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Gnostics in Valuation: Non-Parametric Approach to Multiples Estimation

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Abstract:- Using valuation multiples generates a number of methodological questions in adjusting the multiples to the particular industry. In this paper, we identify the most accurate method of estimation of the multiples for the business valuation. Extensive secondary theoretical research in the theory of valuation was done for analyzing the appropriate multiples. Math Gnostics non-parametric method is used vice versa in comparison with standard statistical location parameters to find the proper mean value. Local and Global Gnostics' functions reached lower deviation of Market Value. The evidence shows that using the Gnostics' location parameters instead of statistical ones allows reached Market Value with higher accuracy. The future research intention is focused on the application of Math Gnostics on the digital economy sector.

Keywords:- *Valuation; Multiples; Math Gnostics; Accuracy*.

I. INTRODUCTION

The crucial task of business valuation of any assets accuracy. As the main sources of accuracy can be simplified identified capital markets imperfections, legislature, and the wide range of behavioral aspects. Value of any assets can be viewed from many philosophical aspects. The result of the valuation procedure supposes to be monetary amount what is done by legislature purposes. On that basis is a request on the highest accuracy of valuation procedure obvious.

Present theory of finance identifies the value of the business with anticipated benefits resulting from the core business. Equation 1 expresses the general relation of value creation. Value is the function of This is the basic principle of valuation, see Equation 1.

$$V=f(U_n) \qquad (1)$$

where V is the value of business, U_n are benefits.

The parading beak down of the classical approach changes the cost/comparison/income/ view to simple one approach. All of the foregoing can be articulated as future benefits. And Value Time Comparability principle allows reaching some result with different valuation approach because from the begging all based on the presumption of future benefits. The estimated value then consists of two parts core kernel value and errors. Application of different valuation principles and techniques differ in the ratio of core value and error on the final result. For future explanation is used Market Value according to IVS and Fair Market Value according to IFRS.

Capital markets imperfections lead to undervalued and overvalued stocks. Errors made by markets cause the differences of market value to intrinsic value, see Equation 2.

$$Market\ Value = V_i \pm \xi_{d,f} \qquad (2)$$

where V_i is intrinsic value of stock, and $\xi_{d,f}$ is error in market value.

Market value is only the value of stocks based on the measure of real-world numbers. But, application and results are connected with a lot of issues. Not all of the capital markets are such a "developed" for catching the "fair value" of businesses. Use the market value with no issues is possible essentially only for big corporations listed on big markets (US, UK, FR, GER, JAP etc.) SME's are principled excluded. From the practical point of view can be identified issues the length of an analyzed period, frequency, date of valuation, and namely average values.

II. HYPOTHESIS ARTICULATION

The research question is methodological. The goal is to find the approach which the most accurately enables to use valuation multiples. The hypothesis is articulated as follows:

H_0: Gnostic location parameter rather than statistical mean allows to estimate valuation multiples which achieve a more accurate estimate of market value.

H_1: Gnostic location parameter does not allow to estimate valuation multiples which achieve a more accurate estimate of market value than the statistical mean.

Testing criteria is a difference in average relative error between two subsets of estimated market values, see Equation 3

$$R_G < R_s \qquad (3)$$

Where R_G is the average relative error of estimated market values with using Math Gnostics location parameter, R_S is the relative error of estimated market values using statistical mean.

The lower values of average relative values indicate the higher accuracy of estimation.

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III. MATERIAL AND METHODS

The deduction is used for research question formulation. From general knowledge about capital markets is articulated research problem of the most fitting method for estimation valuation multiples for the forestry industry. Induction derived the theoretical solution of the most accurate estimation of location parameter for the multiples value in comparison with the value of publicly traded stock on capital markets. For deriving the method is used the comparison of standard statistical approaches of mean value and probability and methods of mathematical Gnostics. Firstly, are summarized mostly used multiples in relative valuation approach recommended by extensive literature research for commodity companies. The multiples are decomposed ale analytically explained with the attention on the usage of location parameters. Secondly, Gnostics' location parameters and functions are decomposed and explained and adjusted for multiples valuation. Further, the theoretical comparison is done and results are discussed and formulas were derived for empirical verification of the accuracy of application of mathematical Gnostics in the valuation of forestry companies via relative approach.

Hypothesis verification according to testing criteria stated in equation (1) starts at data collection. Data used for hypothesis testing are collected form the Warsaw Stock Exchange, Wood, and Paper sector, Forestry Enterprises. Data are valid to 31.10.2017. Variables used are Market Capitalization, Earnings and P/E. Math Gnostics were applied on the sample to find gnostic criteria: Optimal S, the Relative information quality of the fit, LB_z, UB_z, LB_{fin}, UB_{fin}, density's Maximum and Minimum, Robust median, Location parameter, the Mean absolute data fitting error, Maximum absolute data fitting error, Mean absolute error of the probability, and Maximum absolute error of the probability. For statistical analysis were applied standard descriptive statistics.

The valuation in all three approaches (cost, market, income) inherently comes from comparison with other economic subjects. To make the judgment about the economic and financial condition of businesses the group of truly alike firms has to be chosen. The firms should apply the same operating policy, corporate governance, internal structure etc. but it is usually not available characteristics for grouping (Kovanic and Humber, 2013). The breadth of products or services offered, which influence the total market exposure of the business, since a few of the sectors of the market behave in the same way or in a synchronous manner.

Damodaran (Damodaran, 2006) pointed out that classical classification of businesses, see above, and is not sufficient nowadays. Like grouping and comparison, criteria are taken into account economic characteristic in the different point of view. Companies can be similar just only because of economic performance and no meter in what industry they are. It comes from the definition of business like an engine of money.

The fundamental grouping characteristics:

- The type of inputs used (raw material, energy, prefabricated goods, union or non-union labor, etc.).
- The technology employed assembly line, large automated machinery, labor-intensive processes, retail operation, etc.
- The company's image and reputation, and relative position in its markets.
- Size: not only in term of relative dominance in market share, but a large established company has access to resources not available to others. Further a firm with a substantial international presence can advantageously employ the resources available to any of its components to dampen other local economic problems asynchronously.
- The existence of strong industry lobbies to assist in the obtaining better operating conditions as well as other comparative advantages over both local and international competitors.
- Geographic locations determines the distance from sources of raw materials, availability and cost of transport facilities, etc.
- Corporate laws and Market Regulation can provide comparative advantages to operating in certain countries or states.
- Links to the financial sector: in those counties, where banks are not prohibited for owing other types of firms, advantageous sources of non-competitive financing may exist.

According to the findings, see Table 1, is identified as a mostly fitted multiple for a valuation of forestry enterprises P/E ratio, mostly used and published capital market indicator.

Sector	Multiple Used	Rationale
Cyclical Manufacturing, Commodity	PE, Relative PE	Often with normalized earnings
Growth Firms	PEG ratio	Big differences in growth rates
Young Growth Firms w/losses	Revenue Multiples	What choice do you have
Infrastructure	EV/EBITDA	Early losses, big DA
REIT	P/CFE (where CFE = Net Income + Depreciation)	Big depreciation charges on real estate
Financial Services	Price/Book Equity	Marked to market?
Retailing	Revenue Multiples	Margins equalize sooner or later

Table 1. Conventional Usage of Multiples

Kovanic and Humber (Kovanic and Humber, 2013) propose the different approach. To launch the analysis with a group of companies primarily chosen by applying some traditional-formal criteria for industry membership, technology, etc., but leaving the final choice of firms really comparable to robust modeling technique, which provides this information as an outcome of the analysis, rather than having it imposed as an a priori assumption. The explanation of this approach is based on Economics of Information (Math Gnostics) which is a robust modeling apparatus. The solution about conformity of grouping firms and analyzed lies in the hands of the math model, which maximize the information, explored from data. The Gnostics theory of uncertain data is derived from properties of every single measurement. The composition law then obtains the properties of the whole data

sample. The Gnostics theory is based upon two axioms (Wagner at al., 2002).

Date in Math Gnostics consist from ideal value fulfilled by uncertainty factor and scale parameter, see Equation 4.

$$A = A_0 + S\phi \qquad (4)$$

Where A is measured value, A_0 is ideal value, ϕ is standardized uncertainty, and S is scale parameter.

The change of the ideal value influences only the absolute value of the vector while change of the uncertainty rotates the vector in the Minkowskian plane. The curvature of the plane is determined by the scale parameter. The axiom can be written in matrix form, which is isomorphs with Lorentz's transformation known from the special theory of relativity (Wagner at al., 2002).

For the quantifying the error and weights of the data are used formulas in Equation 5 and. Equation 6.

$$f_i = \frac{2}{q_i^2 + \frac{1}{q_i^2}} \qquad (5)$$

$$h_i = \frac{q_i^2 - \frac{1}{q_i^2}}{q_i^2 + \frac{1}{q_i^2}} \qquad (6)$$

where $q_i = \left(\frac{z_i}{z_0}\right)^{1/S}$,

$f_i(1) = 1,\ \lim_{q_i \to 0} f_i = 0,\ \lim_{q_i \to \infty} f_i = 0;\ h_i(1) = 0,\ \lim_{q_i \to 0} h_i = -1,\ \lim_{q_i \to \infty} h_i = +1.$

The second axiom on Gnostics postulates the composition law to be isomorphs with the conservation of energy-momentum from the special theory of relativity. The statistical composition law is isomorphs with the law of conservation of energy from the classical mechanics (Kovanic and Humber, 2013) (Wagner at al., 2002).

According to Gnostics (Kovanic and Humber, 2013) should be a distribution function characterized by the pattern formed by different values of the given dataset (sample). In Gnostics, the probability can be interpreted as the expectation based on the data in the sample. The model should be a continuous distribution function of actual data distribution. We can find it by Discrete Distribution Function (DDF), a family of distribution functions suitable to the model, and the Criterion Function. If the function is extremizationated then ensure the best goodness-of-fit. To obtain Discrete Distribution Function DDF there versions can be applied:

- The Empirical Distribution Function (EDF),
- The Kolmogorov -Smirnov DDF,
- The Maximum Entropy DDF.

All these three approaches to the construction of a discrete distribution function DDF are based on the a priori assumption, that all data have equal importance. The mix of all three previous functions gives us the Weighted Empirical Distribution Function. The Gnostics theory drives four version of the distribution function: ELDF, EGDF, QLD, and QGDF. They represent four applications of the same formula and they are related to the probability distribution of individual datum, see Equation 7.

$$** DF = \frac{1 - h_{**}}{2} \qquad (7)$$

Where $\overline{h}$ is the specific version of data sample´s irrelevance.

The weights and irrelevances and their functions are defined in two kinds of data support. Additive data are arbitrary real numbers form the interval $\mathcal{R}^1 = (-\infty, \infty)$. Multiplicative data are strictly positive numbers from the interval $\mathcal{R}_+ = (0, \infty)$. Thus, theoretically domain of Gnostics distribution function is $\mathcal{R}_+$ however the real data are always bounded within a finite interval.

To unite the manipulations of additive and multiplicative data unified closed interval is introduced, see Equation 8.

$$Z_e = \left[\frac{1}{\exp(1)}, \exp(1)\right] \qquad (8)$$

Local distribution function (ELDF) is a sum of expressions, which have properties of Parzen's kernels. It is only one derived from theory. Can be written in the form Equation 9.

$$ELDF \equiv EL(\mathbb{Z}, Z_0, S) = \frac{1 - \overline{h_E}}{2} \qquad (9)$$

EDGF extremely fits for the test of data sample homogeneity. Conversely, the applicability is not universal. It is suitable only for data samples for which EGDF´s density is non-negative over its full range and which have only one maximum. Quantifying local distributing function (QLDF) according to general formula Equation 5 involves obstacles (irrelevance and improbability what can reach infinite values) and cannot be applied directly. In this case, for QLDF quantified irrelevance, Equation 10.

$$h_{QL} = \frac{\overline{h_G}}{\sqrt{1 + \left(\overline{h_Q}\right)^2}} \qquad (10)$$

Where $\overline{h_Q}$ is the weighted mean of quantifying irrelevances of all of the sample's data. Quantifying irrelevance of k-th datum is $(q^2 - 1/q^2)/2$ and quantifying weight$(q^2 + 1/q^2)/2$.

According to Equation 5. is quantified local distribution function, Equation 11 and 12:

$$QLDF \equiv QL(\mathbb{Z}, Z_0, S) = \frac{(1 - h_{QL})}{2} \qquad (11)$$

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Differentiating us get density, see Equation 12.

$$\frac{dQL}{dZ_0} = \frac{1}{SZ_0} \frac{\overline{f_Q}}{\left(1+\left(\overline{h_Q}\right)^2\right)^{3/2}} \quad (12)$$

Where is $\overline{f_Q}$ is weighted men of the quantifying irrelevances of the samples ´s data.

For single datum, $(N = 1)$ it provides the same results as by the densities of ELDF and EGDF (but not for $N > 1$). QLDF emphasize outliers of the sample.

Then, quantifying global distribution function is, Equation 13.

$$QGDF \equiv QG\left(\mathbb{Z}, Z_{0,}, S\right) = \frac{\left(1-\frac{\overline{h_Q}}{\overline{f_Q}}\right)}{2} \quad (13)$$

In Math Gnostics, the cluster is the group of integrated peaks. Thus, peak equals kernel. Marginal cluster analysis is decomposition of one-dimensional data sample into two or more subgroups which represent themselves in the sample´s density graph as separated clusters (simplest way is to use EL-distributions). Gnostics approach for data (business units) comparison is based on marginal (one-dimensional) analysis. This is the first stage of possible Gnostics analytics approaches. It is based mainly on the application of distribution functions (Kovanic and Humber, 2013).

Gnostics theory in the marginal analysis (Kovanic and Humber, 2013) focuses on four distribution function mentioned above. These distribution functions recommend comparing by five characteristics:

- S – Scale parameter,
- LB – Low bound,
- UB – Upper bound,
- MF – Maximum fidelity fit.

Maximum fidelity fit (MF-criterion) is the generally applicable function for an arbitrary system of a priory data weights. It measures relations between two numbers – the actual value and its required value (estimating the weight).

The optimal values of parameters S_g, LB, UB can be finding by fidelity function, see Equation 14.

$$f_{max} = max_{S_g,LB,UB} \sum_{m=1}^{N} f\left(\frac{EGDF(Z_m)}{E_{MF,m}}\right) \quad (14)$$

Where $f(*)$ is the fidelity data weight $f(*) = \frac{2}{((*)^2+(*)^{-2})}$, Z_m are the sample's data $(m = 1, \dots, N)$, $E_{MF,m}$ is weighted empirical distribution function.

Gnostics theory unequivocally does not use average (arithmetic mean) but local and global distribution function for description and hereinafter for comparison of data. The comparison is the fundamental principle of valuation. To find the right approach is the huge challenge. Standard statistics give us for comparing mean values and choose from the standardized functions loaded by errors. Damodaran (Damodaran, 2006) recommends for comparison not only standard business characteristics but also economic variables based on income. If the business is the engine for the money no matter in what industry the company works. The crucial characteristics are the economics one. In this case, the sample is seen as more broadly conceived. The Math Gnostics bring us the widest conception of data. Because the Gnostics theory is focused especially on small samples and it is not polluted by the law of big numbers. Any data can be taken and hereinafter analyzed its explanatory value. This is the first introduction of this theory to valuation theory and practice. The results are difficult to measure because the methodology is very different.

IV. RESULTS AND DISCUSSIONS

The hypothesis was tested according to the data sample and methods stated in part 2. The value of P/E estimated by Math Gnostics location parameter is 18,34 and the statistical mean found by standard descriptive statistics is 19, 74. The value of enterprise was estimated by the standard methodology used for valuation multiples using both P/E founded by the Gnostic and statistical approach. After it was calculated the value of absolute and relative value. As a reference value was considered the market capitalization of enterprises. As a testing criterion were stated in Equation 3 the difference between the relative error of measurement between the market values of companies and estimated values by Gnostic scale parameter and statistical mean value. According to results see, Tables 1 and 2, is the relative average error in values of companies estimated by Gnostic location parameter -5, 39 and relative average error in values of companies estimated by statistics -13,43. Based on the findings we fail to reject the null hypothesis.

Math Gnostic Variable	Value
Optimal S	1,86
LBz	0,29
UBz= 2.987781	1,00
Lbfin	9,95
Ubfin	33,14
The density's MAXIMUM No. 1 in the point xinfty	0,49
The density's MINIMUM No. 2 in the point xinfty	12,19
The density's MAXIMUM No. 2 in the point xinfty	36,67
Robust median	17,81
Location parameter	18,34
Mean absolute data fitting error	2,43
Maximum absolute data fitting error	7,39
Mean absolute error of the probability fit is	0,07
Maximum absolute error of the probability fit	0,15

Table 2. Math Gnostics Parameters

	Minimum	Maximum	Mean		Std. Deviation	Variance	Skewness		Kurtosis	
	Statistic	Statistic	Statistic	Std. Error	Statistic	Statistic	Statistic	Std. Error	Statistic	Std. Error
P/E	12,40	32,20	19,7400	3,66710	8,19988	67,238	,968	,913	,048	2,000

Table 3. Descriptive Statistics

Figure 1 shows probability distribution as a function of quantiles of the finite data support probability of Global Distribution Function. The distribution functions show high robustness with respect to volatility in the stock market data. We can identify two outliers in Figure 1 what finally do not have the impact on location parameter.

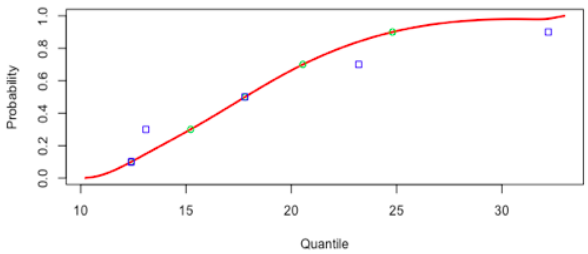

Fig 1:- Probability of EGDF Finite

Figure 2 shows density of Global Distribution Function. The course of the function find from graphical analysis that estimation of Gnostic local parameter is more accurate than the statistical mean, for numerical see Tab 1 and 2.

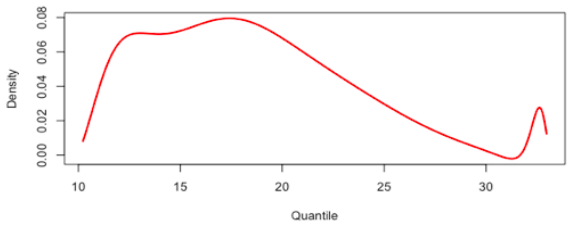

Fig 2:- Density of EGDF Finite

Data are homogenous thus are represented by EGDF function as a single gnostic event – central tendency. Distribution is drawn over the probability density over the finite data support. Figure 3 shows probability distribution as a function of quantiles of the infinite data support.

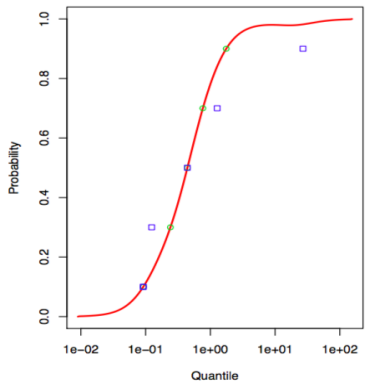

Fig 3:- Probability of EGDF Infinite

Figure 4 shows distribution of the probability density over the infinite data support.

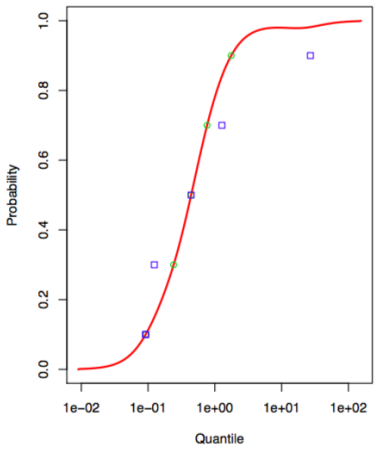

Fig 4:- Density of EGDF Infinite

V. CONCLUSIONS

Relative valuation, the same as more theoretically proved Income valuation approach needs for any enterprise specific access. General usage of multiples without discounts and bonuses can lead to degradation of the valuation practice and. Research findings made proof that using Gnostics variables brings more precise results in a relative valuation approach using multiples. The results implied the further research to apply the Gnostic multiples on non-listed enterprises, using alternative multiples derived from DCF methods and establish a comparative basis for relative valuation of Czech and Slovak Enterprises with CEE countries. The most fitting Gnostic characteristic was estimated Global Distribution Function EGDF.

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