

Estimation of Aquifer Hydraulic Conductivity and Evaluation of Empirical Formulae Based on Grain Size Analysis and Permeameter Test in Yenagoa, Bayelsa State, Nigeria

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Abstract:- Grain size distribution of eight soil samples recovered from boreholes drilled within the Yenagoa metropolis was determined by means of mechanical sieve analysis. From the distribution curves, grading characteristics: d_{10} , d_{25} , d_{30} , d_{60} and d_{75} and their derivatives such as the effective size, uniformity coefficient, coefficient of sorting, coefficient of gradation and porosity were calculated. The hydraulic conductivity of the unconsolidated aquifer materials was first evaluated using six empirical formulae on the basis of the grain size distribution and secondly by the constant head permeameter laboratory test method. Analyses of the results obtained using the various empirical formulae show that only Kozeny-Carman and Hazen formulae reliably estimated the hydraulic conductivities of the various soil samples as compared with constant head method results and were well within known ranges. The Slitchter, Beyer, Terzarghi and USBR empirical formulae, significantly underestimated the hydraulic conductivities of the samples and are probably not within the domain of applicability for the soils analyzed in the study area. Average hydraulic conductivity values determined using Kozeny-Carman equation, Hazen formula and the constant head permeameter test are 193.92 m/day, 102.37m/day and 171.93m/day respectively. The above values indicates that the groundwater yield of the aquiferous materials is adequate for municipal water supply.

Keywords:- Grain size distribution; Sieve analysis; hydraulic conductivity; porosity; permeameter.

I. INTRODUCTION

Physical characteristics of aquifers such as hydraulic conductivity, transmissivity and storativity that control groundwater flow and transport are very important properties and are usually estimated for groundwater flow model calibration. These parameters are also important properties for the assessment of contaminated land, and for safe construction of civil engineering structures. The hydraulic conductivity (K) is a hydro geologic property of the medium which refers to the ease with which a fluid can flow through the medium. It depends upon the porous medium and flowing fluid. Mathematically hydraulic conductivity is:

$$K = k \frac{\rho g}{\mu} \quad (1)$$

Where k = intrinsic permeability of porous medium and ρ and μ are density and dynamic viscosity of the flowing fluid respectively. Hydraulic conductivity is a direct function of average grain size distribution of granular porous media. Therefore, as the average grain size decreases from sand to clay, $K_{\text{sand}} > K_{\text{silt}} > K_{\text{clay}}$ [1]. Hydraulic conductivity determination can be done by different techniques such as field methods (pumping test of wells, auger hole test and tracer test), laboratory methods, namely constant head permeameter (CHP) and falling head permeameter (FHP) methods and calculations from empirical formulae [2]. However, the field methods are limited for accurate estimation of hydraulic conductivity due to aquifer geometry and precise knowledge of hydraulic boundaries as well as the cost of well construction and operations [3]. Alternatively, empirical formulae for estimating the hydraulic conductivity based on grain-size distribution characteristics have been developed and used to overcome these problems. Grain- size distribution methods are comparably less expensive and do not depend on the geometry and hydraulic boundaries of the aquifer. Soil is often made up of grains of many different sizes and textures. Since pore size distribution is very difficult to determine, the potential alternative is the grain size distribution as a substitute which is easy to measure and used for the approximation of hydraulic properties and estimation of hydraulic conductivity [4]. Several formulae have been established by many researchers and scientists based on experimental work using the hydraulic conductivity and grain size relationship, such as Hazen, Kozeny, Carman, Terzaghi, Shepherd, Alyamani and Sen [5-11].

The aim of this study is to estimate the value of hydraulic conductivity across sections of the shallow aquifer and assess its variability within the Yenagoa metropolis. Secondly, the study attempts to evaluate the applicability and reliability of some of the commonly used empirical formulae for the determination of hydraulic conductivity of unconsolidated soil materials.

II. LOCATION OF STUDY

Geographically, the study area is located between latitudes 4° 55' and 5° 05'N and longitudes 6°15' and 6°20'E, (Fig.1). It is within the freshwater swamp geomorphic unit of the Niger Delta Sedimentary Basin of Southern Nigeria. The swamps are vegetated tidal flats formed by a reticulate pattern of interconnected meandering creeks and tributaries of the River Niger and is endowed with the sedimentary rocks characteristic of the Niger Delta. Major access to the area is the Mbiama – Yenagoa road, with other minor network of roads linking the different communities and their environs.

The detailed geology and hydrogeology of the area has been described by a number of researchers. Litho-stratigraphically, the rocks of the study area are divided into the oldest Akata Formation (Paleocene), the Agbada Formation (Eocene) and the youngest Benin Formation (Miocene to Recent [12-14]. The Benin Formation is the water bearing zone of the area. It is overlain by Quaternary deposits (40-150m) thick, and generally consists of rapidly alternating sequences of sands and silty clay which later become increasingly prominent seawards [15-16]. Generally multi-aquifer systems have been identified in the Delta based on strata logs. The first aquifer is mostly unconfined, while the rest are confined. The average depth of boreholes in Yenagoa is between 10 and 40 metres. Deep boreholes in the area tap water from depths up to about 200m or more [17-18]. Rainfall is the major source of recharge for aquifers in the area.

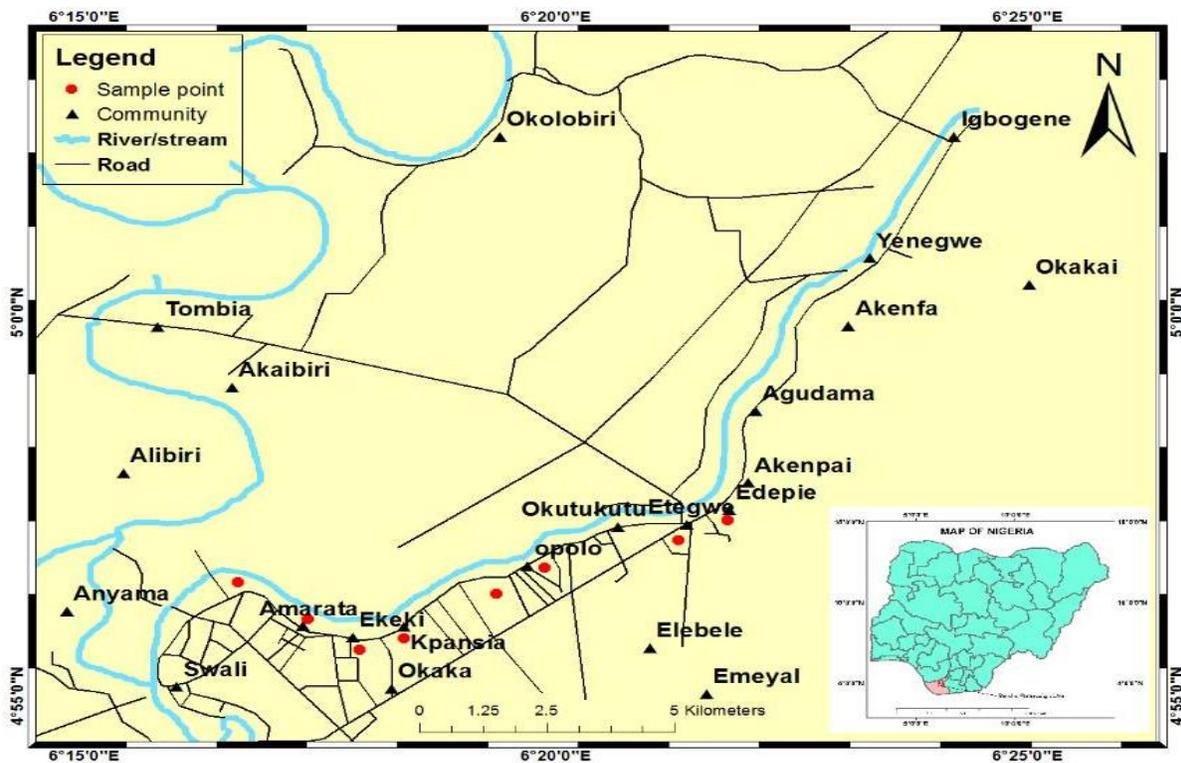


Fig. 1:- Map of Yenagoa showing sample points

III. METHODS OF STUDY

A. Soil boring

The investigation comprised eight boreholes with soil sampling executed using a light shell and auger hand rig. During the boring operations, disturbed samples were collected at 1m interval up to a depth of 20m. Undisturbed samples were also retrieved from the boreholes with conventional open-tube sampler 0.1m in diameter and 0.45m in length. The open-tube sampler consists essentially of a lower end and upper end screwed into a drive head that is attached to the rods of the rig. The head has an overdrive space and incorporates a non-return valve to permit the escape

of air or water as the sample enters the tube. The sampler is driven into the soil by dynamic means using a drop hammer. All samples recovered from the boreholes were examined, identified and roughly classified in the field. Following the completion of borehole drilling, the samples obtained at a subsurface depth of 15m across the eight study sites were isolated and taken for this study. The depth (15m) is strategic because it appears to be the approximate target depth exploited by most private boreholes within the Yenagoa metropolis. This relatively shallow depth although susceptible to surface and near surface contaminants is often the choice drill depth of private boreholes due to the huge financial budget required to drill deep wells. Secondly this choice is informed by the

observation that iron (Fe²⁺) is generally ubiquitous in the immediate underlying layers (20-40 meter) across subsurface sections of the study area.

B. Grain size analysis

The obtained soil samples were subsequently subjected to mechanical sieve analysis in order to construct the grain-size distribution curves according to the standard procedures. The grain-size diameters d₁₀, d₂₅, d₃₀, d₅₀, d₆₀ and d₇₅, were read off from the grain-size distribution curves (Fig. 2) from which the various grading coefficients and porosity values were calculated. The statistical grain-size methods were then employed to determine the hydraulic conductivity values as presented in Table 1. For the evaluation of empirical formulae, the results of hydraulic conductivity derived from the grain size approach were compared with the results obtained from the constant head permeameter test.

C. Constant head permeameter test

The constant head permeameter test method is mostly used for materials with medium to high hydraulic conductivity. In this setup, the soil sample is contained in a vertical cylinder between two porous plates. The inside area of the cylinder gives the cross-sectional area (A) of the soil sample. The length of the soil sample (L) is measured between the porous screens. The soil sample is first made saturated with water by connecting to a source of water supply. The

water continues to flow downwards through the sample until steady flow conditions develop. A constant water level in the supply reservoir is maintained by always having a small overflow taking place at its top. The soil sample is then placed in a constant head chamber which is filled up to the brim at the start of the experiment, so that any further water entering into it must overflow. This overflowing water is collected in a graduated jar over a certain counted period. The volume of water so collected, divided by the time, indicates the discharge (Q), which is the discharge coming out of the soil sample, under a head difference given by the difference between the water level in the supply reservoir and the constant head chamber (H).

Using Darcy’s law, the discharge is expressed by:

$$Q = KiA \tag{2}$$

Where i is the hydraulic gradient

$$K = \frac{QL}{HA} \tag{3}$$

To avoid large errors, the experiment was run over a reasonable period, so that the quantity of water collected in the jar is large in comparison to the least count of the graduated jar.

IV. RESULTS AND DISCUSSIONS

The result of the grain size analysis showing the sample location, particle diameter and the percentage passing is presented in Table 1.

Location	Opening(mm)	0.841	0.707	0.500	0.354	0.210	0.177	0.105	0.063
		Percentage finer							
Edepie	Sample 1	99.9	97.0	86.0	55.0	14.82	12.64	6.6	2.6
Etegwé	Sample 2	97.3	92.0	75.0	39	10.5	9.2	5.8	3.2
Opolo	Sample 3	98.6	90.7	73.0	46.2	12.9	10	6.1	2.9
Biogbolo	Sample 4	100	95.6	81.2	54.8	13.4	11.2	4.7	2.1
Kpansia	Sample 5	83.1	74.5	60.0	28.7	10.0	8.5	7.2	4.0
Ekeki	Sample 6	88.7	81.5	58.9	36.8	17.2	10.0	4.8	2.3
Amarata	Sample 7	92.8	86.7	65.4	46.2	13.4	8.7	4.5	1.8
Onopa	Sample 8	93.6	91.3	77.5	48.9	15.0	12.6	5.4	3.1

Table 1: Results of grain size analysis

The values obtained in Table 1 were used to construct the grain size distribution curve on a semi-logarithmic graph such that particle diameter was plotted on the X-axis and the percentage passing on the Y-axis (Fig. 2).

The quantitative analysis of the grain size distribution curves was based on the determined grading characteristics such as d₁₀, d₂₅, d₃₀, d₆₀ and d₇₅. From these geometric values, the effective size, uniformity coefficient, coefficient of sorting and coefficient of gradation were derived. Uniformity coefficient (Cu) is equal to d₆₀/d₁₀. Soils with Cu less than or

equal to 3 are considered to be “poorly graded” or “uniform”. Coefficient of gradation (Cc) = (d₃₀)²/(d₆₀×d₁₀). For well-graded soils, Cc is approximately equal to 1. The Sorting Coefficient S_O = (d₇₅/d₂₅)^{1/2}. This measure tends to be used more by geologists than engineers. The larger the value of S_O, the more well-graded the soil. The parameter d₁₀ is referred to as the "effective size" of the soil. Empirically, d₁₀ has been strongly correlated with the permeability of fine-grained sandy soils.

BH No.	Effective particle size D ₁₀ (mm)	D ₂₀ (mm)	D ₃₀ (mm)	Mean particle size D ₅₀ (mm)	D ₆₀ (mm)	Coefficient of uniformity	Coefficient of sorting	Coefficient of gradation
1	0.16	0.25	0.28	0.33	0.38	2.38	1.27	0.2056
2	0.2	0.3	0.33	0.38	0.41	2.05	1.25	1.0976
3	0.18	0.27	0.3	0.36	0.41	2.28	1.36	0.9878
4	0.17	0.26	0.28	0.32	0.38	2.24	1.29	1.0464
5	0.21	0.33	0.34	0.43	0.50	2.38	1.48	0.8897
6	0.18	0.22	0.31	0.42	0.51	2.83	1.57	0.5272
7	0.19	0.27	0.3	0.34	0.46	2.42	1.5	0.9592
8	0.15	0.26	0.31	0.36	0.42	2.80	1.46	1.1267

Table 2: Derivatives of particle size distribution test

From the grain-size distribution curves, soil samples were classified using British Standard Soil Classification System. The soils are classified into basic soil-type groups according to size, and the groups further divided into coarse, medium and fine sub-groups.

The percentage composition of the soil samples is presented in Table 3 and shows that samples 5 and 6 have the highest percentage of coarse size grains with 30.5% and 20.3% respectively, while samples 4 and 8 have the greatest percentage of fine grain fractions with 15% and 25.0% respectively. However, all eight samples are basically classified as medium sand because greater proportion of all the samples have grain size diameters between 0.2 – 0.5mm. Also, all the samples show uniform soil condition since uniformity coefficient is less than 3 and the grading curves are designated uniform grading curves as coefficient of gradation ranges from 0.5 to 2.

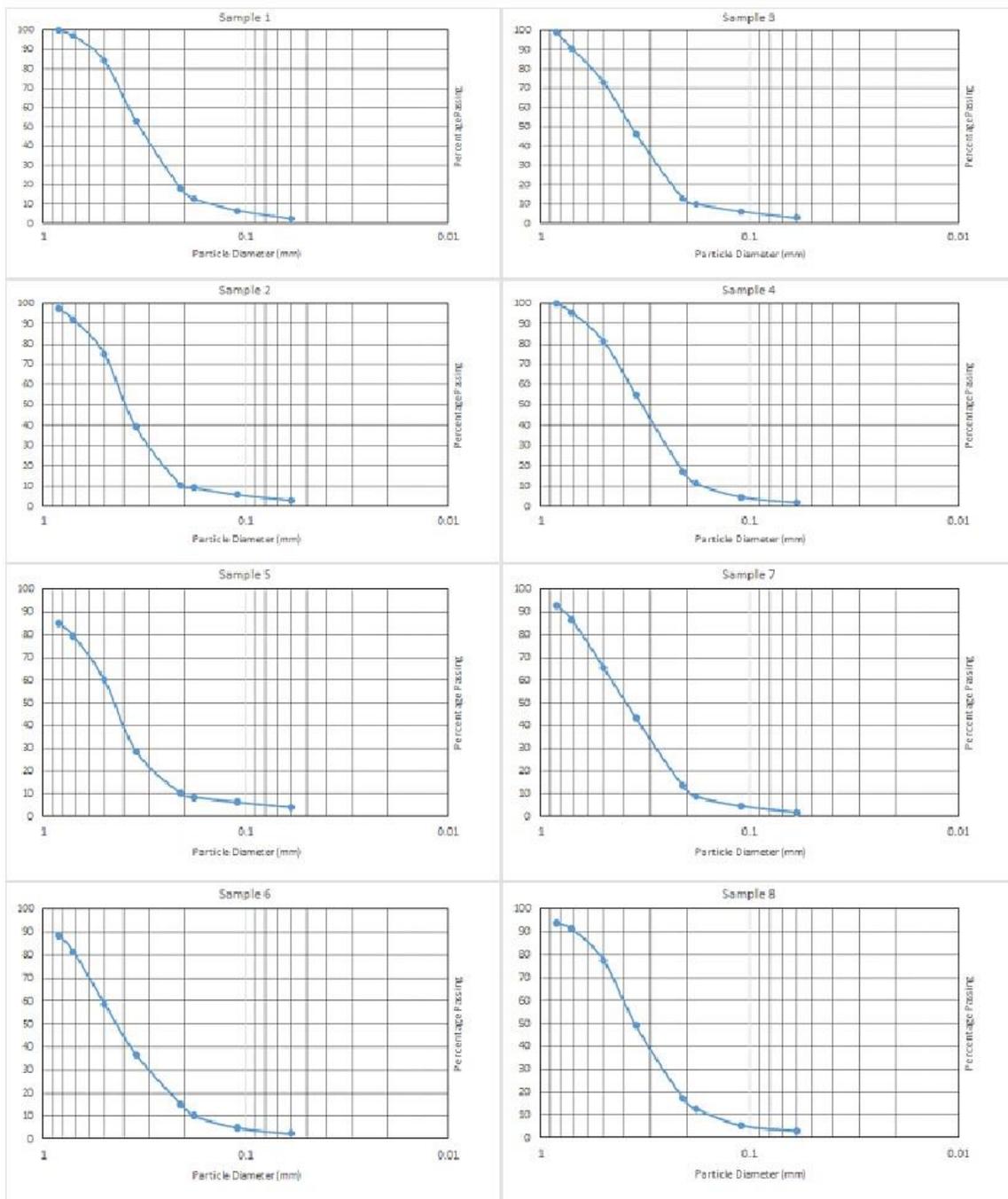


Fig. 2:- Grain size distribution curves of soil samples

Sample	Basic soil type	Composition of soil sample			Sub soil type
		% Fine (0.06-0.2)mm	% Medium (0.2-0.5)mm	% Coarse (0.6-2.0)mm	
1	Sand	12.2	82.2	5.6	Medium sand
2		10.5	81.5	8.0	Medium sand
3		13.2	77.5	9.3	Medium sand
4		15.0	76.3	8.7	Medium sand
5		6.0	63.5	30.5	Medium sand
6		14.6	65.1	20.3	Medium sand
7		11.4	73.5	15.1	Medium sand
8		25.0	66.3	8.7	Medium sand

Table 3: Classification of soil type based on percentage composition of grain sizes

The mathematical expression of the six empirical formulae used in the estimation of hydraulic conductivity in this study and their applicability is presented in table 4.

S/No.	Author	Mathematical formula	Applicability
1	Hazen	$K = \frac{\rho g}{\mu} \times 6 \times 10^{-4} \times [1 + 10(n - 0.26)](d_{10})^2$	Used for the estimation of hydraulic conductivity of uniformly graded soils ranges from fine sand to gravel of diameter 0.1 to 3 mm respectively. This formula basically depends on the effective size of grains.
2	Kozeny-Carman	$K = \frac{\rho g}{\mu} \times 8.3 \times 10^{-3} \times [n^3 / (n - 1)^2](d_{10})^2$	Widely used and accepted for hydraulic conductivity estimation because it depends on both the effective grain size and porosity (number of pores) of the porous media as given below.
3	Breyer	$K = \frac{\rho g}{\mu} \times 6 \times 10^{-4} \times [\log 500/u] \times (d_{10})^2$	Most useful for materials with heterogeneous distributions and poorly sorted grains with a uniformity coefficient between 1 and 20, and effective grain size between 0.06mm and 0.6mm.
4	Slitcher	$K = \frac{\rho g}{\mu} \times 1 \times 10^{-2} \times n^{3.287} \times (d_{10})^2$	This formula is most applicable for grain-size between 0.01mm and 5mm.
5	Tarzaghi	$K = \frac{\rho g}{\mu} \times C_t \times \left(\frac{n - 0.13}{\sqrt[3]{1 - n}}\right)^2 \times (d_{10})^2$	Tarzaghi's formula is most applicable for large-grain sand Where the C_t = sorting coefficient.
6	USBR	$K = \frac{\rho g}{\mu} \times 4.8 \times 10^{-4} \times (d_{10})^{2.3}$	United State Bureau of Reclamation (USBR) formula, estimates hydraulic conductivity from the effective grain size (d_{10}). This formula is suitable for medium-grain sand with a uniformity coefficient less than 5.

Table 4: Empirical formulae and applicability

Where n = porosity and u = coefficient of uniformity

For the evaluation and reliability of empirical formulae, values of hydraulic conductivity derived from the statistical grain size methods were compared with hydraulic conductivity results obtained from the constant head permeameter tests. Table 5 indicates that Hazen and Kozeny-Carman methods reveal hydraulic conductivity values ranging from 79.32 - 137.38 m/day with an average of 102.37 m/day and 96.90 - 423.03 m/day, average of 193.92 m/day respectively. Computed range of values using Breyer, Slitcher, Tarzarghi and USBR methods are recorded as 16.71 - 53.09, 8.81 - 37.74, 18.23 - 39.11, and 12.74 - 32.38 m/day with mean values of 36.27, 14.46, 24.99 and 20.76 m/day respectively. From Table 6, hydraulic conductivity values computed from the constant head permeameter range from 84.41- 414.53 m/day with an average of 171.93 m/day.

In general, results showed that the hydraulic conductivities calculated using the USBR and Slitcher methods are in all cases lower than values from the other methods as well as from the constant head permeameter test results just as reported by some other workers [19-20]; these methods are always considered inaccurate. Averagely, Terzaghi and Breyer methods gave results that were slightly higher than results from Slitcher and USBR but by far less than results from Hazen, Kozeny-Carman and permeameter test. Values obtained using the Kozeny-Carman formula are considered the most accurate for this study and compares most favorably with the results estimated from the permeameter test for six sample locations while results using the Hazen equation were closest to those from the permeameter test for samples 5 and 6. The mean hydraulic conductivity values based on the six empirical formulae used in this paper are represented in Fig. 3, while the variability of the hydraulic conductivity based on the constant head permeameter test across the sampling locations is expressed in Fig. 4. The hydraulic conductivity values shows only slight variation across the sample sites particularly in the eastern and central sections and are roughly between 50 – 150 m/day. Samples 7 and 8 which are located at the western portion of the study area, reveals relatively higher values of 246.15 and 414.53 m/day respectively, suggesting aquifers at the western zone may be more productive than those in the east and central zones of the study area.

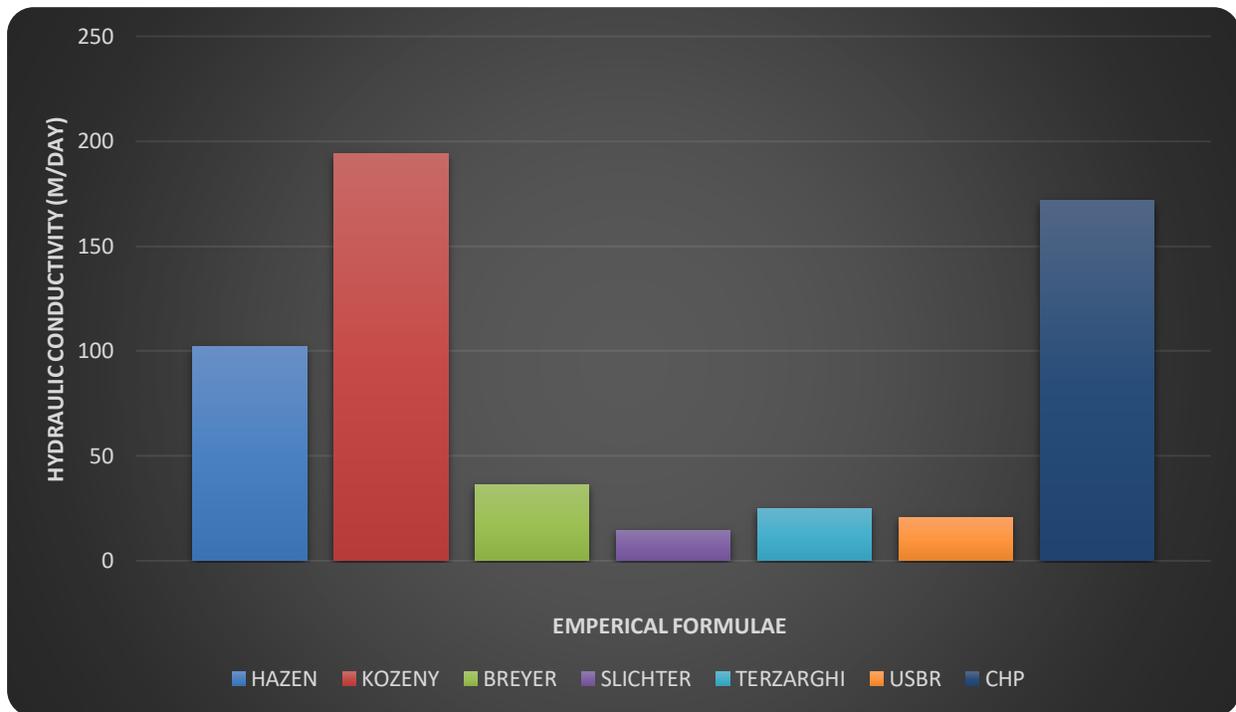


Fig. 3:- Average values of hydraulic conductivity based on empirical formulae

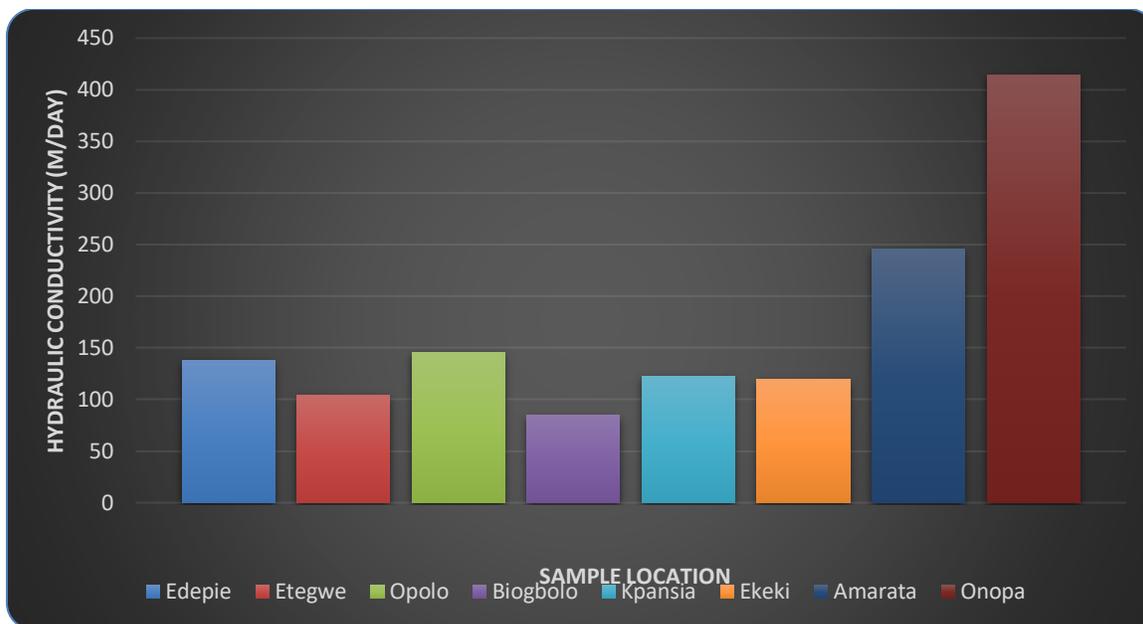


Fig. 4:- Variability of hydraulic conductivity values based on permeameter test across the study locations

BH No.	Porosity $0.255(1+0.83u)$	Hazen	Kozeny-Carman	Breyer	Slitchter	Tarzarghi	USBR
1	0.7577	79.32	135.99	30.83	8.89	18.81	17.10
2	0.6889	109.67	96.90	45.50	10.15	18.23	26.01
3	0.7371	96.93	134.63	39.31	10.37	19.45	20.41
4	0.7280	85.10	108.13	35.20	8.81	16.71	18.72
5	0.7599	137.38	236.41	53.09	15.38	30.84	32.38
6	0.8547	116.68	187.15	16.71	37.74	39.11	12.74
7	0.7674	113.67	229.39	43.30	13.06	25.25	20.41
8	2.8000	80.20	423.03	26.26	11.29	20.60	18.72

Table 5: Hydraulic conductivities calculated from empirical formulae

Soil Sample: Loose sand mixture Specimen diameter D = 6.22cm	Borehole No.							
	1	2	3	4	5	6	7	8
X-Area (m ²)	0.0122	0.0122	0.0122	0.0122	0.0122	0.0122	0.01222	0.0122
Piezometer tap distance L = 10.35cm	10.35	10.35	10.35	10.35	10.35	10.35	10.35	10.35
Piezometer level distance (cm)	5.24	5.4	4.8	4.38	4.84	4.7	5.39	4.75
Duration of sampling (s) t	60	60	60	60	60	60	60	60
Mass of water collected & container (g) M _{wc}	424	417	428	417	423	423	443	484
Mass of container (g) M _c	398	398	398	398	398	398	398	398
Hydraulic gradient i	0.5063	0.5217	0.4638	0.4232	0.4676	0.454106	0.5208	0.4589
Discharge velocity (m/s) v	0.26	0.19	0.3	0.19	0.25	0.25	0.45	0.86
Hydraulic conductivity at ambient temperature (m/day)	138.24	104.11	146.02	84.41	122.77	119.23	246.15	414.53

Table 6: Hydraulic conductivities estimated from constant head permeameter test

CONCLUSIONS

The study estimated hydraulic conductivity (K) values from the analysis of grain-size distribution and constant head permeameter test. The mean hydraulic conductivity results based on calculation using six empirical formulae were of the order: Kozeny-Carman > Hazen >Breyer>Terzaghi> USBR >Slichter. Out of the six empirical equations, only Kozeny-Carman and Hazen formulae reliably estimated the hydraulic conductivities of the various soil samples as compared with constant head method results, whereas the other four methods generally underestimated the hydraulic parameter.

The soils were classified as uniformly graded medium sands as per the percentage particle size composition and grading characteristics of the analyzed samples. Average hydraulic conductivity values determined using Kozeny-Carman equation, Hazen formula and the constant head permeameter test are 193.92 m/day, 102.37m/day and 171.93m/day respectively, suggesting prospective groundwater yield of the aquiferous materials will be adequate for municipal supply.

Lastly, hydraulic conductivity can be reliably estimated using the appropriate empirical formulae based on grain size distribution and grading characteristics. However, to avoid the pitfall of either underestimation or overestimation, the applicability of the formula should be tested on the basis of the semblance of its results with that of other established or traditional methods.

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