# Speed Control of Induction Motors

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Abstract:- The electric drive systems used in industrial applications are increasingly required to meet the higher performance and reliability requirement. Today about 90% of all industrial motor applications are induction motors because they are simple in design, easy to maintain, and less costly than other types of motors. The load on induction motor is not constant, and varies as per load requirement. So speed must change as per load. The speed of induction motors can be controlled by the stator frequency ( $f_s$ ) and voltage ( $V_s$ ). Although there are different methods of speed control of induction motors,

constant  $\frac{V_s}{f_s}$  control is investigated in this paper. Of course the most preferred technique in most variable speed induction motor drive applications is the  $\frac{V_s}{f}$ 

technique. In constant  $\frac{V_s}{f_s}$ , we can vary the supply voltage as well as the supply frequency such that the  $\frac{V_s}{f_s}$  ratio remains constant, and the flux remains constant too.

*Keywords:*— *induction motor, speed control, frequency, voltage, flux.* 

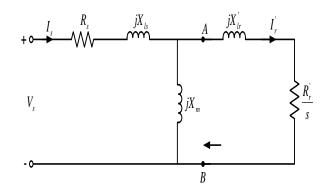
# I. INTRODUCTION

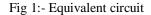
Demand-side energy management is one of the many ways to cope with the increasing demand for energy in industrialized countries. Improving the efficiency of electrical motors and, more specifically, the induction motors is one aspect where there is a significant potential for energy savings due to the large number of electrical motors in the electrical power system of industrialized and developing countries [1]. Induction motors are widely used in many residential, commercial, industrial and utility applications. This is because the motor has low manufacturing costs, wide speed range, high efficiency and robustness [2]. Motor-driven systems use two-thirds of the total electricity consumed by industry [3]. Induction motors, both three- and single-phase, are used extensively for adjustable-speed drives applications. Actually, induction machines dominate the scene of adjustable-speed pump drives because of the cost-driven nature of such a motor application [4]. They consume a large percentage of the electrical demands in industrialized countries [5]. The speed of induction motors can be controlled by varying  $f_s$ , which controls the synchronous speed (and, hence, the motor speed, if the slip is kept small), keeping the flux  $\phi$  constant by varying  $V_s$  in a linear proportion to  $f_s$  [6]. So, we can get different operating zones for various speeds and torques.

Thus variable frequency control is employed to achieve efficient motor breaking and speed reversal, and to reduce the starting current.

# II. MOTOR EQUIVALENT CIRCUITS

The rotor bars are shorted to represent squirrel cage induction motor, or it is wound rotor with shorted terminals. Three-phase squirrel cage induction motors in particular is by far the most representative of the rotating electrical machines used in the present industrial driven equipment because of its roughness and versatility [7]. These machines are structurally very robust and are a primary source of motive power and speed control where dc machines cannot be used [8]. The equivalent circuit is shown in Fig.1.





$$Z_{in} = R_{s} + jX_{s} + \frac{jX_{m} \left(\frac{R'_{r}}{s} + jX'_{lr}\right)}{\frac{R'_{r}}{s} + j(X'_{lr} + X_{m})}$$
(1)

$$I_s = \frac{V_s}{Z_{in}} \tag{2}$$

$$I'_{r} = \frac{jX_{m}I_{s}}{\frac{R'_{r}}{s} + j(X'_{lr} + X_{m})}$$
(3)

Alternatively, we consider The venin equivalent circuit at terminal AB

$$V_{TH} = jX_m x \frac{V_s}{R_s + j(X_{ls} + X_m)} = \frac{V_s X_m}{\sqrt{R_s^2 + X_s}} \angle 90^o - \tan^{-1}\left(\frac{X_s}{R_s}\right)$$
(4)

where 
$$X_s = X_{ls} + X_m$$

$$Z_{TH} = \frac{jX_m (R_s + jX_{ls})}{R_s + j(X_{ls} + X_m)} = \frac{X_m^2 R_s}{R_s^2 + X_s^2} + \frac{jX_m (R_s^2 + X_s X_{ls})}{R_s^2 + X_s^2}$$
(5)

$$Z_{TH} = R_t + jX_t \tag{6}$$

where

$$R_{t} = \frac{X_{m}^{2}R_{s}}{R_{s}^{2} + X_{s}^{2}}; X_{t} = \frac{X_{m}(R_{s}^{2} + X_{s}X_{ls})}{R_{s}^{2} + X_{s}^{2}}$$
(7)

The simplified equivalent circuit referred to the stator is shown in Fig. 2.

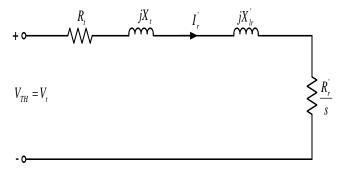


Fig 2:- Simplified equivalent circuit referred to the stator

$$I_{r}^{'} = \frac{V_{t}}{\sqrt{\left(R_{t} + \frac{R_{r}^{'}}{s}\right)^{2} + \left(X_{t} + X_{lr}^{'}\right)^{2}}}$$
(8)

The airgap power is the power dissipated in fictitious rotor resistance  $\frac{R_r}{s}$ 

$$P_a = 3I_r^{2} \frac{R_r}{s} \tag{9}$$

International Journal of Innovative Science and Research Technology

ISSN No:-2456-2165

When we subtract rotor copper loss,  $P_{rcu} = 3I_r^{'2}R_r'$ we get the mechanical output power  $P_m$  given by

$$P_m = P_a - \Pr_{cu} = 3I_r^{'2} \frac{R_r^{'}}{s} - 3I_r^{'2} R_r^{'} = 3I_r^{'2} \frac{R_r^{'}}{s} (1-s)$$
(10)

Synchronous speed is given by

$$\omega_s = \frac{2}{p}\omega = \frac{2}{p}2\pi f = \frac{4\pi f}{p} \tag{11}$$

Where f is the stator supply frequency and p is number of poles.

If the motor speed is  $\omega_m$ , then  $\omega_m$  can be derived from the definition of slip *s* as

$$s = \frac{\frac{2}{p}\omega - \omega_m}{\frac{2}{p}\omega}$$
(12)

$$\omega_m = \frac{2}{p}\omega(1-s) \tag{13}$$

The torque can be estimated using measured stator currents and estimated stator flux [9]. Over the years, considerable technology advances have been made in the motor and drive areas resulting in major improvements in their performance [10].

Mechanical Torque is

$$T_{m} = \frac{P_{m}}{\omega_{m}} = \frac{3I_{r}^{'2} \frac{R_{r}^{'}}{s}(1-s)}{\frac{2}{p}\omega_{s}(1-s)} = \frac{3P}{2\omega_{s}}I_{r}^{'2} \frac{R_{r}^{'}}{s}$$

$$T_{m} = \frac{3P}{2\omega_{s}}\frac{R_{r}^{'}}{s}\frac{V_{t}^{2}}{\left(R_{t} + \frac{R_{r}^{'}}{s}\right)^{2} + \left(X_{t} + X_{lr}^{'}\right)^{2}}$$
(14)
(15)

Minimum/maximum torque  $T_{\rm m\ max/min}$  is obtained by differentiating Tm with respect to slip s to obtain slip at which  $T_{\rm m}$  is maximum or minimum by equating the differential to zero:

$$\frac{d}{ds}T_{m} = \frac{d}{ds} \left[ \frac{3P}{2\omega_{s}} \frac{R_{r}}{s} \frac{V_{t}^{2}}{\left(R_{t} + \frac{R_{r}}{s}\right)^{2} + \left(X_{t} + X_{lr}\right)^{2}} \right] = 0$$

$$\frac{R_r^2}{s^2} - R_t^2 - \left(X_t + X_{lr}^{'}\right)^2 = 0$$

$$S_{\text{max/min}} = \pm \frac{R_r^{'}}{\sqrt{R_t^2 + \left(X_t + X_{lr}^{'}\right)^2}}$$
(16)

Positive sign gives  $T_{\text{max}}$  and negative sign gives  $T_{\text{min}}$ .

Substituting  $S_{\text{max}}$  and  $S_{\text{min}}$  in the equation for torque gives

$$T_{\max/\min} = \frac{3P}{4\omega_s} \frac{V_t^2}{R_t \pm \sqrt{R_t^2 + (X_t + X_{lr})^2}}$$
(17)

Plus sign gives  $T_{\text{max}}$  and negative sign gives  $T_{\text{min}}$ 

When  $\omega_m = 0$ , i.e, *s*=1, we have starting torque  $T_{ms}$  as

$$T_{m} = \frac{3P}{2\omega_{s}} R_{r}^{'} \frac{V_{t}^{2}}{\left(R_{t} + R_{r}^{'}\right)^{2} + \left(X_{t} + X_{lr}^{'}\right)^{2}}$$
(18)

At synchronous speed, i.e., s = 0, and  $T_m = 0$  as shown in Fig. 3 and Fig. 4

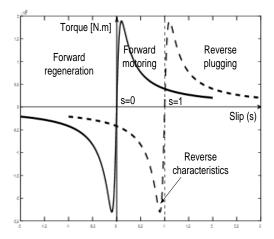


Fig 3:- Torque-slip characteristics

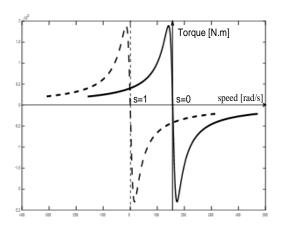


Fig 4:- Torque-speed characteristics

The reverse characteristic is obtained by the reversal of the phase sequence of the motor terminals [11].

## III. VARIABLE FREQUENCY CONTROL

#### A. Constant Flux Control

Equation (12) shows that the synchronous speed is directly proportional to the supply frequency. Therefore the synchronous speed and the motor speed can be controlled below and above the normal full-load speed by changing the supply frequency. If the stator drop is neglected the voltage E induced in the stator is equal to the supply voltage  $V_s$ . It can be shown to be:

$$E = V = k f \phi \tag{19}$$

where k = constant, and  $\phi = \text{air-gap flux}$ 

From equation (19), any reduction in f without a change in  $V_s$  causes an increase in  $\phi$  which will saturate the motor, leading to increase in magnetizing current, distortion of line current and voltage, core loss, stator copper loss, and production of high-pitched noise. However, decrease of  $\phi$  should be avoided in order to retain the torque capability of the motor. Therefore, the variable frequency control below the rated frequency is generally carried out by reducing the machine phase voltage along with the frequency in such a manner that the flux is maintained constant.

In constant flux control, the magnetizing current is kept constant from zero speed to rated speed and then rotor current or mechanical power is maintained constant above rated speed. Fig. 5 is used for this analysis.

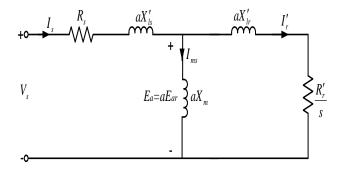


Fig 5:- Equivalent circuit for constant flux control

$$a = \frac{f}{f_r} \tag{20}$$

where  $f_r$  is the rated frequency. The motor reactances,  $X_{ls}$ ,  $X'_{lr}$  and  $X_m$  are measured at rated frequency ( $f_r$ ).  $E_{ar}$  is rated induced emf.

$$I_{ms} = \frac{aE_{ar}}{jaX_m} = \frac{E_{ar}}{jX_m}$$
(21)

 $I_{\rm ms}$  is therefore kept constant at its rated value  $I_{\rm ms} = \left| \frac{E_{\rm ar}}{X_{\rm m}} \right|$ 

Under this condition, rotor current is

$$I'_{r} = \frac{aE_{ar}}{\frac{R'_{r}}{s} + jaX'_{lr}} = \frac{E_{ar}}{\frac{R'_{r}}{as} + jX'_{lr}} = \frac{E_{ar}}{\sqrt{\left(\frac{R'_{r}}{as}\right)^{2} + X'_{lr}^{2}}}$$
(22)

The torque becomes

$$T = \frac{3I_{r}^{\prime 2} \left(\frac{1-s}{s}\right) R_{r}^{\prime}}{\frac{2}{P} a \omega_{s} (1-s)} = \frac{3P}{2\omega_{s}} \frac{R_{r}^{\prime}}{as} I_{r}^{\prime 2}$$
(23)

$$T = \frac{3P}{2\omega_s} \frac{R'_r}{as} \frac{E_{ar}^2}{\left(\frac{R'_r}{as}\right)^2 + X'_{lr}^2}$$
(24)

Slip is defined as

$$s = \frac{a\omega_s - \frac{P}{2}\omega_m}{a\omega_s}$$

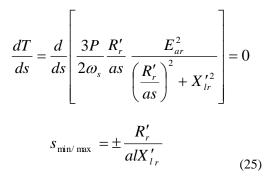
$$as\omega_s = a\omega_s - \frac{P}{2}\omega_m = \omega_{sl} = slip \ speed$$

(as) is kept constant from zero frequency to rated frequency. This keeps both  $|I'_r|$  and T constant from zero speed to rated speed or zero frequency to rated frequency or from a=0

to a=1=
$$\frac{f_r}{f_r}$$

If (as) is kept constant then since  $\omega_s = \text{constant}$ , slip speed in rad/s is also kept constant.

Slip at which maximum/minimum torque occurs is given by



Substituting in *T* we have that

$$T_{\max/\min} = \pm \frac{3P}{4\omega_s} \frac{E_{ar}^2}{X'_{lr}} = \text{Constant}$$
(26)

If we take many values of *a* between 0 and 1.0 ( $0 \le a \le$  1.0), the nature of speed-torque curves for the variable frequency operation at a constant flux are shown in Fig. 6, both for the motoring and braking operations. Thus the torque-speed characteristics are approximately parallel straight lines.

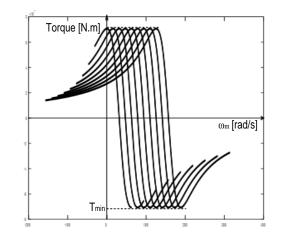


Fig 6:- Speed-torque curves with variable frequency controlconstant flux control.

For a>1, rotor current is maintained at its rated value, that is

$$I'_{r} = \frac{E_{ar}}{\frac{R'_{r}}{s} + jaX'_{lr}} = \text{Constant}, \qquad (27)$$

 $P_{\rm m}$  is constant since  $I'_r$  is constant and s is constant.

$$T_m = \frac{P_m}{\omega_m} \tag{28}$$

Since  $P_{\rm m}$  is constant,  $T_{\rm m}$  decreases linearly with a or  $\omega_{\rm m}$ 

$$P_{m} = 3I_{r}^{\prime 2} \frac{(1-s)}{s} R_{r}^{\prime}$$

$$T_{m} = \frac{3P}{2\omega_{s}} \frac{R_{r}^{\prime}}{s} \frac{E_{ar}^{2}}{\left(\frac{R_{r}^{\prime}}{s}\right)^{2} + \left(aX_{lr}^{\prime}\right)^{2}}$$
(29)
(30)

For maximum torque

$$\frac{dT_m}{ds} = \frac{d}{ds} \left[ \frac{3P}{2\omega_s} \frac{R'_r}{s} \frac{E_{ar}^2}{\left(\frac{R'_r}{s}\right)^2 + X_{lr}^{\prime 2}} \right] = 0$$
$$s = \pm \frac{R'_r}{aX'_{lr}}$$
(31)

Thus,

$$T_{\max/\min} = \pm \frac{3P}{4\omega_s} \frac{E_{ar}^2}{aX'_{lr}}$$
(32)

and slip is always close to the rated value. Above base speed, Ear is kept constant. In most induction machines,  $\frac{R'_r}{r} >> aX'_{lr}$ .

$$I'_r = \frac{sE_{ar}}{R'_r} \tag{33}$$

$$T_{m} = \frac{3P}{2\omega_{s}} \frac{R_{r}'}{s} \frac{s^{2} E_{ar}^{2}}{{R_{r}'}^{2}} = \frac{3P}{2\omega_{s}} \frac{s E_{ar}^{2}}{R_{r}'}$$
(34)

B. Constant 
$$\frac{V_s}{f_s}$$
 Control

Constant flux control is difficult to implement because it is not feasible to accurately monitor the flux (or  $I_m$ ) and keep it constant. Therefore in the field, practical methods are

used for field control. One of the methods is to keep  $\frac{V_s}{f_s}$ 

constant at the rated value and we call this constant  $\frac{V_s}{f}$ 

drive. The equivalent circuit used for analysis of this drive is shown in Fig. 7.

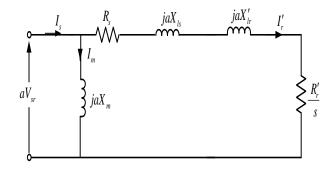


Fig 7:- Per phase approximate equivalent circuit referred to the stator

International Journal of Innovative Science and Research Technology ISSN No:-2456-2165

where,

$$\frac{V_s}{f_s} = \frac{V_{sr}}{f_{sr}} = k$$

 $V_{\rm sr}$  is rated stator terminal voltage per phase, and  $f_{\rm sr}$  is rated stator frequency.

 $V = aV_{sr}$  is applied to the stator at frequency  $f = af_r$  where  $0 \le a \le 1$ 

$$I'_{r} = \frac{aV_{sr}}{\sqrt{\left(R_{s} + \frac{R'_{r}}{s}\right)^{2} + a^{2}\left(X_{ls} + X'_{lr}\right)^{2}}}$$
(35)

Air-gap power,

$$P_a = 3I_r^{\prime 2} \frac{R_r^{\prime}}{s} \tag{36}$$

Mechanical power,

$$P_m = 3I_r'^2 \frac{R_r'}{s} (1-s)$$
(37)

Motor speed,

$$\omega_m = \frac{2}{p} a\omega \left(1 - s\right) \tag{38}$$

Developed Torque,

$$T_{m} = \frac{P_{m}}{\omega_{m}} = \frac{3p}{2\omega} \frac{R'_{r}}{as} \frac{V_{sr}^{2}}{\left(\frac{R_{s}}{a} + \frac{R'_{r}}{as}\right)^{2} + \left(X_{ls} + X'_{lr}\right)^{2}}$$
(39)

The typical torque-speed characteristics are shown in Fig. 8. As the frequency is reduced, *a* decreases and the slip for maximum torque increases. It can be observed that the maximum torque ( $T_{max}$ ) decreases as *a* or  $\omega_m$  decreases while minimum torque ( $T_{min}$ ) increase with decrease in *a* or  $\omega_m$ . By varying both the voltage and frequency, the torque and speed can be controlled. The voltage at variable frequency can be obtained from three-phase inverters or cycloconverters. The cycloconverters are used in very large power applications, such as locomotives and cement mills, where the frequency [10].

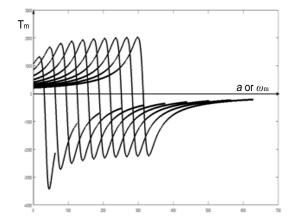
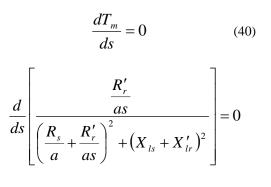


Fig 8:- Speed-torque curves with variable frequency control

- constant 
$$\frac{V_s}{f_s}$$
.

The maximum or minimum torque occurs at slip given by



Thus

$$s_{\max/\min} = \frac{\frac{R'_{r/a}}{a}}{\sqrt{\left(\frac{R_{s/a}}{a}\right)^{2} + \left(X_{ls} + X'_{lr}\right)^{2}}}$$
(41)

And so  $T_{\text{max/min}} = \frac{3p}{4\omega} \frac{V_{sr}^2}{\frac{R_s}{a} \pm \sqrt{\left(\frac{R_s}{a}\right)^2 + \left(X_{ls} + X_{lr}'\right)^2}}$ (42)

At low frequencies,  $\frac{R_s}{a} >> (X_{ls} + X'_{lr})$ , making  $T_{\text{max}}$ 

approach zero as *a* approaches zero and  $T_{\min}$  approaches  $\infty$  as *a* approaches zero. This is different from the constant flux drive where  $|T_{\min}| = |T_{\max}|$  for all *as*. Compare Fig. 6 and Fig. 8.The variation of various variables such as developed torque (T), power (P<sub>m</sub>), terminal voltage (V), and rotor current (I<sub>r</sub>) with per unit frequency a are shown in shown in Fig. 9.

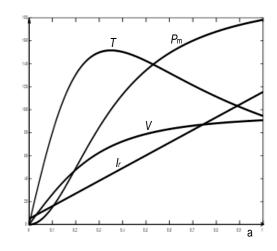


Fig 9:- Plots of T,  $P_m$ , V, and  $I_r$  versus per unit frequency a

For  $a \ge 1 V_s$  must be kept constant at  $V_{sr}$  which is the rated terminal voltage to prevent insulation damage. This implies keeping operation constant at rated current or power to prevent overheating. For a >1, the equivalent circuit is shown in Fig. 10.

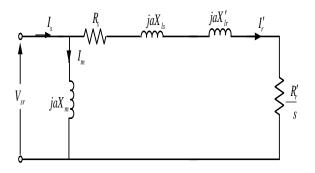


Fig 10:- per phase approximate equivalent circuit referred to the stator

This gives  $T_{\rm m}$  as

$$T_m = \frac{3p}{2\omega} \frac{R'_r}{as} {I'_r}^2 \tag{43}$$

$$T_{m} = \frac{3p}{2\omega} \frac{R'_{r}}{as} \frac{V_{sr}^{2}}{\left(R_{s} + \frac{R'_{r}}{s}\right)^{2} + a^{2} \left(X_{ls} + X'_{lr}\right)^{2}}$$
(44)

The torque-speed characteristics for various values of a are shown in Fig. 11.

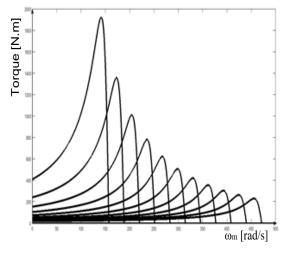


Fig 11:- Speed-torque curves with variable frequency control (*a*> 1)

Fig. 11 shows that the torque tends to zero as a increases. Thus, for a given torque demand, the speed can be controlled by changing the frequency. The three-phase inverter can vary the frequency at a fixed voltage.

And maximum/minimum torque is given by

$$T_{\max/\min} = \frac{3p}{4a\omega} \frac{V_{sr}^2}{R_s \pm \sqrt{R_s^2 + a^2 (X_{ls} + X_{lr}')^2}}$$
(45)

For a > 1,  $T_{\text{max}}$  and  $T_{\text{min}}$  for  $a(X_{ls} + X'_{lr}) >> R_s$  are equal in magnitude, given by

$$T_{\max/\min} = \frac{3p}{4\omega} \frac{V_{sr}^{2}}{\pm a^{2} \left(X_{ls} + X_{lr}'\right)}$$
(46)

but less than that for constant flux drive. This implies higher critical speed at which operating T is equal to pull-out torque. To make  $\frac{V_s}{f_s}$  = constant control perform like constant flux control, we adjust terminal voltage according to the equation:

$$V_s = V_o + k f_s \tag{47}$$

where kh as the value

$$k = \frac{V_{sr} - V_o}{f_{sr}} \tag{48}$$

$$V_s = V_o + \frac{V_{sr} - V_o}{f_{sr}} f_s \tag{49}$$

where  $V_0$  is the stator terminal voltage required to produce rated flux ( $I_{msr}$ ) at zero speed.

#### IV. CONCLUSION

In this paper, the speed control of squirrel-cage induction motor has been investigated. If the ratio of voltage V

to frequency  $\frac{V_s}{f_s}$  is kept constant at its rated value, the

flux remains constant, and the maximum torque, which is independent of frequency, can be maintained approximately constant. However at low frequencies, the air-gap flux is reduced due to the drop in the stator impedance and the ratio has to be increased to maintain the torque level. This is

known as  $\frac{V_s}{f_s}$  or *volts/hertz* control. It is the most popular

method of induction motor speed control.

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