7 Fractional Triple Laplace Transform and its Properties

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Abstract:- In this article, we introduce definition of a fractional triple Laplace transform of order α , 0 < α ≤ 1 , for fractional differentiable functions. Some main properties and inversion theorem of fractional triple Laplace transform are established. Further, the connection between fractional triple Laplace and fractional triple Sumudu transforms are presented.

***keywords*:-** *Fractional Riemann-Liouville Derivatives,triple Laplace transform ,triple Sumudu transform, Mittag Leffler function.*

I. INTRODUCTION

There are various integral transforms in the literature which are used in astronomy, physics and also in engineering. The integral transforms were vastly applied to obtain the solution of differential equations, therefore there are different kinds of integral transforms like Mellin, Laplace, Fourier, and so on. Partial differential equations are considered one of the most significant topics in mathematics and others. There are no general methods for solve these equations. However, integral transform method is one of the most familiar method in order to get the solution of partial differential equations [1,2]. In [3,4] triple Laplace transform was used to solve third order partial differential equations. Moreover the relation between them and their applications to differential equations have been determined and studied by [5,6]. In this study we focus on triple Laplace transforms. First of all, we start to recall the definition of triple Laplace transform as follows

II. DEFINITION

Definition 1 := Let f be a continuous function of three variables; then, the triple Laplace transform of $f(x,y,t)$ is defined by

$$L_{x,y,t}[f(x,y,t)] = F(p,s,k) = \int\int\int_0^{\infty} e^{-px}e^{-sy}e^{-kt}f(x,y,t)dx\,dy\,dt. \tag{1}$$

where, $x,y,t > 0$ and p,s,k are Laplace variables, and

$$f(x,y,t) = \frac{1}{2\pi i}\int_{\alpha-i\infty}^{\alpha+i\infty} e^{px}\left[\frac{1}{2\pi i}\int_{\beta-i\infty}^{\beta+i\infty} e^{sy}\left[\frac{1}{2\pi i}\int_{\mu-i\infty}^{\mu+i\infty} e^{kt}F(p,s,k)dk\right]ds\right]dp \tag{2}$$

is the inverse Laplase transform.

III. FRACTIONAL DERIVATIVE VIA FRACTIONAL DIFFERENCE

Definition:= Let $g(t)$ be a continuous function and not necessarily differentiable function, then the forward operator $FW(h)$ is defined as follows

$$FW(h)\,g(t) := g(t+h),$$

where $h > 0$ denote a constant discretization span.

Moreover, the fractional difference of $g(t)$ is known as

$$\Delta^{\alpha} g(t) = (FW-h)^{\alpha} g(t) = \sum_{m=0}^{\infty}(-1)^m({}^{\alpha}C_m)g[t+(\alpha-m)h]\ where\ 0<\alpha<1.$$

and the α-derivative of $g(t)$ is known as

$$g^{(\alpha)}(t) = \lim_{h\to 0}\frac{\triangle^{\alpha} g(t)}{h^{\alpha}}.$$

See the details in [9,10].

IV. MODIFIED FRACTIONAL RIEMANN-LIOUVILLE DERIVATIVE

The author in [10] proposed an alternative definition of the Riemann-Liouville fractional derivative
Definition:= Let $g(t)$ be a continuous function, but not necessarily differentiable, then

- Let us presume that $g(t)=K$, where K is a constant, thus α- derivative of the function $g(t)$ is

$$\begin{cases} D_t^\alpha K = K\Gamma^{-1}(1-\alpha)t^{-\alpha}, & \alpha < 0 \\ = 0, & otherwise. \end{cases}$$

- On the other hand, when $g(t) \neq K$ hence

$$g(t) = g(0) + (g(t) - g(0)),$$

and fractional derivative of the function $g(t)$ will be known as

$$D^a g(t) = D_t^a g(0) + D_t^a (g(t) - g(0)),$$

at any negative α,(α < 0) one has

$$D_t^\alpha (g(t) - g(0)) = \frac{1}{\Gamma(-\alpha)} \int_0^t (t-\tau)^{-\alpha-1} g(\eta) d\eta, \ \alpha < 0,$$

while for positive α, we will put

$$D_t^a (g(t) - g(0)) = D_t^a g(t) = D_t (g^{(a-1)}).$$

When $m < \alpha < m+1$, we place

$$g^\alpha(t) = (g^{(\alpha-m)}(t))^m, \ m \leq \alpha < m+1, \ m \geq 1.$$

V. INTEGRAL WITH RESPECT TO $(dt)^a$

The next lemma show the solution of fractional differential equation

$$dy = g(t)(dt)^\alpha, \ t \geq 0, \ y(0) = 0, \tag{3}$$

by integration with respect to $(dt)^a$.

Lemma 5.1 *If g(t) is a continuous function, so the solution of (3) is defined as the following*

$$y(t) = \int_0^t g(\eta)(d\eta)^\alpha, \ y(0) = 0$$
$$= \alpha \int_0^t (t-\tau)^{\alpha-1} g(\eta) d\eta, \ 0 < \alpha < 1, \tag{4}$$

for more results and various views on fractional calculus,see for example [8,9,10,11].

VI. FRACTIONAL TRIPLE SUMUDU TRANSFORM

We defined Triple Sumudu transform of the function depended on three variables . Analogously, fractional triple Sumudu transform was defined and some properties were given as the following.

Definition:= The fractional triple Sumudu transform of function $f(x,y,t)$ is known as

$$S_\alpha^3 f(x,y,t) = G_\alpha^3(u,v,w) = \int\int\int_0^\infty E_\alpha(-(x+y+t)^\alpha) f(ux,vy,tw)(dx)^\alpha (dy)^\alpha (dt)^\alpha,$$

where $u,v,w \in C$, $x,y,t>0$ and $E_\alpha(x) = \sum_{m=0}^\infty \frac{x^m}{\Gamma(\alpha m+1)}$ is the Mittag-Leffler function.

VII. TRIPLE LAPLACE-SUMUDU DUALITY OF FRACTIONAL ORDER

Definition:= Let *f*(*x*,*y*,*t*) denote a function which vanishes for negative values of t.Its triple Laplace's transform of order α (or its α *th* fractional Laplace's transform) is define by the following expression:

$$L_\alpha f(x,y,t) = F_\alpha(u,v,w) = \int\int\int_0^\infty E_\alpha(-(ux+vy+wt)^\alpha) f(x,y,t)(dx)^\alpha (dy)^\alpha (dt)^\alpha$$

$$= \lim_{M\to\infty} \int\int\int_0^M E_\alpha(-(ux+vy+wt)^\alpha) f(x,y,t)(dx)^\alpha (dy)^\alpha (dt)^\alpha. \quad (5)$$

provided that integral exists.

Theorem 7.1 *If the Laplace transform of fractional order of a function f(x,y,t) is* $L_a[f(x,y,t)]=F_a(u,v,w)$ *and the Sumudo transform of this function is*

$S_a[f(x,y,t)]=G_a(u,v,w)$,

then

$$G_a(u,v,w)= \frac{1}{u^a v^a w^a} F_a\left(\frac{1}{u}, \frac{1}{v}, \frac{1}{w}\right),\ 0<a<1 \quad (6)$$

Proof. By the definition of fractional triple Sumudu transformation,

$$G_\alpha(u,v,w) = S_\alpha[f(x,y,t)] = \lim_{M\to\infty} \int\int\int_0^M E_\alpha(-(x+y+t)^\alpha) f(ux,vy,wt)(dx)^\alpha (dy)^\alpha (dt)^\alpha \quad (7)$$

$$= \lim_{M\to\infty} \alpha^3 \int\int\int_0^M (M-x)^{\alpha-1}(M-y)^{\alpha-1}(M-t)^{\alpha-1} E_\alpha(-(x+y+t)^\alpha) f(ux,vy,wt)\,dx\,dy\,dt. \quad (8)$$

By using the changes of variable ux $ux \longrightarrow x',\ vy \longrightarrow y',\ wt \longrightarrow t'$

$$= \frac{1}{u^\alpha v^\alpha w^\alpha} \lim_{M\to\infty} \int_0^{Mu}\int_0^{Mv}\int_0^{Mw} (Mu-x')^{\alpha-1}(Mv-y')^{\alpha-1}(Mw-t')^{\alpha-1} E_\alpha\left(-\left(\frac{x'}{u}+\frac{y'}{v}+\frac{t'}{w}\right)^\alpha\right)$$

$$\times f(x',y',t')dx'\,dy'\,dt'$$

$$= \frac{1}{u^\alpha v^\alpha w^\alpha} \int\int\int_0^\infty E_\alpha\left(-\left(\frac{x'}{u}+\frac{y'}{v}+\frac{t'}{w}\right)^\alpha\right) f(x',y',t')(dx')^\alpha (dy')^\alpha (dt')^\alpha$$

$$= \frac{1}{u^\alpha v^\alpha w^\alpha} F_\alpha\left(\frac{1}{u}, \frac{1}{v}, \frac{1}{w}\right) \quad (9)$$

VIII. SOME PROPERTIES OF FRACTIONAL TRIPLE SUMUDU TRANSFORM

we recall some properties of fractional triple Sumudu transform

$S_a f(ax,by,ct)=G_a(au,bv,cw)$,

$S_a f(x-a,y-b,t-c)=E_a(-(a+b+c)^a)G_a(au,bv,cw)$,

$$S_\alpha \partial_t^\alpha f(ax,by,ct) = \frac{G_\alpha(u,v,w) - \Gamma(1+\alpha)f(0,x,y)}{u^\alpha v^\alpha} \quad (10)$$

where ∂_t^α is denoted to fractional partial derivative of order α.

IX. MAIN RESULT

The main results in this work are present in the following section,

X. TRIPLE LAPLACE TRANSFORM OF FRACTIONAL ORDER

$$L_\alpha[f(x,y,t)] = F_\alpha(p,s,k) = \int\int\int_0^\infty E_\alpha(-(px+sy+kt)^\alpha) f(x,y,t)(dx)^\alpha(dy)^\alpha(dt)^\alpha \tag{11}$$

where $s,p,k \in C$ and $E_a(x)$ is Mittag- Leffler function.

Corollary 10.1 By using the Mittag-Leffler property then we can rewrite the formula (11) as the following:

$$L_\alpha[f(x,y,t)] = F_\alpha(p,s,k) = \int\int\int_0^\infty E_\alpha(-(px)^\alpha)E_\alpha(-(sy)^\alpha)E_\alpha(-(kt)^\alpha) f(x,y,t)(dx)^\alpha(dy)^\alpha(dt)^\alpha \tag{12}$$

Remark 10.2 In particular case, fractional triple Laplace transform (11) turns to triple Laplace transform (1) when α=1.

XI. SOME PROPERTIES OF FRACTIONAL TRIPLE LAPLACE TRANSFORM

In this section, various properties of fractional triple Laplace transform are discussed and proved such as linearity property, change of scale property and so on.

1. Linearity Property:

Let $f_1(x,y,t)$ and $f_2(x,y,t)$ be functions of the variables x,y and t, then

$$L_a[a_1 f_1(x,y,t)+a_2 f_2(x,y,t)]=a_1 L_a[f_1(x,y,t)]+a_2 L_a[f_2(x,y,t)]. \tag{13}$$

where a_1 and a_2 are constant.

Proof. We can simply get the proof by applying the definition (11).

2. Changing of Scale Property:

If $L_a[f(x,y,t)]=F_a(p,s,k)$ hence $L_a[f(ax,by,ct)]= \frac{1}{a^a b^a c^a} F_a(\frac{p}{a}, \frac{s}{b}, \frac{k}{c})$.

Wherever a,b and c are constants.
Proof.

$$L_\alpha[f(ax,by,ct)] = \int\int\int_0^\infty E_\alpha(-(px+sy+kt)^\alpha) f(ax,by,ct)(dx)^\alpha(dy)^\alpha(dt)^\alpha. \tag{14}$$

We set u = ax, v = by and w = ct in the equality (14),therefore we obtain

$$L_\alpha[f(ax,by,ct)] = \frac{1}{a^\alpha b^\alpha c^\alpha}\int\int\int_0^\infty E_\alpha\left(-\left(\frac{pu}{a}+\frac{sv}{b}+\frac{kw}{c}\right)^\alpha\right) f(u,v,w)(du)^\alpha(dv)^\alpha(dw)^\alpha \tag{15}$$

$$= \frac{1}{a^\alpha b^\alpha c^\alpha} F_\alpha\left(\frac{p}{a},\frac{s}{b},\frac{k}{c}\right) \tag{16}$$

3. Shifting Property:

Let $L_a[f(x,t)]=F_a(p,s,k)$ then $L_a[E_a(-(ax+by+ct)^a) f(x,y,t)]=F_a(p+a,s+b,k+c)$.

Proof. $L_\alpha[E_\alpha(-(ac + by + ct)^\alpha)f(x,y,t)] = \int\int\int_0^\infty E_\alpha(-(ax + by + ct)^\alpha)E_\alpha(-(px+sy+kt)^\alpha)f(x,y,t)(dx)^\alpha(dy)^\alpha(dt)^\alpha$

by using the equality

$$E_\alpha(\lambda(x+y+t)^\alpha) = E_\alpha(\lambda x^\alpha)E_\alpha(\lambda y^\alpha)E_\alpha(\lambda t^\alpha)$$

which implies that

$$L_\alpha[E_\alpha(-(ax+by+ct)^\alpha)f(x,y,t)] = \int\int\int_0^\infty E_\alpha(-((a+p)x+(b+s)y+(c+k)t)^\alpha)f(x,y,t)(dx)^\alpha(dy)^\alpha(dt)^\alpha.$$

Hence

$$L_\alpha[E_\alpha(-(ax+by+ct)^\alpha)f(x,y,t)] = F_\alpha(p+a, s+b, k+c)$$

4. Multiplication by $x^a y^a t^a$*:*
Let

$$L_\alpha[f(x,y,t)] = F_\alpha(p,s,k) = \int\int\int_0^\infty E_\alpha(-((px)+(sy)+(kt))^\alpha)f(x,y,t)(dx)^\alpha(dy)^\alpha(dt)^\alpha \tag{17}$$

then

$$L_\alpha[x^\alpha y^\alpha t^\alpha f(x,y,t)] = \frac{\partial^{3\alpha}}{\partial p\partial s\partial k}L_\alpha[f(x,y,t)] \tag{18}$$

Proof.

$$L_\alpha[x^\alpha y^\alpha t^\alpha f(x,y,t)] = \int\int\int_0^\infty x^\alpha E_\alpha(-px)^\alpha y^\alpha E_\alpha(-sy)^\alpha t^\alpha E_\alpha(-kt)^\alpha f(x,y,t)(dx)^\alpha(dy)^\alpha(dt)^\alpha \tag{19}$$

By using the fact $D_s^\alpha(E_\alpha(-p^\alpha x^\alpha)) = -x^\alpha E_\alpha(-p^\alpha x^\alpha)$*, then*

$$\begin{aligned} &L_\alpha[x^\alpha y^\alpha t^\alpha f(x,y,t)] \\ &= \int\int\int_0^\infty \frac{\partial^\alpha}{\partial p^\alpha}E_\alpha(-(px)^\alpha)\frac{\partial^\alpha}{\partial s^\alpha}E_\alpha(-(sy)^\alpha)\frac{\partial^\alpha}{\partial k^\alpha}E_\alpha(-(kt)^\alpha)f(x,y,t)(dx)^\alpha(dy)^\alpha(dt)^\alpha \\ &= \int\int\int_0^\infty \frac{\partial^{3\alpha}}{\partial p^\alpha\partial s^\alpha\partial k^\alpha}(E_\alpha(-(px)^\alpha))E_\alpha(-(sy)^\alpha)E_\alpha(-(kt)^\alpha)f(x,y,t)(dx)^\alpha(dy)^\alpha(dt)^\alpha \\ &= \frac{\partial^{3\alpha}}{\partial p^\alpha\partial s^\alpha\partial k^\alpha}L_\alpha[f(x,y,t)] \end{aligned} \tag{20}$$

XII. THE CONVOLUTION THEOREM OF THE FRACTIONAL TRIPLE LAPLACE TRANSFORM

Theorem 12.1 *The triple convolution of order α of functions f(x,y,t), and g(x,y,t) can be defined as the expression*

$$(f(x,y,t)***_\alpha g(x,y,t)) = \int_0^x\int_0^y\int_0^t f(x-\eta, y-\theta, t-\gamma)g(\eta,\theta,\gamma)(d\eta)^\alpha(d\theta)^\alpha(d\gamma)^\alpha, \tag{21}$$

therefore one has the equality

$$L_\alpha[(f***_\alpha g)(x,y,t)] = L_\alpha[f(x,y,t)]L_\alpha[g(x,y,t)]. \tag{22}$$

Proof. we apply the definition of fractional triple laplace transform and fractional triple convolution above, the we obtain.

$$L_\alpha[(f***_\alpha g)(x,y,t)] = \int\int\int_0^\infty E_\alpha(-(px)^\alpha)E_\alpha(-(sy)^\alpha)E_\alpha(-(kt)^\alpha)(f**$$

$$*_\alpha g)(x,y,t)(dx)^\alpha(dy)^\alpha(dt)^\alpha,$$

$$= \int\int\int_0^\infty E_\alpha(-(px)^\alpha)E_\alpha(-(sy)^\alpha)E_\alpha(-(kt)^\alpha)\left[\int_0^x\int_0^y\int_0^t\right.$$

$$f(x-\eta,y$$
$$\left. -\theta,t-\gamma)g(\eta,\theta,\gamma)(d\eta)^\alpha(d\theta)^\alpha(d\gamma)^\alpha\right](dx)^\alpha(dy)^\alpha(dt)^\alpha, \tag{23}$$

let $u=x-\eta,\ v=y-\theta, w=t-\gamma$ and taking the limit from 0 to ∞ ,it gives

$$= \int\int\int_0^\infty E_\alpha(-p^\alpha(u+\eta)^\alpha)E_\alpha(-s^\alpha(v+\theta)^\alpha)E_\alpha(-k^\alpha(w+\gamma)^\alpha)$$

$$\times\int\int\int_0^\infty f(u,v,w)g(\eta,\theta,\gamma)(d\eta)^\alpha(d\theta)^\alpha(d\gamma)^\alpha,$$

$$= \int\int\int_0^\infty E_\alpha(-p^\alpha u^\alpha)E_\alpha(-s^\alpha v^\alpha)E_\alpha(-k^\alpha w^\alpha)f(u,v,w)(du)^\alpha(dv)^\alpha(dw)^\alpha$$

$$\times\int\int\int_0^\infty E_\alpha(-p^\alpha\eta^\alpha)E_\alpha(-s^\alpha\theta^\alpha)E_\alpha(-k^\alpha\gamma^\alpha),$$

$$= L_\alpha[f(x,y,t)]L_\alpha[g(x,y,t)].$$

XIII. INVERSION FORMULA OF TRIPLE FRACTIONAL LAPLACES TRANSFORM

Firstly, we will set up definition of fractional delta function of three variables as follows
Definition:= Three variables delta function $d_a(x\text{-}a,y\text{-}b,t\text{-}c)$
of function order α , 0 < α ≤ 1, can be defined as next formula

$$\int_R\int_R\int_R g(x,y,t)d_a(x\text{-}a,y\text{-}b,t\text{-}c)(dx)^a(dy)^a(dt)^a=a^3g(a,b,c). \tag{24}$$

In special case, we have

$$\int_R\int_R\int_R g(x,y,t)d_a(x,y,t)(dx)^a(dy)^a(dt)^a=a^3g(0,0). \tag{25}$$

Example 13.1 *We can obtain fractional triple Laplace transform of* $d_a(x\text{-}a,y\text{-}b,t\text{-}c)$ *as follows*

$$L_\alpha[\delta_\alpha(x-a,y-b,t-c)] = \int\int\int_0^\infty E_\alpha(-(px+sy+kt)^\alpha)\delta_\alpha(x-a,y-b,t-c)(dx)^\alpha(dy)^\alpha(dt)^\alpha,$$
$$=a^3E_a(\text{-}(pa+sb+kc)^a). \tag{26}$$

In particular, we have $L_a[d_a(x,y,t)]=a^3$.

XIV. RELATIONSHIP BETWEEN THREE VARIABLES DELTA FUNCTION OF ORDER α AND MITTAG-LEFFLER FUNCTION

The relation between $E_a(x+y+t)^a$ and $d_a(x,y,t)$ is clarified by the following theorem
Theorem 14.1 *The following formula holds*

$$\frac{a^3}{(M_a)^{3a}}\int_R\int_R\int_R E_a(i(\text{-}wx)^a)E_a(i(\text{-}hy)^a)E_a(i(\text{-}qt)^a)(dw)^a(dh)^a(dq)^a=d_a(x,y,t), \tag{27}$$

where M_a *satisfy the equivalence* $E_a(i(M_a)^a)=1$*, and it is called period of the Mittag-Leffler function.*
Proof. We test that (27) is in agreement with

$$\alpha^3 = \int_R \int_R \int_R E_\alpha(i(\omega x)^\alpha) E_\alpha(i(\eta y)^\alpha) E_\alpha(i(\theta t)^\alpha) \delta_\alpha(x,y,t)(dx)^\alpha (dy)^\alpha (dt)^\alpha, \tag{28}$$

we replace δα(x, y, t) in above equality by (27) to get

$$\alpha^3 = \int_R \int_R \int_R (dx)^\alpha (dy)^\alpha (dt)^\alpha \int_R \int_R \int_R \frac{\alpha^3}{(M_\alpha)^{3\alpha}} E_\alpha(i(\omega x)^\alpha) E_\alpha(i(\eta y)^\alpha) E_\alpha(i(\theta t)^\alpha)$$

$$\times E_\alpha(i(-ux)^\alpha) E_\alpha(i(-vy)^\alpha) E_\alpha(i(-wt)^\alpha)(du)^\alpha (dv)^\alpha (dw)^\alpha,$$
$$= \int_R \int_R \int_R (dx)^\alpha (dy)^\alpha (dt)^\alpha \int_R \int_R \int_R \frac{\alpha^3}{(M_\alpha)^{3\alpha}} E_\alpha(ix^\alpha(\omega - u)^\alpha) E_\alpha(iy^\alpha(\eta - v)^\alpha) E_\alpha(it^\alpha(\theta - w)^\alpha)(du)^\alpha (dv)^\alpha (dw)^\alpha,$$

$$\int_R \int_R \int_R (dx)^\alpha (dy)^\alpha (dt)^\alpha \int_R \int_R \int_R \frac{\alpha^3}{(M_\alpha)^{3\alpha}} E_\alpha(i(-xp)^\alpha) E_\alpha(i(-ys)^\alpha) E_\alpha(i(-tk)^\alpha)(dp)^\alpha (ds)^\alpha (dk)^\alpha,$$

$$= \int_R \int_R \int_R \delta_\alpha(x,y,t)(dx)^\alpha (dy)^\alpha (dt)^\alpha,$$

$$= \alpha^3.$$

Note that one has as well

$$\frac{\alpha^3}{(M_\alpha)^{3\alpha}} \int_R \int_R \int_R E_\alpha(i(-\omega x)^\alpha) E_\alpha(i(-\eta y)^\alpha) E_\alpha(i(-\theta t)^\alpha)(d\omega)^\alpha (d\eta)^\alpha (d\theta)^\alpha = \delta_\alpha(x,y,t). \tag{29}$$

XV. INVERSION THEOREM OF TRIPLE FRACTIONAL LAPLACE TRANSFORM

Theorem 15.1 *Here we recall the fractional triple Laplace transform (11) for convenience*

$$L_\alpha[f(x,y,t)] = F(p,s,k) = \int \int \int_0^\infty E_\alpha(-(px+sy+kt)^\alpha) f(x,y,t)(dx)^\alpha (dy)^\alpha (dt)^\alpha, \tag{30}$$

and its inverse formula define as

$$f(x,y,t) = \frac{1}{(M_\alpha)^{3\alpha}} \int \int \int_{-i\infty}^{i\infty} E_\alpha((px+sy+kt)^\alpha) F_\alpha(p,s,k)(dp)^\alpha (ds)^\alpha (dt)^\alpha. \tag{31}$$

Proof. Substituting (30) into (31) and using the formula(27) ,respectively, we obtain in turn

$$f(x,y,t) = \frac{1}{(M_\alpha)^{3\alpha}} \int \int \int_{-i\infty}^{i\infty} E_\alpha(px)^\alpha E_\alpha(sy)^\alpha E_\alpha(kt)^\alpha (dp)^\alpha (ds)^\alpha (dt)^\alpha$$

$$\times \int \int \int_0^\infty E_\alpha(-(px+sy+kt)^\alpha) f(u,v,w)(du)^\alpha (dv)^\alpha (dw)^\alpha,$$

$$= \frac{1}{(M_\alpha)^{3\alpha}} \int \int \int_0^\infty f(\eta,\theta,\gamma)(d\eta)^\alpha (d\theta)^\alpha (d\gamma)^\alpha$$

$$\times \int \int \int_{-i\infty}^{i\infty} E_\alpha(p^\alpha(x-\eta)^\alpha) E_\alpha(s^\alpha(y-\theta)^\alpha) E_\alpha(k^\alpha(t-\gamma)^\alpha)(dp)^\alpha (ds)^\alpha (dk)^\alpha,$$

$$= \frac{1}{(M_\alpha)^{3\alpha}} \int \int \int_0^\infty \frac{(M_\alpha)^{3\alpha}}{\alpha^3} f(\eta,\theta,\gamma)\delta_\alpha(\eta - x, \theta - y, \gamma - t)(d\eta)^\alpha (d\theta)^\alpha (d\gamma)^\alpha$$

$$= \frac{1}{\alpha^3} \int \int \int_0^\infty f(\eta,\theta,\gamma)\delta_\alpha(\eta - x, \theta - y, \gamma - t)(d\eta)^\alpha (d\theta)^\alpha (d\gamma)^\alpha,$$

$$= f(x,y,t).$$

XVI. CONCLUSION

In this present work, fractional triple Laplace transform and its inverse are defined, and several properties of fractional triple Laplace transform have been discussed which are consistent with triple Laplace transform when α = 1. Moreover convolution theorem is presented.

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REFERENCES

[1]. D. G. Duffy, Transform methods for solving partial differential equations (CRC press, 2004).
[2]. T. A. Estrin and T. J. Higgins, Journal of the Franklin Institute 252, 153â“167 (1951).
[3]. A. KÄ±lÄ±cÂ¸man and H. Gadain, Journal of the Franklin Institute 347, 848â“862 (2010).
[4]. A. KÄ±lÄ±cÂ¸man and H. Eltayeb, Applied Mathematical Sciences 4, 109â“118 (2010).
[5]. H. Eltayeb and A. KilicÂ¸man, Malaysian Journal of Mathematical Sciences 4, 17â“30 (2010).
[6]. A. KÄ±lÄ±cÂ¸man and H. Eltayeb, ISRN Applied Mathematics 2012 (2012).
[7]. H. Eltayeb and A. KilicÂ¸man, Abstract and applied analysis 2013 (2013).
[8]. S. G. Samko, A. A. Kilbas, and O. I. Marichev, Fractional integrals and derivatives (Theory and Applications, Gordon and Breach, Yverdon, 1993).
[9]. I. Podlubny, Fractional differential equations: an introduction to fractional derivatives, fractional differential equations, to methods of their solution and some of their applications, Vol. 198 (Academic press, 1998).
[10]. R. Hilfer, Applications of fractional calculus in physics (World Scientific, 2000).
[11]. K. Oldhman and J. Spanier, The fractional calculus: Theory and applications of differentiation and integration to arbitrary order (Academic Press, New York, 1974).