Sadik Transform in Control Theory

Sadikali Latif Shaikh
Department of Mathematics,
Maulana Azad College of Arts, Science and Commerce,
Dr. Rafiq Zakaria Campus, Aurangabad. (M.S.). 431001
India

Abstract:- Similar to the Laplace transform, a new transform named Sadik transform is introduced. Proved properties of Sadik transform for derivative of function. Also proved shifting theorem for Sadik transform. Obtained transfer function of dynamical system in control theory using Sadik transform. Solved some applications in control theory by Sadik transform. Proved that the Laplace transform and all the integral transform similar to the Laplace transform available in the literature are particular cases of the Sadik transform.

***Keywords**:- Integral transforms, Laplace transform, Control Theory, Transfer functions, dynamical Systems, Ordinary Differential Equations.*

I. INTRODUCTION

Integral transforms particularly the Laplace transform plays a crucial role in control engineering. Since the Laplace transform convert convolution product into ordinary multiplication therefore it can be used to create transfer function in control theory. So the Laplace transform is a precious tool in the control theory.

While designing a control system, it is mandatory to understand that how the required system behaves with respect to different controller designs. For this, we designs system of dynamical equations representing controller system, and solve these equations to reach dynamic response. In the literature there are three different domains in which the dynamic response of the system is studied for making control design. These three domains are Laplace domain, frequency domain and the state space.

In this paper we are introducing a new integral transform named Sadik transform [4]. The beauty of the Sadik transform is that the Laplace transform is a particular case of Sadik transform. Sadik transform generate v-domain instead of s-domain, v-domain of Sadik transform has the strength to handle the problems in extreme simplified way. The purpose of the Sadik transform is to solve all dynamical equations in control theory in the possible easiest way. All interesting properties of this new transform and how it works in the control theory, we will discuss in the next section.

II. PRELIMINRIES

We start with definition of the Laplace transform.
Definition2.1 (Laplace transform)
If, $f(t)$ is a piecewise continuous on $0 \le t \le a$ for $a > 0$, and it is an exponential order of α then the Laplace transform of $f(t)$ is denoted by $F(s)$ and defined by,

$$F(s) = L[f(t)] = \int_0^\infty e^{-st} f(t)\, dt$$

If the integral exist
Where, s is a complex parameter.
Definition2.2 (Sadik transform)

If,

A. $f(t)$ is piecewise continuous on the interval $0 \le t \le A$ for any $A > 0$.
B. $|f(t)| \le K.e^{at}$ When $t \ge M$, for any real constant a and some positive constant K and M.

Then Sadik transform of $f(t)$ is defined by

$$F(v^\alpha, \beta) = S[f(t)] = \frac{1}{v^\beta} \int_0^\infty e^{-tv^\alpha} f(t) dt$$

Where,

v Is complex variable,
α is any non- zero real numbers, and
β is any real number.
Note: If we put $\alpha = 1$ and $\beta = 0$ in the definition2.2 then the Sadik transform converts into the Laplace transform.

III. SOME PROPERTIES OF THE SADIK TRANSFORM

Property3.1: If $f'(t)$ is first derivative of $f(t)$ then Sadik transform of $f'(t)$ is

$$S[f'(t)] = v^\alpha F(v^\alpha, \beta) - v^{-\beta} f(0)$$

Where,

$F(v^\alpha, \beta)$ is a Sadik transform of $f(t)$.
Property3.2: If $f''(t)$ is a second derivative of $f(t)$ then Sadik transform of $f''(t)$ is

$$S[f''(t)] = v^{2\alpha} F(v^\alpha, \beta) - v^{\alpha-\beta} f(0) - v^{-\beta} f'(0)$$

Where,

$F(v^\alpha, \beta)$ is a Sadik transform of $f(t)$.
Property3.3: Shifting Theorem of Sadik transform

If

$S[f(t)] = F(v^\alpha, \beta)$ then $S[e^{at} f(t)] = F(v^\alpha - a, \beta)$

IV. IMPULSE RESPONSE AND TRANSFER FUNCTIONS OF LINEAR SYSTEMS USING SADIK TRANSFORM

Here we consider that Sadik transform of the impulse response is a transfer function of linear system.

Consider the input- output relation of a linear dynamical system is described by the differential equation with constant real coefficients

$$c^{(n)}(t)+a_n\,c^{(n-1)}(t)+a_{n-1}\,c^{(n-2)}(t)+\cdots+a_2\,c^{(1)}(t)$$

$$+a_1\,c(t)=b_{m+1}\,r^{(m)}(t)+b_m\,r^{(m-1)}(t)+\cdots+b_2\,r^{(1)}(t)+b_1\,r(t)$$

The coefficients $a_1,a_2,\ldots,a_n$ and $b_1,b_2,\ldots,b_{m+1}$ are real constants and $n\geq m$.

Above differential equation is used for the analysis and design of control systems.

To get the transfer function of the linear system represented by above differential equation we take Sadik transform by assuming zero initial conditions, we get

$$\left(v^{n\alpha}+a_n v^{(n-1)\alpha}+a_{n-1}v^{(n-2)\alpha}+\cdots+a_1\right)C(v^\alpha,\beta)$$

$$=\left(b_{m+1}v^{m\alpha}+b_m v^{(m-1)\alpha}+b_{m-1}v^{(m-2)\alpha}+\cdots+b_1\right)R(v^\alpha,\beta)$$

Where,

$$S[c(t)]=C(v^\alpha,\beta)$$

$$S[r(t)]=R(v^\alpha,\beta)$$

Therefore, the transfer function between $r(t)$ and $c(t)$ is denoted by $G(v^\alpha,\beta)$ and is given by

$$G(v^\alpha,\beta)=\frac{C(v^\alpha,\beta)}{R(v^\alpha,\beta)}$$

That is,

$$G(v^\alpha,\beta)=\frac{\left(b_{m+1}v^{m\alpha}+b_m v^{(m-1)\alpha}+b_{m-1}v^{(m-2)\alpha}+\cdots+b_1\right)}{\left(v^{n\alpha}+a_n v^{(n-1)\alpha}+a_{n-1}v^{(n-2)\alpha}+\cdots+a_1\right)}$$

In the above equation if we put $\alpha=1$ and $\beta=0$ then $G(v^1,0)$ is nothing but a transfer function obtained by the Laplace transform. For the convenience and for the simplicity we can change and fix any values of α and β according to situation of the problem.

Suppose that an input response is $c(t)=e^{-t}$ then its transfer function by Sadik transform is

$$C(v^\alpha,\beta)=\frac{v^{-\beta}}{v^\alpha+1}$$

For instant suppose that $\alpha=1$ and $\beta=1$ then,

$$C(v^1,1)=\frac{1}{(v+1)\,v}$$

Now the poles of $C(v^1,1)$ are 0 and -1 but if we consider transfer function of the same input by the Laplace transform then it is obtained by

$$C(s)=\frac{1}{s+1}$$

And this transfer function has only one pole at $s=-1$.

Poles of the transfer function are important to analyze the response of system in frequency domain, for better analyze we can find different poles of transfer function by changing values of α and β in the Sadik transform.

V. APPLICATIONS IN CONTROL THEORY

The following example in mechanical modelling elaborate the effectiveness of a new transform (Sadik transform)

Example 4.1: (Spring-mass system with viscous damping problem)

The following differential equation is the equation of motion for an ideal spring- mass system with damping

$$\frac{d^2x}{dt^2}+3.\frac{dx}{dt}+2.x=0$$

Subject to condition that $x=1,\ \frac{dx}{dt}=0$ at t=0.

Solution:

Taking Sadik transform of equation and using given initial conditions, we get

$$v^{2\alpha}X(v^\alpha,\beta)-v^{\alpha-\beta}x(0)-v^{-\beta}x'(0)+3\,v^\alpha X(v^\alpha,\beta)-3-v^{-\beta}x(0)+2X(v^\alpha,\beta)=0$$

Therefore,

$$X(v^\alpha,\beta)=\frac{v^{\alpha-\beta}+3.v^{-\beta}}{(v^{2\alpha}+3\,v^\alpha+2)}$$

$$X(v^\alpha,\beta)=\frac{v^{-\beta}(v^\alpha+3)}{(v^\alpha+2)(v^\alpha+1)}$$

Now for the simplicity we choose $\alpha=1$ and $\beta=1$ (you may choose another values of α and β) . Hence,

$$X=\frac{1}{(v)}\left[\frac{2}{v+1}-\frac{1}{v+2}\right]$$

That is,

$$X = \left[\frac{2v^{-1}}{v+1} - \frac{1.v^{-1}}{v+2}\right]$$

Now carefully taking inverse Sadik transform, we get,

$$x(t) = 2e^{-t} - e^{-2t}$$

It is required solution in time domain.

Example2:

A shaft of inertia J is rotated for an angle θ due to applied torque T against a bearing friction f. find the transfer function of the system.

Solution: (By the Laplace transform method)

The differential equation form of the above statement is

$$T(t) = J\,\ddot{\theta} + f\,\dot{\theta}$$

Taking the Laplace transform, we get

$$T(s) = J\left[s^2\theta(s) - s\theta(0) - \theta'(0)\right] + f[s - \theta(0)]$$

Assuming initial conditions zero, we get

$$T(s) = \theta(s).[J\,s^2 + f.s]$$

Therefore,

$$\frac{\theta(s)}{T(s)} = \frac{1}{J\,s^2 + f.s}$$

As the applied torque is input and the output angular displacement is an output of the system. Here the poles of the transfer function are $s = 0, s = \frac{-f}{J}$.

Solution: (By the Sadik transform method)

The differential equation form of the above statement is

$$T(t) = J\,\ddot{\theta} + f\,\dot{\theta}$$

Applying the Laplace transform, we get

$$T(v^\alpha,\beta) = J\left[v^{2\alpha}\theta(v^\alpha,\beta) - v^{\alpha-\beta}\theta(0) - v^{-\beta}\dot{\theta}(0)\right]' + f[v^\alpha\theta(v^\alpha,\beta) - v^{-\beta}\theta(0)]$$

Assuming initial conditions zero, we get

$$T(v^\alpha,\beta) = \theta(v^\alpha,\beta).[J\,v^{2\alpha} + f.v^\alpha]$$

Therefore,

$$\frac{\theta(v^\alpha,\beta)}{T(v^\alpha,\beta)} = \frac{1}{J\,v^{2\alpha} + f.v^\alpha}$$

This is required transfer function in v-domain and this transfer function can be modified by putting values of α and β , it is an advantage of Sadik transform.

VI. CONCLUSION

A new transform named Sadik transform works effectively to solve dynamical problems in control theory. Instead of the Laplace transform we may use Sadik transform for better understanding the problems in control theory. Also more details of Sadik transform will be investigate in further research work.

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