

Substituting the decomposition series $\sum_{n=0}^{\infty} y_n(x)$ for $y(x)$ into (27) gives

$$\sum_{n=0}^{\infty} y_n(x) = 0 + L^{-1}\left(6(10+2x^3+x^6)e^{-3y}\right) , \tag{28}$$

Identifying the Zeroth component $y_0(x)$ by all terms that are not included under the inverse operator $L^{-1}$ and following the above discussion leads to the recursive relation

$$y_{n+1} = L^{-1}\left(6\left(10+2x^3+x^6\right)A_n\right), \tag{29}$$

$$y_0 = 1 \qquad n \geq 0$$

The Adomian polynomials for the exponential nonlinearity $e^{-3y}$ are given as

$$A_0 = e^{-3y_0}$$

$$\cdots \qquad A_1 = -3e^{-3y_0}y_1$$

$$A_2 = \frac{9}{2}e^{-3y_0}y_1^2 - 3y_2e^{-3y_0}$$

$$A_3 = -\frac{9}{2}e^{-3y_0}y_1^3 + 9y_2y_1e^{-3y_0} - 3e^{-3y_0}y_3 \tag{30}$$

Using (29), the first several calculated solution components are

$$y_0(x) = 1$$

$$y_1(x) = x^3 + \frac{1}{28}x^6 + \frac{1}{165}x^9$$

$$y_2(x) = -\frac{15}{28}x^6 - \frac{3}{70}x^9 - \frac{523}{56056}x^{12} - \frac{13}{61600}x^{15} - \frac{1}{62700}x^{18}$$

$$y_3(x) = -\frac{57}{154}x^9 + \frac{9}{196}x^{12} + \frac{402069}{34652800}x^{15} + \frac{15237}{26626600}x^{18} + \frac{111965597}{2074851178400}x^{21} + \frac{38719}{33473440000}x^{24} + \frac{7}{}$$

$$y_4(x) = -\frac{2295}{8008} - \frac{140751}{2932160}x^{15} - \frac{195616251}{14484870400}x^{18} - \frac{23440386933}{22446117293600}x^{21} - \frac{3317125515507}{28217976026240000}x^{2}4 - \frac{}{}$$

and so on. This gives the approximation of $y(x)$ in a series form by

$$y(x) = x^3 - \frac{1}{2}x^6 + \frac{1}{3}x^9 - \frac{1}{4}x^{12} + \dots \tag{35}$$

| X | Exact | Madm | Error |
|---|---|---|---|
| 0.0 | 0 | 0 | 0 |
| 0.1 | 0.0009995 | 0.0009995 | 0 |
| 0.2 | 0.00796817 | 0.00796817 | 0 |
| 0.3 | 0.02664193 | 0.02664193 | 0 |
| 0.4 | 0.06203539 | 0.06203515 | 2.4E-07 |
| 0.5 | 0.11778304 | 0.11777634 | 6.7E-06 |
| 0.6 | 0.19556678 | 0.19546849 | 9.829E-05 |
| 0.7 | 0.29490592 | 0.29397091 | 0.00093501 |
| 0.8 | 0.41343328 | 0.40695629 | 0.00647699 |
| 0.9 | 0.54754321 | 0.51221299 | 0.03533022 |
| 1.0 | 0.69314718 | 0.532662 | 0.16048518 |

Table 3: Comparison of numerical errors

Problem 4. Consider the following nonlinear Emden Fowler type equation

$$y^{iv} + \frac{20}{x}y''' + \frac{120}{x^2}y'' + \frac{240}{x^3}y' + \frac{120}{x^4}y = 5160 + x^{10} - y^2, \tag{36}$$

with exact solution = $x^5$ $y(0) = 0, y'(0) = 0, y'''(0) = 0$

Obtained by substituting $n$ = 5 in (10) and by setting $f(x)g(y) = 5160 + x^{10} - y^2$
First Eq.(36) can be written as

$$L^{-1} = x^5 \int_0^x \int_0^x \int_0^x \int_0^x x^{-5}(.)dxdxdxdx, \tag{37}$$

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Operating on both sides of (37), and using the initial conditions at $x=0$, yields (38)

$$Ly = 5160 + x^{10} - y^2$$

$$L^{-1}(Ly) = L^{-1}\left(5160 + x^{10} - y^2\right)$$

$$y(x) = y(0) + L^{-1}\left(5160 + x^{10} - y^2\right)$$

Substituting the decomposition series) into (28) gives

$$\sum_{n=0}^{\infty} y_n(x) \text{ for } y(x$$

$$\sum_{n=0}^{\infty} y_n(x) = 0 + L^{-1}\left(5160 + x^{10}\right) - L^{-1}\left(y^2\right), \tag{39}$$

Identifying the Zeroth component $y_0(x)$ by all terms that are not included under the inverse operator $L^{-1}$ and following the above discussion leads to the recursive
relation (40)

$$y_0 = L^{-1}\left(5160 + x^{10}\right)$$

$$y_{n+1} = -L^{-1}(A_n), \qquad n \geq 0$$

The Adomian polynomials for the exponential nonlinearity $y^2$ are given as

$$A_0 = y_0^2$$
$$A_1 = 2y_0y1$$
$$A_2 = y_1^2 + 2y_0y_2$$
$$A3 = 2y1y2 + 2y0y3$$
$$A_4 = y_2^2 + 2y_1y3 + 2y_0y_4$$
$$A_5 = 2y_2y_3 + 2y_1y_4 + 2y_0y_5 \tag{41}$$
$$\cdots$$

Using (40), the first several calculated solution components are

$$y_o(x) = \frac{43}{42}x^5 + \frac{1}{93024}x^{14}$$

$$y_1(x) = -\frac{1849}{164094336}x^{14} - \frac{43}{959951865600}x^{23} - \frac{1}{13716433630126080}x^{32}$$

$$y_2(x) = \frac{79507}{1693355090918400}x^{23} + \frac{17397241}{825681297015884574 72000}x^{32} + \frac{1453}{5115853913755562385408000}x^{41} + \text{——}$$

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and so on. This gives the approximation of $y(x)$ in a series form by

$$y(x) = \frac{43}{42}x^5 - \frac{5}{9652608}x^{14} + \frac{43}{19921824599040}x^{23} + \frac{11377591}{825681297015884574720000}x^{32} + \frac{1453}{\quad}$$

| | Exact | Madm | Error |
|---|---|---|---|
| 0.0 | 0 | 0 | 0 |
| 0.1 | 0.00001 | 0.00001024 | 2.4E-07 |
| 0.2 | 0.00032 | 0.00032762 | 7.62E-06 |
| 0.3 | 0.00243 | 0.00248786 | 5.786E-05 |
| 0.4 | 0.01024 | 0.01048381 | 0.00024381 |
| 0.5 | 0.03125 | 0.03199405 | 0.00074405 |
| 0.6 | 0.07776 | 0.07961143 | 0.00185143 |
| 0.7 | 0.16807 | 0.17207166 | 0.00400166 |
| 0.8 | 0.32768 | 0.33548188 | 0.00780188 |
| 0.9 | 0.59049 | 0.60454917 | 0.01405917 |
| 1.0 | 1 | 1.02380901 | 0.02380901 |

Table 4: Comparison of numerical errors

Problem 5. Consider the following nonlinear Emden Fowler type equation (43)

$$y'' - \frac{2}{x}y' + \frac{2}{x^2}y = 2x + x^6 - y^2$$

$$y(0) = 0, y'(0) = 0$$

with exact solution = $x^3$

Obtained by substituting $n = -1$ and by setting $f(x) + g(y) = 2x + x^6 - y^2$. First Eq. (43) can be written as

$$Ly = 2x + x^6 - y^2 \tag{44}$$

Operating

$$L^{-1} = x\int_0^x \int_0^x x^{-1}(.)dxdx,$$

on both sides of (44),and using the initial conditions at $x = 0$, yields

$$L^{-1}(Ly) = L^{-1}\left(2x + x^6 - y^2\right)$$

$$y(x) = y(0) + L^{-1}\left(2x + x^6 - y^2\right) \tag{45}$$

Substituting the decomposition series $\sum_{n=0}^{\infty} y_n(x)$ for $y(x)$ into (45) gives

) into (45) gives

$$\sum_{n=0}^{\infty} y_n(x) = L^{-1}\left(2x + x^6\right) - L^{-1}\left(y^2\right),$$

Identifying the Zeroth component $y_0(x)$ by all terms that are not included under the inverse operator $L^{-1}$ and following the above discussion leads to the recursive
relation

$$y_o = L^{-1}\left(2x + x^6\right) \tag{47}$$

0

$$y_{n+1} = -L^{-1}(A_n), n \geq$$

The Adomian polynomials for the exponential nonlinearity $y^2$ are given as

$$A_0 = y_0^2$$

$$A1 = 2y0y1$$

$$A_2 = y_1^2 + 2y_0y_2$$

$$A_3 = 2y_1y_2 + 2y_0y_3 \tag{48}$$

Using (47), the first several calculated solution components are

$$y_o(x) = x^3 + \frac{1}{42}x^8$$

$$y_1(x) = -\frac{1}{42}x^8 - \frac{1}{2772}x^{13} - \frac{1}{479808}x^{18}$$

$$y_2(x) = \frac{1}{2772}x^{13} + \frac{1}{146608}x^{18} + \frac{169}{3657576384}x^{23} + \frac{1}{7073329536}x^{28}$$

$$y_3(x) = -\frac{25}{5277888}x^{18} - \frac{95}{914394096}x^{23} - \frac{215}{233419874688}x^{28} - \frac{1405}{349598896238592}x^{33} + \dots$$

$$y_4(x) = \frac{211}{3657576384}x^{23} + \frac{3725}{2567618621568}x^{28} + \frac{277055}{17829543708168192}x^{33} + \frac{602144755}{6661581097508048904192}x^{38} + \dots$$

and so on. This gives the approximation of $y(x)$ in a series form by

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$$y(x) = x^3 + \frac{1723}{2567618621568}x^{28} + \frac{1093503725}{133231621950160978083884}x^{38} + \frac{1975}{171437920270848}x^{33} + \ldots \quad (49)$$

| X | Exact | ADM | ERROR |
|---|---|---|---|
| 0.000000 | 0.000000 | 0.000000 | 0.00000 |
| 0.100000 | 0.001000 | 0.001000 | 0.00000 |
| 0.200000 | 0.008000 | 0.008000 | 0.00000 |
| 0.300000 | 0.027000 | 0.027000 | 0.00000 |
| 0.400000 | 0.064000 | 0.064000 | 0.00000 |
| 0.500000 | 0.125000 | 0.125000 | 0.00000 |
| 0.600000 | 0.216000 | 0.216000 | 0.00000 |
| 0.700000 | 0.343000 | 0.343000 | 0.00000 |
| 0.800000 | 0.512000 | 0.512000 | 0.00000 |
| 0.900000 | 0.729000 | 0.729000 | 0.00000 |
| 1.000000 | 1.000000 | 1.000000 | 0.00000 |

Table 5: Comparison of numerical errors

Problem 6. Consider the following linear Emden Fowler type equation

$$y'' - \frac{2}{x}y' + \frac{2}{x^2}y = (x^2 + 2x)e^x \quad , \quad (50)$$

$$y(0) = 0, y'(0) = 0$$

with exact solution = $x^2e^x$ $\qquad Ly = (x^2 + 2x)e^x$

Obtained by substituting $n = -1$ and by setting $f(x) = (x^2 + 2x)e^x$ **.** First Eq. (50) can be written as Operating

$$L^{-1} = x\int_0^x \int_0^x x^{-1}(.)dxdx, \quad (51)$$

on both sides of (51),and using the initial conditions at $x = 0$, yields

$$L^{-1}(Ly) = L^{-1}\Big((x^2+2x)e^x\Big)$$

$$y(x) = y(0) + L^{-1}\Big((x^2+2x)e^x\Big) \tag{52}$$

Substituting the decomposition series $\sum_{n=0}^{\infty} y_n(x)$ for $y(x)$ into (52) gives

) into (52) gives

$$\sum_{n=0}^{\infty} y_n(x) = L^{-1}\Big(2x + x^6\Big) - L^{-1}\Big(y^2\Big), \tag{53}$$

Identifying the Zeroth component $y_0(x)$ by all terms that are not included under the inverse operator $L^{-1}$ and following the above discussion leads to the recursive
relation

$$y_o = x^3 + \frac{1}{2}x^4 + \frac{1}{6}x^5 + \frac{1}{24}x^6 + \frac{1}{120}x^7 + \frac{1}{720}x^8 + \dots$$

This gives the approximation of $y(x)$ in a series form by

$$y(x) = x^2 e^x$$

## Chapter 3:- Conclusion

In this paper, we proposed an efficient modification of the standard Adomian decomposition method for solving Emden Fowler Type Equations of various order the proposed method takes care of solitary Boundary value problems (BVPs). The advantage of using the proposed algorithm of this paper is clearly demonstrated for Examples 1-6. In comparison with exact solutions this method gives us a very better approximation. The study showed that the modified decomposition method is simple and easy to use and produces reliable results with few iterations used.

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