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Numerical Modelling of Aircraft Wing Deflection Under Different Loading Conditions

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Abstract:- This paper analyses deflection of an aircraft wing under various loading (uniformly distributed loading and point concentrated loading) conditions at the edges and the nodes and boundary conditions (fixed support, roller support and free) using JAVA code. The JAVA code was used to develop a computer interactive model. Classical plate theory and plane stress assumptions were made. Finite element analysis was used to carry out analysis on the wing which was taken as a thin plate. The wing was discretized with triangular and rectangular meshing. The aircraft wing considered is Airbus A320 wing. The interactive model developed analyzed different cases with different boundary conditions and loading conditions. Two cases were analyzed and deflection (lateral displacement) along X – axis, Y – Axis and XY – Axis for each case were presented. It was discovered that the greater the load, the higher the deflection obtained and for edges and nodes with fixed and roller support, higher loadings has to be applied for deflection to occur.

***Keywords**:- Aircraft Wing, Plate theory, Finite Element Analysis, Loadings, Deflection.*

I. INTRODUCTION

Structural components in engineering such as buildings, bridges, automobiles, aircraft bodies etc. are subjected to different stresses and loading systems. These stresses cause structural failure and deformation consequently in these components, therefore structural analysis before their construction is the most delicate and of course a pre – requisite operation that must be carried out.

The aircraft wings are the primary lift producing device for an aircraft. The aircraft wings are designed aerodynamically to generate lift force which is required in order for an aircraft to fly. Besides generating the necessary lift force, the aircraft wings are used to carry the fuel required for the mission by the aircraft, can have mounted engines or can carry extra fuel tanks or other armaments. The basic goal of the wing is to generate lift and minimize drag as far as possible. When the airflow passes the wing at any suitable angle of attack, a pressure differential is created. A region of lower pressure is created over the top surface of the wing while, a region of higher pressure is created below the surface of the wing [1].

It is important that all the different types of loads that the aircraft wing will bear be well estimated, and then the structural response to those loads be carefully calculated. To carefully calculate the response of the aircraft wing to estimated or measured loadings, it is important to use structural analysis techniques to which considerable confidence can be assigned [2]. Of all the numerical methods for such complex analysis, finite element method demonstrates greater potential for such analysis.

The use of finite element analysis in structural analysis have been increasing over the years as it has proven an alternative to mathematical analysis mostly giving a close approximation. The finite element technique of plane stress analysis has been presented in different papers. The technique has gained considerable recognition with application to problems associated with the aircraft industry. In all of these papers, however, the technique has been applied to problems associated with isotropic materials. Reference [3] examined the framework method and the stiffness element method closely and determined their applicability in handling problems in orthotropic plane stress. The concept of fracture mechanics and numerical approaches was introduced in [4] to solve interacting cracks problems in solid bodies which involves elastic crack interaction. A new computational fracture mechanics algorithm was developed by adopting stress singularity approach in finite element (FE) formulation. As a conclusion, the FE formulated approach was found to be at agreeable accuracy with analytical formulation.

Some other researchers have also worked on plate bending analysis using finite element method but used softwares with finite element working environment. This involves first getting the mathematical model of such problems and using the software to model the problem and a keen comparison will be made to highlight the best procedure although, the programming model always proved more efficient. The design of the rigid RCC (Roller Compacted Concrete) pavements was developed gradually in [5] through methods established by various organizations for the determination of the necessary thicknesses of roadways. In the study, a numerical 3D modelling was used by introducing to the computer code "Abaqus" the behaviour law of the RCC. Finally, the results of 3D modelling were compared with those obtained by the various other methods. The comparison shows good correspondences although the 3D modelling gives results slightly lower than those given by the 2D methods in stresses. A study to validate a numerical method of analysis that can predict the damage tolerance of these reinforced panels was presented in [6]. Therefore, using a fracture mechanics approach, several models (different by the geometry and the types of reinforcement constraints) were simulated with the finite element solver ABAQUS. The stress intensity factor

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trend obtained numerically as a function of crack growth was used to determine the fatigue crack growth rate, obtaining a good approximation of the experimental crack propagation rate in the skin.

Bending of an isotropic rectangular plate under various loading conditions was studied and presented by [7] using MATLAB code and commercial finite element software ANSYS. Classical plate theory (CPT) and plane stress assumption were used. Also, four node rectangular nonconforming elements were used. The presented code was able to consider residual stresses, self-strains, body forces, distributed and concentrated forces and distributed moments. Furthermore, it analyzed different boundary conditions for the plate. The results of finite element analysis were compared with each other and also by CPT results and a good agreement between them was noticed. On the other hand, the convergence of results was investigated by increasing the number of elements.

The analysis of static analysis of an isotropic rectangular plate with various boundary conditions and various types of load applications was carried out in [8]. Finite element analysis was carried out for an isotropic rectangular plate by considering the master element as a four nodded quadrilateral element. Numerical analysis (finite element analysis, FEA) was been carried out by developing programming in mathematical software MATLAB and the results obtained from MATLAB gave good agreement with the results obtained by classical method - exact solutions. Also, the material was modeled in ANSYS and comparison was done between the results obtained from FEA numerical analysis, and ANSYS results with classical method - exact solutions. Numerical results showed that, the results obtained by finite element analysis and ANSYS simulation results were in close agreement with the results obtained from exact solutions from classical method.

This paper aims at presenting the analysis of deflection of an aircraft wing under different loading and boundary conditions using finite element method. The finite element analysis was done and was written in JAVA code as an interactive computer model.

II. MATHEMATICAL MODEL

A. Governing Equations

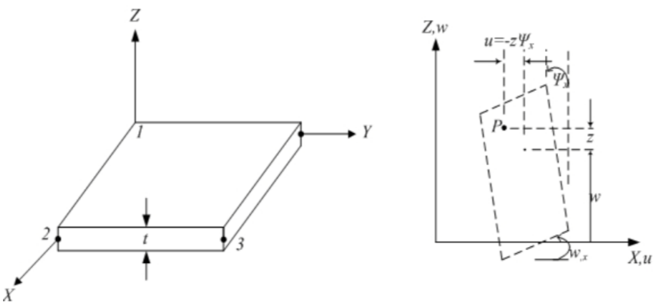

Fig 1:- Thin Plate Element [9]

Upon bending, particles that were on the mid-surface z = 0 undergo a deflection w(x, y) along z. The slopes of the mid-surface in the x and y directions are ∂w/∂x and ∂w/∂y. The rotations of the material normal about x and y are denoted by θx and θy, respectively. For small deflections and rotations, the foregoing kinematic assumption relates these rotations to the slopes:

$$\theta_x = \frac{\partial w}{\partial y}, \qquad \theta_y = \frac{\partial w}{\partial x} \tag{1}$$

The displacements $\{u_x, u_y, u_z\}$ of a plate particle P (x, y, z) not necessarily located on the mid-surface are given by

$$u_x = -z\frac{\partial w}{\partial x} = z\theta_y, \qquad u_y = -z\frac{\partial w}{\partial y} = -z\theta_x, \qquad u_z = w \tag{2}$$

Where, w is the deflection of the middle plane of the plate in the z direction. Further the relationship between, the strain and deflection is given by,

$$e_{xx} = \frac{\partial u_x}{\partial x} = -z\frac{\partial^2 w}{\partial x^2} = -zk_{xx}$$

$$e_{yy} = \frac{\partial u_y}{\partial y} = -z\frac{\partial^2 w}{\partial y^2} = -zk_{yy}$$

$$e_{zz} = \frac{\partial u_z}{\partial z} = -z\frac{\partial^2 w}{\partial z^2} = 0$$

$$2e_{xy} = \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} = -2z\frac{\partial^2 w}{\partial x \partial y} = -2zk_{xy}$$

$$2e_{xz} = \frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x} = -\frac{\partial w}{\partial x} + \frac{\partial w}{\partial x} = 0$$

$$2e_{yz} = \frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y} = -\frac{\partial w}{\partial y} + \frac{\partial w}{\partial y} = 0$$

Here,

$$k_{xy} = \frac{\partial^2 w}{\partial x^2}, \qquad k_{yy} = \frac{\partial^2 w}{\partial y^2}, \qquad k_{xy} = \frac{\partial^2 w}{\partial x \partial y} \tag{3}$$

Are the curvatures of the deflected surface [9].

B. Constitutive Equations

From Hooke's law

$$\sigma = [D]\varepsilon \tag{4}$$

Where,

$$[D] = \frac{E}{(1-v^2)}\begin{bmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & \frac{1-v}{2} \end{bmatrix}$$

Here, [D] is equal to the value defined for 2D solids in plane stress condition (i.e.) $\sigma_z = 0$ [10].

C. Calculations of Moments and Shear Forces

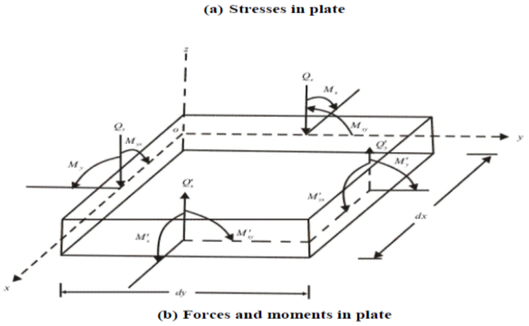

Fig 2:- Forces and moments in plate [10]

Let's consider a plate element of $dx \times dy$ and with thickness (t). The plate is subjected to external uniformly distributed load p. For a thin plate, body force of the plate can be converted to an equivalent load and therefore, consideration of separate body force is not necessary.

$$\sigma = -z\,[D]\,\Delta w$$

It is observed from the above relation that the normal stresses are varying linearly along thickness of the plate. Hence the moments on the cross section can be calculated by integration.

$$M = \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \int_{-t/2}^{t/2} \sigma z dt = -\left(\int_{-t/2}^{t/2} z^2 dt \right) [D]\Delta w = \frac{t^3}{12}[D]\Delta w \tag{5}$$

On expansion, we have

$$M_x = \frac{Et^3}{(12 - v^2)}\left(\frac{\partial^2 w}{\partial x^2} + v\frac{\partial^2 w}{\partial y^2}\right) = D_p(\chi_x + v\chi_y) \tag{6}$$

$$M_y = \frac{Et^3}{(12 - v^2)}\left(\frac{\partial^2 w}{\partial y^2} + v\frac{\partial^2 w}{\partial x^2}\right) = D_p(\chi_y + v\chi_x) \tag{7}$$

$$M_{xy} = \frac{Et^3}{12(1 + v)}\frac{\partial^2 w}{\partial_x \partial_y} = -D_p\frac{(1 - v)}{2}\chi_{xy} \tag{8}$$

Where Dp is known as flexural rigidity of the plate is given by

$$D_p = \frac{Et^3}{12(1 - v^2)}$$

Let consider the bending moments vary along the length and breadth of the plate as a function of *x* and *y*. Thus, if *Mx* acts on

one side of the element, $M_x = M_x + \frac{\partial M_x}{\partial_x} dx$ acts on the opposite side. Considering equilibrium of the plate element, the equations for forces can be obtained as

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + p = 0 \tag{9}$$

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_y}{\partial y} = Q_x \tag{10}$$

$$\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} = Q_y \tag{11}$$

Substituting the equations for moments, we have,

$$Q_x = -D_p\frac{\partial}{\partial x}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)$$

$$Q_y = -D_p\frac{\partial}{\partial y}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)$$

Substituting the force equations into the moment's equations, we have,

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{p}{D_p} \tag{12}$$

Source: Introduction to Plate Bending problems [10]

D. Governing Equation for Deflection of Plates

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\nabla^2 w = \frac{q}{D}$$

$$\nabla^2(\nabla^2 w)\frac{q}{D}$$

$$\nabla^4 = \frac{q}{D}$$

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D} \tag{13}$$

Source: Theory of Plates [11]

III. METHODOLOGY

A. Aircraft Case Study

The case study of Airbus A320 was considered. Airbus A320 (2011) single aisle passenger aircraft is one of the most modern aircraft in Airbus industry. It is a commercial aircraft with high passenger capacity, and it is mostly used for domestic routes. Airbus is one of the leading aircraft manufacturers in the world with the most modern and comprehensive product line with unbeatable fuel efficiency. It has a swept wing according to shape, middle wing according to the position of the wing on the aircraft and bi – wing according to the number of wings [12].

B. Airbus A320 Wing Geometry

Geometric Scaling Of Airbus A 320 Wing:

Semi Span	3000 mm
Root Chord	1180 mm
Tip Chord	192 sq.mtrs
Wing Area	763 mm
Mac (Location From Root)	250
Sweep Angle L.E	0º
Sweep Angle T.E	170º
Twist Root	3.7º
Twist Tip	0.8º

Table 2: Scaled Dimension of Wing (1:5) [1]

The original scaled dimensions (1:5) of the A320 wing are shown in Table 2 above. We know the values of root and tip chord lengths, sweep angles at the leading and trailing edges. The A320 wing AutoCAD drawing is shown in Figure 3.

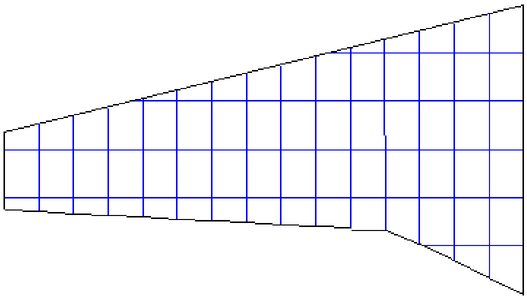

Fig 3:- Final Meshed Model of Wing

By using AutoCAD software, the wing was designed according to required specifications. We have divided the wing into 15 stations at a distance of 200 mm. The very purpose of the division of wing is to give varying thickness at each division.

S/No	Chord (mm)	Thickness (mm)
0	1186.6	141.98
1	1083.029	130.17
2	979.42	118.61
3	875.3	105.24
4	783	94.11
5	740.56	89.69
6	698.1	83.91
7	656	78.85
8	613.5	73.76
9	571	68.6
10	529.1	63.58
11	486.2	58.44
12	444	53.36
13	401.5	48.26
14	359	43.211
15	192	23.07

Table 3: Chord and Thickness at Each Station (1:5) [1]

C. Finite Element Analysis

Programming

The application of computer by the finite element method analysis of stresses for plate bending was written in a programming language known as JAVA. The first part is the main program that carries out the computation of stress analysis on the aircraft wing which is taken as a plate. The second part is the Graphics User Interface (GUI) was created that made the program a computer interactive model which drives the finite element method program. This part forms the interface that generates data input. The computer interactive model has the following steps for problem solving:

- Modelling: Includes the system geometry definition and material property selection. In this step user can draw either 2D representation of the problem.
- Meshing: This step involves discretizing the model according to predefined geometric element.
- Solution: This step involves applying boundary conditions and loads to the system and solves the problem.
- Post processing: This involves plotting nodal solutions (unknown parameters), which may be of displacements/stresses/reactive forces etc.

IV. RESULT AND DISCUSSION

A. Case One

The properties inputed include Young's Modulus E = 72 × 109 Pa, Poisson ratio **υ** = 0.3, thickness t = 0.5 m, load = 10KPa (Uniform Loading along Y) and geometry.

SIDE NO	DIMENSION	BOUNDARY
1	5900 mm	Fixed
3	1606.9072 mm	Free
4	4060.7881 mm	Free, Loaded
5	4219.7208 mm	Free
6	11149.6092 mm	Free, Loaded
7	11008.3554 mm	Free

Table 4 Case One: Geometry and Boundary Condition

V. RESULTS

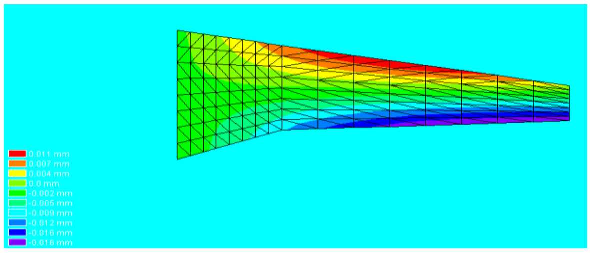

Fig 4:- Case One: Displacement along X – Axis

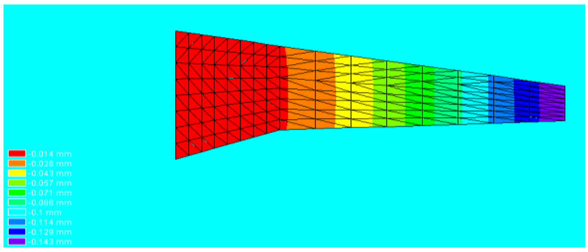

Fig 5:- Case One: Displacement along Y – Axis

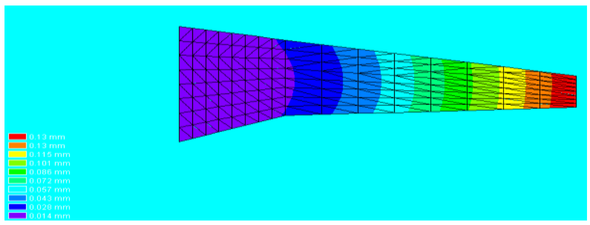

Fig 6:- Case One: Displacement along XY – Axis

The maximum deflection along X – axis under the boundary conditions presented is 0.011 mm and the minimum deflection is – 0.016 mm, the maximum deflection along Y – axis - 0.014 mm, – 0.143 mm minimum deflection and the maximum and minimum deflection obtained along XY – axis as 0.13 mm and 0.014 mm respectively.

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B. Case Two

The properties inputed include Young's Modulus E = 72 × 109 Pa, Poisson ratio **υ** = 0.3, thickness t = 0.5 m, load = 15 KPa (Uniform Loading along X) and geometry.

SIDE NO	LOADING	BOUNDARY
1	0	Free
3	0	Free
4	0	Roller Support
5	15 KPa	Free
6	0	Roller Support
7	15 KPa	Free

Table 5 Case Two: Loading and Boundary Condition

Results

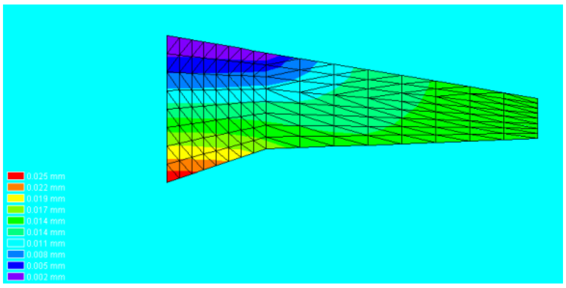

Fig 7:- Case Two: Displacement along X – Axis

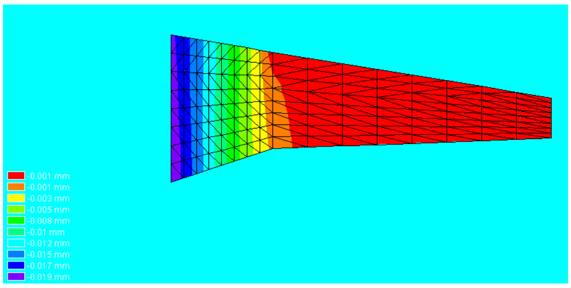

Fig 8:- Case Two: Displacement along Y – Axis

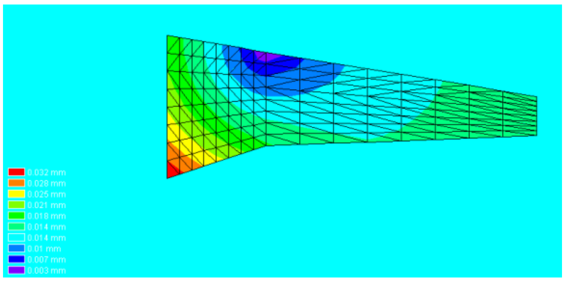

Fig 9:- Case Two: Displacement along XY – Axis

For the second case, the maximum deflection along X – axis under the boundary conditions presented is 0.025 mm and the minimum deflection is 0.002 mm, the maximum deflection along Y – axis - 0.001 mm, – 0.0019 mm minimum deflection and the maximum and minimum deflection obtained along XY – axis as 0.032 mm and 0.003 mm respectively.

VI. CONCLUSION

The research considered analysis of deflection of an aircraft wing under different loading conditions using finite element method. JAVA code was used to develop an interactive computer model which analyses the deflection of an aircraft wing under various loading (uniformly distributed loading and point concentrated loading) conditions at the edges and the nodes and support conditions (fixed support, roller support and free end). Two different cases were analyzed and deflection (lateral displacement) along X – axis, Y – Axis and XY – Axis for each case were presented. It was discovered that the greater the load, the higher the deflection obtained and for edges and nodes with fixed and roller support, higher loadings has to be applied for deflection to occur.

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LIST OF SYMBOLS AND ABBREVIATIONS

t – Thickness (mm or m)
σ – Stress (Pa or N/m)
∂ **-** Partial differential
υ – Poisson's ratio
θ – ply angle
ε – Strain (Pa)
γ **-** Shear strain (Pa)

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χ - Curvature along respective directions
E – Modulus of elasticity (GPa)
M – Bending moment
x – x- direction
[1] y – y-direction
[2] z – z-direction
[3] w - Deflection
[4] [K] – Stiffness Matrix (Pa)
[5] GUI – Graphics User Interface
[6] q – External load applied to the plate