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Free Vibration of a Cracked and Misaligned Rotor

Subham Kumar Gupta[1],
UG Student, Department of Mechanical Engineering,
Koneru Lakshmaiah Education Foundation, Vaddeswaram,
Guntur, Andhra Pradesh, India –522502

Parvej Alam[2]
UG Student, Department of Mechanical Engineering,
Koneru Lakshmaiah Education Foundation, Vaddeswaram,
Guntur, Andhra Pradesh, India –522502

Abstract:- In this paper an analytical method is used meant for the vibration study of a cracked and misaligned rotor which is given by Chondros theoretically. By considering a simply supported beam which acts like a turbine shaft with crack and without crack a comparison is made between their natural frequencies which varies along with the crack ratio and position of the crack. The existence of cracks causes changes in the mechanical properties, physical properties and stiffness of the structure. Furthermore, it leads to the alteration in energetic retort of the whole beam. The results obtained from finite element analysis can be compared theoretically by the governing equation given by Chondros. The analysis was performed using ANSYS 18.1 software. Here the shaft crack is considered in the form of spring.

***Keywords**:- Natural frequencies, crack formation, simply supported beam, ANSYS.*

I. INTRODUCTION

Fatigue cracks are the prompt source of dire failure in shafts. Researchers have been developing a pool proof strategy in identifying the cracks in dynamic machines. These cracks minimize the stiffness of the body. This crack can be modelled in ANSYS or any analyzing software and the natural frequencies can be calculated. This analysis results can be compared with the experimentation or theoretically. In this paper the result obtained from ANSYS can be compared with the cracked theory given by Chondros. [1] This paper has presented the analysis of a cracked beam which is then compared with the numerical integration results which satisfactorily gives the same results. [2] Vibration when integrated with other condition monitoring tools like oil analysis, wear debris analysis gives perfect monitoring program for rotating equipment in process industries and concluded that vibration monitoring technique can reduce the breakdowns of power plants. [3] This paper shows the fault recognition in rotating of Turbine shaft by investigating its vibration data using FFT technique that is well known in checking vibration of any rotating equipment's and this paper presents an rationalized review of a diversity of vibration feature mining techniques that have demonstrated success when applied to rotating machineries. [4] This appraisal focused on health monitoring of rolling elements like bearings. Bearing Condition Monitoring using signal processing methods, using artificial intelligent diagnostic techniques were discussed. [5] In this paper, smart condition monitoring concept has been outlined. During presentation few case studies presented to support the present smart condition monitoring hypothesis. Some cases of bearing parameter estimation and crack force estimation covered. [6] This paper shows the behavior of turbine shaft under normal as well as defective condition & its effects on its performance. The detection in the fault in shaft and the relation with respect of faults has been studied. [7] This paper summarized the consequences of an appraisal of vibration analysis procedures as a technique for identification of gear faults. The utmost effective vibration procedures are deliberated and experimentally equated, regarding an industrial gearbox. [8] Refined the differential equation and boundary conditions of the damaged beam as a one-dimensional gamut. The constant cracked beam vibration philosophy developed at this point. A steel beam with a multi-edge crack was also examined. [9] This paper analyses the rotating appliance condition monitoring based on vibration data analysis. After the analysis of major, well established and sophisticated approaches, new unaided approaches based on per-mutative investigation are also momentarily referenced. [10] In this paper, a reasoned technique for vibration analysis of a fractured simply supported beam is examined. By considering a non-linear model for the lethargy crack, the governing equation of motion of the fractured beam is explained by perturbation method. [11] The crack was created as an incessant flexibility by means of the displacement field near the crack, found with rupture mechanics methods. The consequences of two autonomous evaluations of the lowest natural frequency of perpendicular vibrations for beams with a single-edge flaw are presented. Experimental results from Aluminum beams with lethargy cracks are very nigh to the values supposed. [12] The paper presents a common industrial case study and observations. Vibration attitude of turbo-generator (TG) set during unlade shutdown condition. Fatigue tests to interpret the knife-edge crack propagation straight at low ferment loads. [13] In this paper a mode of free vibration analysis of multi-cracked propeller is obtainable. The cracks are expected to be in the first mode of flaw, i.e. the inaugural mode. Based on the Timoshenko beam philosophy, the frequency equation can be assembled by accumulating the transfer matrix of each segment of the multi-stepped and multiple cracked rotor, and then resolve the frequencies as well as the consistent mode shapes of the cracked rotor. The paraphernalia of both analogous distances along axis and/or locations of cracks are measured in free vibration analysis. An algorithm and statistical examples are comprised. [14] In this paper, natural frequencies and mode shapes of fractured beam are attained using the finite element technique. The authors made a shape function that can utterly satisfy the local flexibility circumstances at the crack locations, which can give more precise vibration modes. [15] this paper presents relation between vibration amplitude and on the crack shaft, depth was created, this helps in determine the depth of the crack by measuring the vibration data in terms of peaks. By observing the generated peaks on the obtained curve of vibration increases with respect to the depth of the crack due to reduction of material from the shaft and it decreases the stiffness of the shaft.

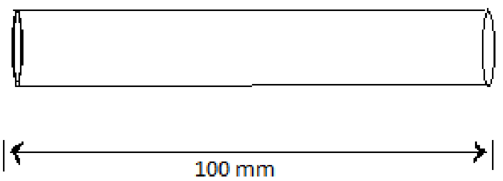

Fig 1:- uncracked rotor shaft

II. NUMERICAL APPROACH (FINITE ELEMENT METHOD)

In finite element method, the first three natural frequencies and its mode shapes of a cracked beam has been identified under simply supported beam boundary condition in ANSYS APDL solver (version 18.1). The three-dimensional rotor is modelled like a simply supported beam with solid brick elements. A sample of the meshed element shown below Fig 2.

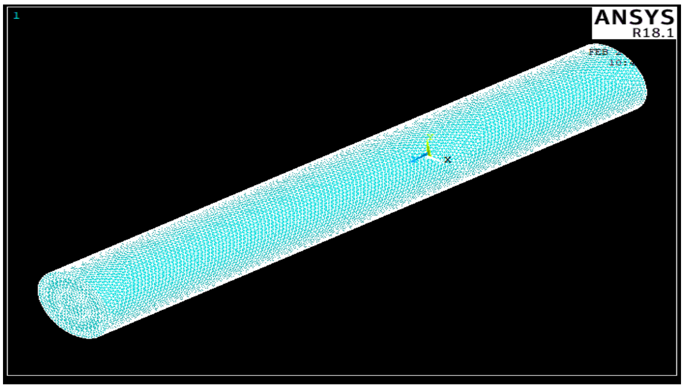

Fig 2:- Meshed rotor with crack

Theoretically these obtained natural frequencies can be validated with the equation given below, unlike crack models have been projected and explored computationally and experimentally. Here, a crack model from Chondros [2] is used. The ability of this crack model to forecast changes in beam natural frequencies has shown good pact with other models in the literature as well as experimental results.

The idea of the basic crack model is seen in Fig. 1. If the crack is only clearly seen in its neighborhood, the crack is shown as a local alteration in rotor flexibility. The location of the rotor which comprises the crack is modeled as a torsion spring with a local compliance different than that of the undamaged rotor as shown in Fig. 1. The local compliance of the rotor is then

$$\text{Compliance, } C = \frac{6\pi(1-\vartheta^2)h.\emptyset(\frac{a}{h})}{E.I}$$

Where, ϑ = Poisson's ratio
E = Young's Modulus
I = Moment of Inertia
a = Depth of the crack
h = Diameter of the rotor
$\eta = a/h$

$$\emptyset = 0.6272\left(\frac{a}{h}\right)^2 - 1.04533\left(\frac{a}{h}\right)^3 + 4.5948\left(\frac{a}{h}\right)^4 - 9.9736\left(\frac{a}{h}\right)^5 + 20.2948\left(\frac{a}{h}\right)^6 - 33.0351\left(\frac{a}{h}\right)^7 + 47.1063\left(\frac{a}{h}\right)^8 - 40.7556\left(\frac{a}{h}\right)^9 + 19.6\left(\frac{a}{h}\right)^{10}$$

The basic equation for the cracked rotor is given by:

$$\frac{d^4\omega}{d\eta^4}T + \left(\frac{\rho A}{EI}\right)p^2L^4\omega = 0$$

The characteristic equation is

$$m^4 - p^4 = 0$$

$$\Rightarrow (m^2 - p^2)(m^2 + p^2) = 0$$

$$\Rightarrow m = \pm p\ ;\ m = \pm ip$$

$$\omega(\eta) = Ae^{p\eta} + Be^{-p\eta} + C\cos p\eta + D\sin p\eta \qquad (1)$$

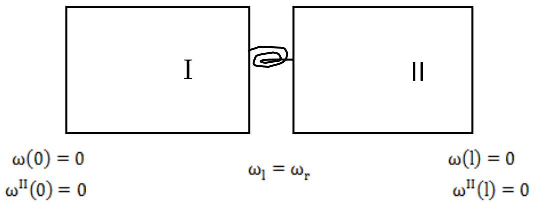

Fig 3:- Crack model of the rotor with end conditions

$$\frac{\partial\omega_l}{\partial\eta} - \frac{d\omega_r}{d\eta} = EIC\frac{\partial^2\omega_l}{\partial\eta^2}$$

$$\frac{\partial^2\omega_l}{\partial\eta^2} = \frac{\partial^2\omega_r}{\partial\eta^2}$$

$$\frac{\partial^3\omega_l}{\partial\eta^3} = \frac{\partial^3\omega_r}{\partial\eta^3}$$

Equation (1) can also be written as

$$\omega(\eta) = A\cosh(p\eta) + B\sinh(p\eta) + C\cos(p\eta) + D\sin(p\eta) \qquad (2)$$

On substituting the boundary conditions

$$\omega(0) = 0 \Rightarrow A + C = 0$$

$$\omega^{II}(0) = 0 \Rightarrow A - C = 0$$

$$\Rightarrow A = 0 \text{ and } C = 0$$

$$\omega(\eta) = B\sin(p\eta) + D\sin(p\eta) \qquad (3)$$

$$\text{Let } \varepsilon = 1 - \eta$$

$$\omega(\varepsilon) = B\sin(p\varepsilon) + D\sin(p\varepsilon) \qquad (4)$$

Boundary Conditions:

$$\omega_l(\eta_c) = \omega_r(\varepsilon_c) \qquad (5)$$

$$\omega_l{}^{II}(\eta_c) = \omega_r{}^{II}(\varepsilon_c) \qquad (6)$$

$$\omega_l{}^{II}(\eta_c) = -\omega_r{}^{II}(\varepsilon_c) \qquad (7)$$

$$\omega_l{}^{I}(\eta_c) + \omega_r{}^{I}(\varepsilon_c) = \frac{K}{l}\omega_l{}^{I}(\eta_c) \qquad (8)$$

On Applying Boundary Conditions

(5) =>$B\sinh(\rho\eta_c) + D\sin(\rho\eta_c) = B_I\sinh(\rho\eta_c) + D_I\sin(\rho\eta_c)$

(6)=>

$P^2\,[B\sinh(\rho\eta_c) - D\sin(\rho\eta_c)] = P^2\,(B_I\sinh(\rho\eta_c) - D_I\sin(\rho\eta_c)]$

(7)=>

$P^3\,[B\cosh(\rho\eta_c) - D\cos(\rho\eta_c)] = -P^3\,(B_I\cosh(\rho\eta_c) - D_I\cos(\rho\eta_c)]$

(8)=>

$$P[B\cosh(\rho\eta_c) + D\cos(\rho\eta_c) + B_I\cosh(\rho\eta_c) + D_I\cos(\rho\eta_c)] = \frac{K}{I}\rho^2[B\sinh(\rho\eta_c) - D\sin(\rho\eta_c)]$$

On writing in a matrix form:

$$\begin{bmatrix} \sinh(\rho\eta_c) & \sin(\rho\eta_c) & -\sinh(\rho\varepsilon_c) & -\sin(\rho\varepsilon_c) \\ \sinh(\rho\eta_c) & \sin(\rho\eta_c) & -\sinh(\rho\varepsilon_c) & \sinh(\rho\varepsilon_c) \\ \cosh(\rho\eta_c) & -\cos(\rho\eta_c) & \cosh(\rho\varepsilon_c) & -\cosh(\rho\varepsilon_c) \\ \{\cosh(\rho\eta_c) - \frac{K\rho}{I}\sinh(\rho\eta_c)\} & \{\cosh(\rho\eta_c) + \frac{K\rho}{I}\sinh(\rho\eta_c)\} & \cosh(\rho\varepsilon_c) & \cosh h(\rho\varepsilon_c) \end{bmatrix} \begin{bmatrix} B \\ D \\ B_I \\ D_I \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

On doing Row operations for the above matrix:

I – II: $2\sin(\rho\eta_c)D - 2\sin(\rho\varepsilon_c)D_I = 0$ (9)

I + II: $2\sin(\rho\eta_c)B - 2\sinh\sin(\rho\varepsilon_c)B_I = 0$ (10)

III – IV:

$$\frac{K\rho}{I}\sinh(\rho\eta_c)B - \left\{2\cos(\rho\eta_c) + \frac{K\rho}{I}\sin(\rho\eta_c)\right\}D - 2\cos(\rho\varepsilon_c)D_I \quad (11)$$

III + IV:

$$\{2\cosh(\rho\eta_c) - \frac{K\rho}{I}\sinh(\rho\eta_c)\}B{+}{:}\ \frac{K\rho}{I}\sin(\rho\eta_c)D + 2\cosh(\rho\varepsilon_c)B_I = 0 \quad (12)$$

$$D = \frac{f_2}{f_1}D_I$$

$$B = \frac{f_4}{f_3}B_I$$

$$f_5B - f_6D - f_7D_I = 0$$

$$f_8B + f_9D + f_{10}B_I = 0$$

$$\frac{f_5f_4}{f_3}B_I - \frac{f_6f_2}{f_1}D_I - f_7D_I = 0$$

$$\frac{f_5f_4}{f_3}B_I - \{\frac{f_6f_2}{f_1} + f_7\}D_I = 0 \quad (13)$$

$$\frac{f_8f_4}{f_3}B_I + \frac{f_9f_2}{f_1}D_I + f_{10}B_I = 0$$

$$\left[\frac{f_8f_4}{f_3} + f_{10}\right]B_I + \frac{f_9f_2}{f_1}D_I = 0 \quad (14)$$

Eliminating B_I & D_I from (13), (14)

$$\frac{f_5f_4f_9f_2}{f_3f_1} + \left[\frac{f_8f_9}{f_3} + f10\right]\left[\frac{f_6f_2}{f_1} + f7\right] = 0$$

$$\frac{f_5f_4f_9f_2 + f_8f_9f_6f_2}{f_3f_1} + \frac{f_7f_8f_9}{f_3} + \frac{f_6f_2f_{10}}{f_1} + f_{10}f_7 = 0$$

$$\frac{f_2f_9(f_4f_5 + f_6f_8)}{f_1f_3} + \frac{f_7f_8f_9}{f_3} + \frac{f_2f_6f_{10}}{f_1} + f_7f_{10} = 0$$

$$F = f_2f_9(f_4f_5 + f_6f_8) + f_1f_7f_8f_9 + f_2f_3f_6f_{10} + f_1f_3f_7f_{10} = 0 \quad (15)$$

Where,

$$f_1 = \sin(\rho\eta_c)$$

$$f_2 = \sin(\rho\varepsilon_c)$$

$$f_3 = \sin h(\rho\eta_c)$$

$$f_4 = \sin h(\rho\varepsilon_c)$$

$$f_5 = \frac{K\rho}{I}\sinh(\rho\eta_c)$$

$$f_6 = 2\cos(\rho\eta_c) + \frac{K\rho}{I}\sinh(\rho\eta_c)$$

$$f_7 = 2\cos(\rho\varepsilon_c)$$

$$f_8 = 2\cos h(\rho\eta_c) - \frac{K\rho}{I}\sinh(\rho\eta_c)$$

$$f_9 = \frac{K\rho}{I}\sinh(\rho\eta_c)$$

$$f_{10} = 2\cosh(\rho\varepsilon_c)$$

Equation (15) can be solved by to calculate natural frequency of cracked rotor.

III. RESULTS AND DISCUSSION

The following natural frequencies values for cracked and misaligned shaft shown below in the table are obtained from ANSYS 18.1 software. These results are compared with the theoretical experiment done by Chondros[11] which approximately gives the same result obtain in ANSYS software with an error rate of 3-6 %, so this analysis is valid for a approximate results in any of the terotechnology department to carry out the condition monitoring

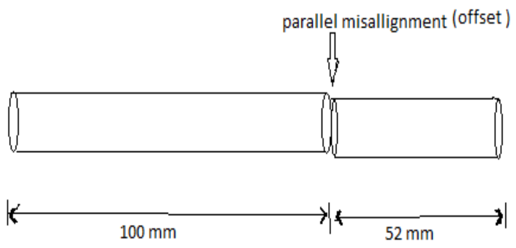

Fig 4:- parallel misalignment

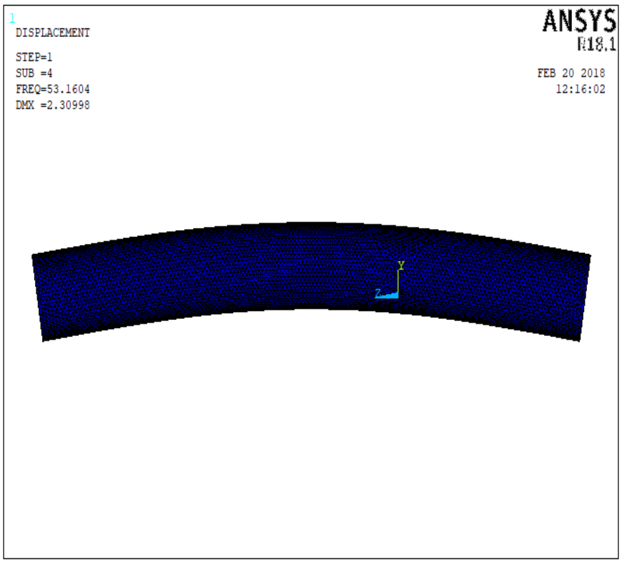

Fig 5:- 1st mode shape of aligned rotor

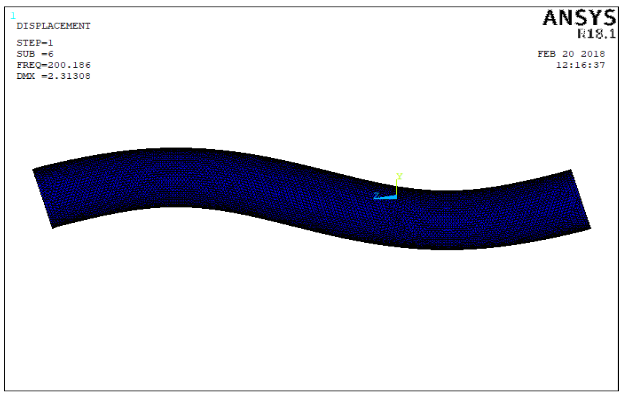

Fig 6:- 2nd mode shape of aligned rotor

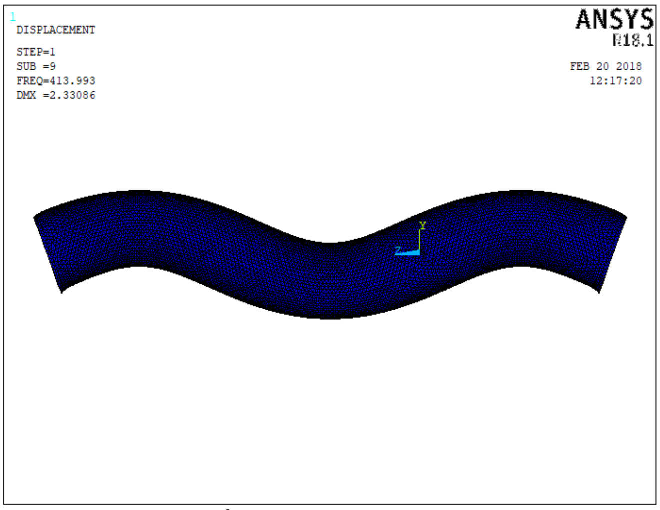

Fig 7:- 3rd mode shape of aligned rotor

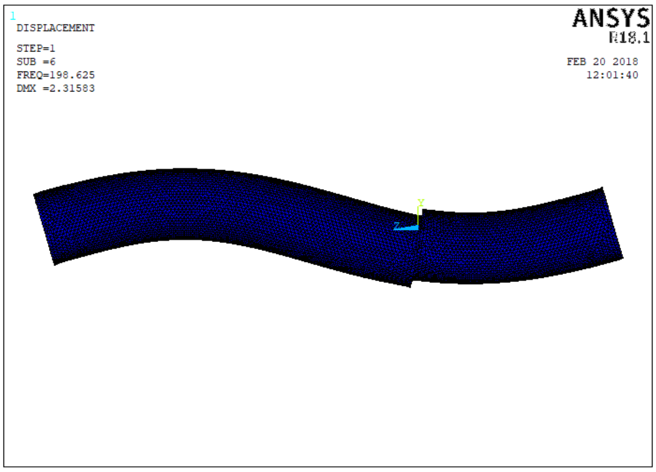

Fig 8:- mode shape of parallelly misaligned shaft

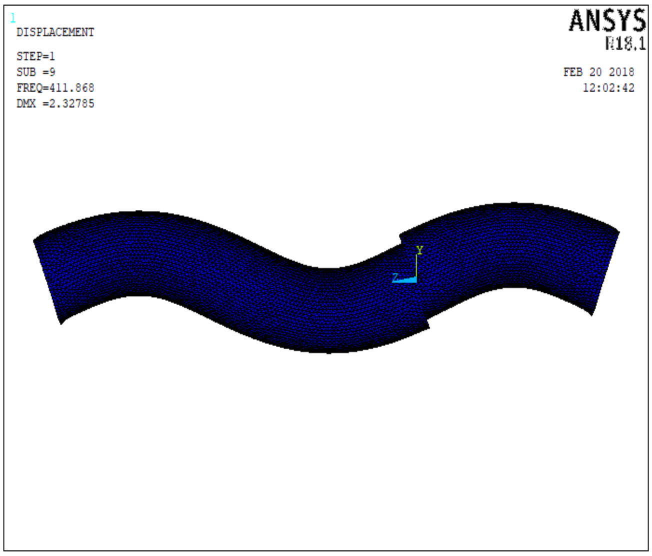

Fig 9:- mode shape of misaligned shaft.

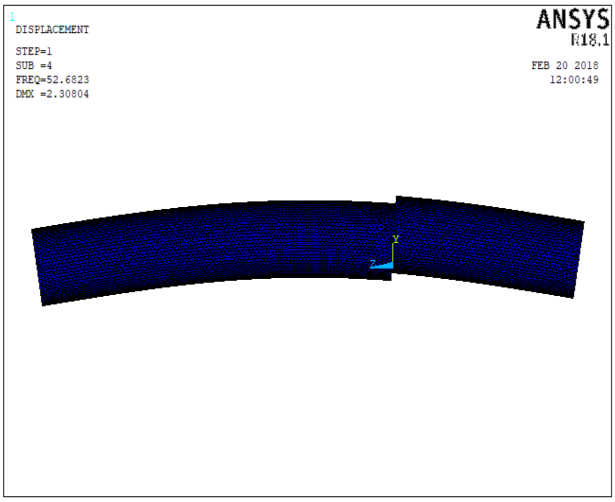

Fig 10:- mode shape of parallelly misaligned shaft

	Natural frequencies		
Offset distance	ω_1	ω_2	ω_3
At (0,0) coinciding	54.201	204.95	524.30
0.1mm	53.941	204.06	524.44
0.2mm	53.722	202.78	524.35
0.3mm	53.694	202.73	524.27
0.4mm	54.048	204.08	524.42
0.5mm	53.683	202.69	524.22
0.6mm	53.677	202.67	524.18
0.7mm	53.652	307.34	524.19
0.8mm	53.642	307.20	524.13
0.9mm	53.637	307.53	524.12
1mm	53.597	202.37	524.10
2mm	53.35	201.54	523.9

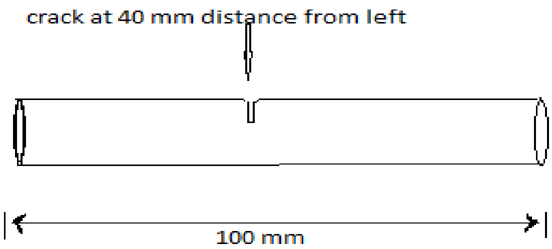

Fig 11:- crack in the rotor shaft

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	Natural frequencies		
Crack position	ω1	ω2	ω3
uncracked	125	443	866
At 10 mm distance	123	434	843
At 20 mm distance	121.79	426	839
At 30 mm distance	120.19	426.42	851.89
At 40 mm distance	118.76	432.97	845.78
At 50 mm distance	118	436	833
At 60 mm distance	118.99	432.95	846.70
At 70 mm distance	120.61	426.29	838.5
At 80 mm distance	121.74	426.29	838.5
At 90 mm distance	123.64	434.92	844.11

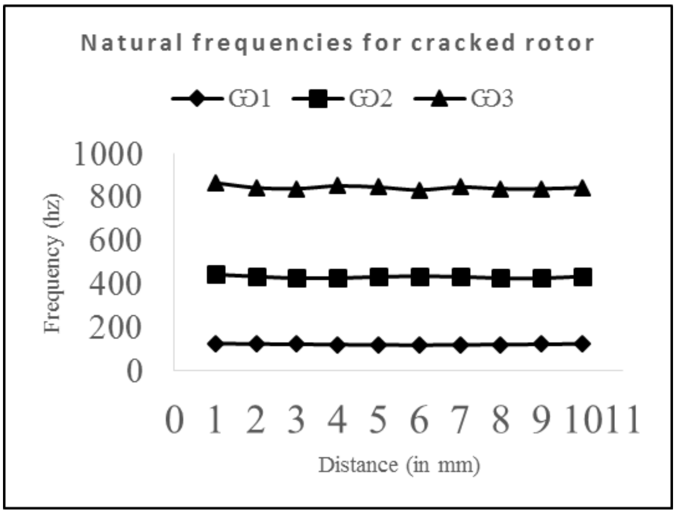

Fig 12:- frequency variation in a cracked rotor.

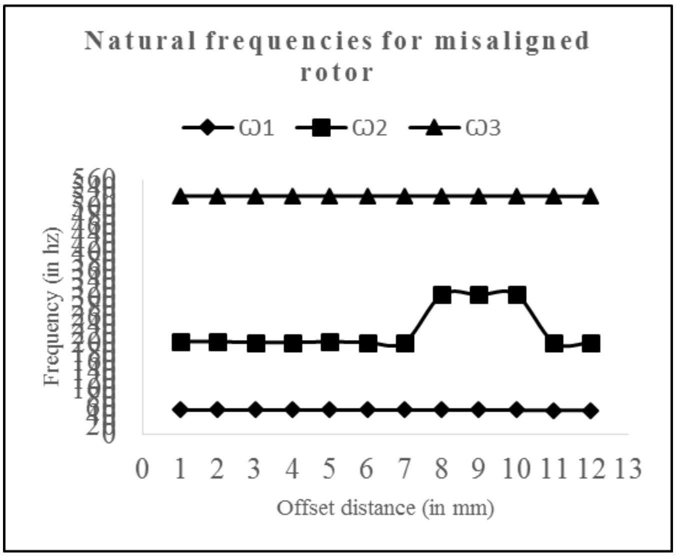

Fig 13:- frequency variation in a misaligned rotor

The following graphs (a) & (b) shows the deviation of frequency with respect to location of the crack in the turbine rotor as the crack location and depth of the crack increases frequency decreases, in case of parallel misalignment also we can observe that frequency changes with respect to the offset in between one axis to another. So, it can be a major problem in any of the power industries that will shut down the work due to ceasing of the machinery.

IV. CONCLUSION

Natural frequencies of a cracked rotor decrease by varying the crack location of a simply supported beam. In this paper it is proven theoretically and analytically that frequency drop is maximum at the center of the simply supported beam. Further as the crack location increases as well as by varying the depth of crack frequency decreases. In this paper the effect of parallel misalignment are also shown, when the offset angle increases, frequency decreases. For small cracks there will be little variation in the natural frequencies but when crack depth and offset angle increases it hints to ceasing of machinery.

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