# Testing of Hypothesis by using Generalized Inverse of Matrix

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**Abstract:- This paper focussed on Testing of Hypothesis by using Generalized Inverse of Matrix . In the recent paper (1), On the generalized inverse of a matrix, the generalized inverse matrix wasapplied to solution of systems of equations that are linearly dependent and unbalanced. This paper is an extensive study of (1). It manages Testing of Hypothesis by using Generalized Inverse of Matrices.**

***Keywords*:-** *Generalized Inverse of a matrix, ANOVA, linear models, least square, , Variance, Fullrank partitioning.*

## I. INTRODUCTION

The work displayed in this paper is an expansion of the prior paper [1]. Here the summed up opposite of a lattice is connected to models which are not of full rank in nature.Different techniques exist for comprehending frameworks of concurrent straight condition; some of them are: (end strategy, push lessening technique, in reverse substitution strategy and so forth). Require that the arrangement of direct conditions be straightly autonomous. Consider the possibility that the arrangement of conditions is straightly needy. Summed up opposite can unravel straightly needy and unbalance arrangement of equations.{See Paper[ 1]}

A framework has a reverse just on the off chance that it is square and still, after all that lone on the off chance that it is nonsingular. Typically particular and rectangular grids don't have opposite. As of late needs have been felt in various zones of connected Mathematics for some sort of halfway backwards of a network that is solitary or even rectangular. Such converse are called summed up reverse. The idea of summed up converse was presented first by Moore in 1920 and autonomously rediscovered by Penrose in 1955. Penrose demonstrated that, for each limited grid A (square or rectangular) of Real (or complex) components, there is an extraordinary framework X fulfilling the four conditions. AXA=A, XAX=X,

$(AX)^* = AX$, $(XA)^* = XA$,

Where $A^*$ denotes the conjugate transpose of A

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- *Historical Background of Genralized Inverse Matrix*

The idea of a summed up opposite appears to have been first say in print in 1903 by Fredholm, where a specific summed up reverse called by him pseudo converse as a necessary administrator was given. A few examinations have fretted about the Generalized reverse networks, strikingly among them were: Hurwitz (1912), He described all pseudo backwards and utilized the limited dimensionality of invalid administrators of Fredholm administrators, officially certain in Hilbert's discourses in 1904 of summed up Green capacities were subsequently considered by various creators, specifically, Myller (1906), Westfall (1909), Bounitzky (1909), Elliott (1928) , Reid (1931). Bjerhanmer (1951), Penrose (1951)Relevant productions are the work done by Moore (1920), Siegel (1937), Tseng, Murray and Von Neumann (1936), Alkinson (1950), Adetunde et al; (2008).

## II. TESTING OF HYPOTHESIS BY USING GENERALIZED INVERSE OF MATRICES

The model we shall be dealing with is, $y = xb + c$

Where y is an nx1 vector of observations $y_i$

The following assumptions are made

$E \approx (0, \sigma^2 I)$ *and* $Y \approx (Xb, \sigma^2 I)$

$$y = xb + E$$

Which can be derived by the least squares method, to get

$$x^{T}xb = x^{T}y$$

*Example*

An experiment is to estimate about the effect on the type of tree on its weight of the cashew fruit on four different cashew fruit given the same condition recorded the following weight of its fruit at harvest as

| Weight of 10 tree | Type (one) | Type(Two) | Type(Three) | Type(Four) |
|---|---|---|---|---|
| | 25kg | 48 kg | 30kg | 50 kg |
| | 30kg | 90 kg | 32kg | |
| | 28 kg | 85 kg | | |
| | 45 kg | | | |
| Total | 128 kg | 223 kg | 62 kg | 50 kg |

To calculate the effect of the type of tree on the weight of fruit we assume that the observation $y_{i,j}$ is the sum of four types.

$$y_{i,j} = k + \beta_I + c_{i,j}$$

Where ,kis the population mean of the weight of plant, $\beta_I$ is the effect of the type I on weight , $c_{i,j}$ is the random error term. To develop the normal equations, we write down 10 observations in terms of the equation of the model

$$25 = y_{11} \quad = k + \beta_I + c_{11}$$
$$30 = y_{12} \quad = k + \beta_I + c_{12}$$
$$28 = y_{13} \quad = k + \beta_I + c_{13}$$
$$45 = y_{14} \quad = k + \beta_I + c_{14}$$
$$48 = y_{21} \quad = k + \beta_I + c_{21}$$
$$90 = y_{22} \quad = k + \beta_I + c_{22}$$
$$85 = y_{23} \quad = k + \beta_I + c_{23}$$
$$30 = y_{31} \quad = k + \beta_I + c_{31}$$
$$32 = y_{32} \quad = k + \beta_I + c_{32}$$
$$50 = y_{41} \quad = k + \beta_I + c_{41}$$

This is written in matrix form as

$$\begin{bmatrix} 25 \\ 30 \\ 28 \\ 45 \\ 48 \\ 90 \\ 85 \\ 30 \\ 32 \\ 50 \end{bmatrix} = \begin{bmatrix} y_{11} \\ y_{12} \\ y_{13} \\ y_{14} \\ y_{21} \\ y_{21} \\ y_{11} \\ y_{11} \\ y_{11} \\ y_{11} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mu \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{bmatrix} + \begin{bmatrix} c_{11} \\ c_{12} \\ c_{13} \\ c_{14} \\ c_{21} \\ c_{21} \\ c_{11} \\ c_{11} \\ c_{11} \\ c_{11} \end{bmatrix}$$

*We know that ,x=xb+ c*can be derived by least square to give

$x^T xb = x^T y$

$$X^T X = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 10 & 4 & 3 & 2 & 1 \\ 4 & 4 & 0 & 0 & 0 \\ 3 & 0 & 3 & 0 & 0 \\ 2 & 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$X^T Y = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 25 \\ 30 \\ 28 \\ 45 \\ 48 \\ 90 \\ 85 \\ 30 \\ 32 \\ 50 \end{bmatrix} = \begin{bmatrix} 463 \\ 128 \\ 223 \\ 62 \\ 50 \end{bmatrix}$$

Matrix $x^T x$ has determinant equal to 0 anddon't find the rank, therefore matrix $x^T x$has does not unique inverse, hence the equation don't be express as

$$b = (x^T x)^{-1} (x^T y)$$

Since $(x^T x)^{-1}$ does not exist.

To get one of the solution, we need to find any generalized inverse I of $x^T x$ and write the corresponding solution as $b^0 = I x^T y$ where I is a generalized inverse of $x^T x$ .

choosing

$$I = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1/4 & 0 & 0 & 0 \\ 0 & 0 & 1/3 & 0 & 0 \\ 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

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$$\begin{bmatrix} \mu \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1/4 & 0 & 0 & 0 \\ 0 & 0 & 1/3 & 0 & 0 \\ 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 463 \\ 128 \\ 223 \\ 62 \\ 50 \end{bmatrix} = \begin{bmatrix} 0 \\ 32 \\ 74.3 \\ 31 \\ 0 \end{bmatrix}$$

The expectation of $b^0$ is given as

$$E(b^0) = Gx^T E(x)$$
$$E(y) = xb$$
$$\therefore E(b^0) = sx^T xb$$
$$E(b^0) = sb$$

Where S = $Ix^Tx$ hence $b^0$ is an unbiased estimator of sb but not of b

$$S = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1/4 & 0 & 0 & 0 \\ 0 & 0 & 1/3 & 0 & 0 \\ 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 10 & 4 & 3 & 2 & 1 \\ 4 & 4 & 0 & 0 & 0 \\ 3 & 0 & 3 & 0 & 0 \\ 2 & 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The variance of $b^0$given

$$Var(b^0) = Var(\mathrm{I}x^T y) = \mathrm{I}x^T Var(y)\, x\, I^T = Ix^T xI^T \sigma^2$$

For a full rank model $Var(b) = (x^1 x)^{-1\sigma 2}$, by an appropriate choice of I, $Ix^TxI^T\sigma^2$ can reduce further to $I\sigma^2$

Estimating E(y)

Corresponding to the vector of observations y, we have the vectors of estimated expected values

$\hat{E(y)}$.

$\hat{E(y)} \equiv \hat{y} = xb^0 = x\, Ix^T y$

This vectors is invariant to the choice of whatever generalized inverse of $x^Tx$is used for G, because $xx^T$

is invariant. This means that no matter what solution of the normal equations is used for $b^0$ the vector

$\hat{y} = XGX^{1}Y$ will always be the same.

$$\begin{bmatrix} y_{11} \\ y_{12} \\ y_{13} \\ y_{14} \\ y_{21} \\ y_{21} \\ y_{11} \\ y_{11} \\ y_{11} \\ y_{11} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 32 \\ 74.3 \\ 31 \\ 0 \end{bmatrix} = \begin{bmatrix} 32 \\ 32 \\ 32 \\ 32 \\ 74.3 \\ 74.3 \\ 74.3 \\ 31 \\ 31 \\ 0 \end{bmatrix}$$

*y is the vector of* expected *values*..
To demonstrate the invariance of *y* to the choice of G. Consider

$$G = \begin{bmatrix} 1 & -1 & -1 & -1 & 0 \\ -1 & -5/4 & 1 & 1 & 0 \\ -1 & 1 & 4/3 & 1 & 0 \\ -1 & 1 & 1 & 3/2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

H = $GX^TX$, hence we have

$$G = \begin{bmatrix} 1 & -1 & -1 & -1 & 0 \\ -1 & -5/4 & 1 & 1 & 0 \\ -1 & 1 & 4/3 & 1 & 0 \\ -1 & 1 & 1 & 3/2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 10 & 4 & 3 & 2 & 1 \\ 4 & 4 & 0 & 0 & 0 \\ 3 & 0 & 3 & 0 & 0 \\ 2 & 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

From the above results, it demonstrates that *y* is always the same no matter the G used.

The sum of squares regression is also invariant to the choice of G

## III. PARTITIONING THE TOTALL SUM OF SQUARES

Partitioning the total sum of square for the full rank model is the same for the model not of full rank. The only difference is that there is utility in corrected sums of squares and products of the x – variables.

$$SS_T = y^1y - N\bar{y}^2$$

$$SS_R = (b^0)^1 x^1y - N\bar{y}^2$$

$$SS_E = SS_T - SS_R$$

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$$y^T y = \begin{pmatrix}25 & 30 & 28 & 45 & 48 & 90 & 85 & 30 & 32 & 50\end{pmatrix}\begin{bmatrix}25\\30\\28\\45\\48\\90\\85\\30\\32\\50\end{bmatrix} = 23887$$

$$y'y = 23887$$

$$N = 10, \bar{y} = 46.3$$

$$SS_T = 23887 - 10(46.3)^2$$

$$SS_T = 2450$$

$$(b^0)^T X^T Y = \begin{bmatrix}50 & -338 & 16.9 & -19 & 0\end{bmatrix}\begin{bmatrix}463\\128\\223\\62\\50\end{bmatrix} = -17523.3$$

$SS_R = -17523.3 - 49733.5 = -21437$

$SS_R = -21437$

$SS_R$ is invariant to the choice of G, to show the invariance of $SS_R$, we consider

$$b^0 = \begin{bmatrix}0\\32\\74.3\\31\\0\end{bmatrix}$$

$$(b^0)^T X^T Y = \begin{bmatrix}0 & 32 & 74.3 & 31 & 0\end{bmatrix}\begin{bmatrix}463\\128\\223\\62\\50\end{bmatrix} = 22587$$

$S_T = 22587 - 49733.5$

$= -27146.5$

Since $SS_R$ is the same, no matter the G used we say that $SS_R$ is invariant to G $SS_E = SS_T - SS_R$

$$= 2450 + 21437$$

=23887

Test the hypothesis

$H_0$ :Xb = 0

$H_1$ :Xb$\neq$ 0

For the full rank model the test hypothesis is

$H_0$ : b = 0

$H_1$ :b$\neq$ 0

But for a model not of full rank b is not estimable, hence the hypothesis $H_0$ : b = 0

$H_1$ : b $\neq$ 0

Cannot be tested because b is a non-estimable function

*ANOVA table*

| Source | Df | SS | MS | F |
|---|---|---|---|---|
| Regression<br>Residual Total | 3<br>6<br>9 | – 21437<br>23887<br>2450 | 3981<br>272 | 0.068 |

$R^2$ = 2450/23887 = 0.102

The total variation explained by the model is 10.2%, the overall model is not significant, meaning that the weight of the cashew fruitdoes not depend on the type of cashew tree.

## IV. CONCLUSION

In this paper, the technique for summed up backwards had been connected on direct models which isn't of full rank. Proof has appeared from our outcome that summed up backwards can not be disregarded since it assumes an imperative part in models not of full rank. In particular, the utilization of summed up opposite of a network empowers us to unravel frameworks of direct conditions that are unbalance and straightly subordinate effortlessly.

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