Some Facts About Prime Numbers

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Abstract:- Here, via this study of prime numbers, properties of them will be explored. It will be shown that the higher order prime numbers also follow some pattern and so as the gap between them. New numbers can be created by simply adding the prime numbers which can be proven by the concept of prime killers. And it can be shown that the gap between primes is just square of natural logarithm of some real number.

Keywords:- *Prime numbers, gap between prime numbers, goldbach conjecture.*

I. INTRODUCTION

The numbers which are not divisible by any other numbers except one and itself called prime numbers. In number series prime numbers are distributed very mysteriously. From the centuries all the scientists tried to find the patterns of the prime numbers and properties of higher order prime numbers. Prime numbers are very useful in many aspects of mathematics and physics for their unique characteristics.

By following them carefully patterns can be explored. First of all most of the prime numbers end with 1,3,7,9, as they are distributed in table1.

0th	0	1	2	3	4	5	6	7	8	9
1st	10	11	12	13	14	15	16	17	18	19
2nd	20	21	22	23	24	25	26	27	28	29
3rd	30	31	32	33	34	35	36	37	38	39
4th	40	41	42	43	44	45	46	47	48	49
5th	50	51	52	53	45	55	56	57	58	59

Table1 A Distribution of prime numbers.
Bold numbers are primes.

II. DISCUSSION

In table1 all the prime numbers are categorized by putting them in some columns. Now the numbers begin with one are in 1st column, the numbers begin with two are in 2nd column and so on. The nth column, if n is equals to (3k+1) [where k=0,1,2,3,…..], contains not more than four prime numbers ends with 1,3,7,9 and if n equals to (3k+2) [where k= k=0,1,2,3,…..] then it contains not more than two prime numbers end with 3, 9 and if n equals to (3k+3) [where k= k=0,1,2,3,…..] then it also contains not more than two prime numbers end with 1,7. Together these three rules are (3k+m) rule; k can be 0 or any positive number and m is 1, 2 or 3.

- *Grids*

The above properties of the three columns are repeated up to the infinity, calling them together a grid. As in table1 1st, 2nd and 3rd column together are a grid and so as 4th, 5th and 6th. In a grid there is not more than eight primes out of thirty numbers. As the number of grid increases, the number of prime's decreases for the prime killers discussed later. So out of x numbers there are not more than (8x/30) [where x is any positive integers] numbers are primes.

- *Prime killers*

As the number of grids increases the numbers which can be prime according to (3k+m) rule are get replaced by conjugates because the primes, generated previously in the grids, multiply with all primes and kill the potential primes. Like 49, 77,121 and all the other numbers are can be prime by the (3k+m) rule but they are not as all these number equals to prime multiplied by prime and these types of numbers are prime killers.

Now if $P_1, P_2, P_3, \ldots\ldots P_n$ be the all sets of prime numbers up to nth prime number then

$$\begin{pmatrix} P_4^{\,2} & P_4 P_5 & P_4P_6 \ldots\ldots\ldots\ldots\ldots\ldots P_4P_n \\ 0 & P_5^{\,2} & P_5P_6 \ldots\ldots\ldots\ldots\ldots\ldots P_5P_n \\ 0 & 0 & P_6^{\,2} \ldots\ldots\ldots\ldots\ldots\ldots P_6P_n \\ 0 & 0 & 0 \ldots\ldots\ldots\ldots\ldots\ldots P_n^{\,2} \end{pmatrix}$$

The above whole set represents the prime killers.

- *The gap between prime numbers,*

The gap between prime numbers is totally determined by the multiples of 2, 3, 5 and prime killers.

Let the gap between two primes P_a and P_b[where a, b are huge numbers] is x.

Now we can see that in a grid there exist 15 multiples of 2 out of 30, 3 multiples of 5 out of 30 and 4 multiples of 3 out of 30. So out of x there are (x/2), (3x/30) and (4x/30) multiples of 2, 5 and 3.

Let the prime killers in the gap x is determined by up to q numbers of prime numbers, then from the prime killer set it can be said that there will be maximum $(q^2/2)$ numbers of prime killers in the gap.

And now the gap,

$x = (x/2)+(3x/30)+(4x/30)+(q^2/2)$
or, $x= (15/8)q^2$
or, $x \approx 2q^2$ [where q is any positive integer]

So it can be said that, $(P_b – P_a) \approx 2(\text{any positive integer})^2$ (1)

- *The extension of the prime gap theory*

From prime number theorem we know that $P_n \approx n \ln n$ [where n is large number and ln is natural logarithm] . Let the

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number of primes [i.e. q] which contribute to the prime killers in the gap x is for the prime numbers P_l to P_{l+1}(where l is high).

$$\begin{aligned} \text{So, } q &= P_{l+1} - P_l \\ &\approx (l+1) \ln (l+1) - l \ln l \\ &\approx \ln l \text{ (as l is high)} \end{aligned}$$

Now (1) can be written as, $(P_b – P_a) \approx 2(\ln l)^2$ (2)

III. CONCLUSION

Above discussion about prime number is mainly for the gap between any two primes and the characteristics about them. It may give the reason for the goldbach conjecture and alignment of higher order prime numbers. Like from (1) it can be said that if we add two primes then it always give a conjugate.

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