Learning and Teaching of Mental Computation: Using a Schematization of Mental Multiplication and Analysis of Learners' Productions using IRM

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Abstract: -Mental computation is the most common form of computation used in everyday life. It is used for quick calculations and estimations. Mental computation refers to the process of working out and obtaining exact or approximate answers mentally. The mental computation, with its significant educational challenges including the formation of future spontaneous citizen and able to participate in a debate of great scope, is a set of operations done mentally without intervention of a calculator, with a possibility to resort to the written supports. Several methods of mental multiplication have been elaborated. When calculating mentally, students select from a range of strategies, depending on the numbers used. In this article, we propose a new procedure of mental computation of two whole factors multiplication based on a schematic representation and a wise choice of simple or double reference.

***Keywords**:- Mental computation, schematic representation, mental multiplication, IRM.*

I. INTRODUCTION

Mental computation is a process or activity that allows a series of operations to be performed mentally. In truth, all calculation is "mental": purely mental computation, mental computation with persistent or temporary written traces, and written computation with mental stages. There are also instrumented computations with charts, abacuses or other instruments; Moreover, even in a calculator with calculator, there is a mental component if only to control orders of magnitude and detect typing errors. In addition to its classic pedagogical function of developing the ability to conduct rapid reasoning on the winning strategy choice of automated or thought-out computing dimensions [1], mental computation has, among other things, an indispensable social function in life. Daily to obtain discreetly and quickly an exact result and an order of magnitude to control itself. But estimation is an important part of the mathematics curriculum. For example, it makes it possible to check the consistency of the results when solving problems with a calculator. In fact, we often have to do quick and slow calculations at times when we have no paper, no pencil, no calculator at hand. Thus, mental computations then having great practical utility contributes to the preparation of learners to active life. The literature in mathematics didactics also shows that "students with difficulties in mathematics are usually in mental computation". Many mathematic educators and researchers [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22] emphasized the importance of mental computation.

Moreover, necessarily involving the knowledge of numbers and ma-thematic operations, mental computation not only calls for memory, but develops it. Then, requiring constant attention and cannot be done in a mechanical way, as is often the case in the written calculation, the mental computation is also an important way to develop the sense of the number and to acquire a better understanding of position value and mathematical operations. The student who is skilled in mental arithmetic will be more adept at grasping the links between digital data and transforming them.

Finally, mental computation helps to structure the brain, to shape the way it thinks, to boost memory, the spirit of analysis and synthesis. It thus contributes to the formation of mathematical talent, that is logical thought, the capacity for rapid generalization, the reversibility of mathematical reasoning, the flexibility of thought, as evidenced by the existence of calculators prodigies including the famous German boy Rüdiger Gamm [23]. In front of a multiplication or a division which can seem complex, it will for example be to ask in what measure it is possible to simplify the calculation and / or to break it down to facilitate the task. It is also recognized that head counting shortcomings negatively impact the future of learners. Beyond learning the tables of subtraction, addition and multiplication, the calculation of head has more of an interest, namely: to set up automatisms, to refine its reasoning, to get used to manipulate the numbers, better known the properties of numbers. Studies have shown that the more one will train one's brain to remember, the more automatisms will appear and one's capacity for mental arithmetic will be increased tenfold. Some scientists have been able to determine that mental computation activates brain areas related to spatial attention [24]. The representation of the numbers would thus be comparable to the spatial representation. Good for the brain, it would fight against aging and delay diseases such as Alzheimer. It also allows you to run your neurons at any time while having fun and to do mathematics without realizing it. Consequently, thus contributing inevitably to the development of human intelligence if only through one of its nine constituent capacities namely “the faculty of learning quickly” [25], the mental computation has quite good.

All this explains the stakes of mental mathematics training. To progress, you have to train, put in place various strategies for things to become natural. For several decades, researchers in the field of mathematics education in developed countries have emphasized the importance of mental computation strategies. (e.g. [26, 27, 28, 29]). Mental computation strategies have also been given much attention by researchers in cognitive psychology (e.g. [30, 31]) and in cognitive neuroscience (e.g. [32, 33, 34]). Two aspects of mental computations are divided into two parts: automated calculation and thoughtful calculation, which will be described later. The terms, from one era to another, have varied somewhat. In first approximation, one may be tempted to oppose mental calculus to written or instrumented calculus. But practicing a mental calculation does not mean that everything happens without writing. The expression “mental computation” does not imply that no written support can intervene in the instruction, in the formulation of the result, or even intermediate results [35]. What is designated by the term of written computation or of operation posed requires the knowledge of the tables and the management of the reservoirs, thus of the mental computation already but still too light. The automated calculation aims at the automatic mobilization of results and procedures (called numerical facts) that are supposed to be difficult to memorize, such as addition tables, multiplication tables, a few doubles, squares, multiplying an integer by 10 or 100. In this case, the requirement of speed will be a criterion of success. Before being automated, the results are constructed by reasoning, and therefore “reflected”. The daily and progressive training will lead the student to memorize little by little these numerical facts without recourse to the thoughtful calculation [36]. Whereas the thoughtful (or reasoned) calculation consists of the pupil implementing procedures that are the result of a reasoned treatment linked to the numbers at stake. Indeed, the thoughtful calculation calls upon the elaboration and the use of intermediate procedures to obtain the result and may involve writing, as students may need to keep a written record of the computation steps. Students must adapt their reasoning to the context and develop the characterization of numbers. Speed, without being completely discarded, can be considered as a criterion of success. As for the mental multiplication of two factors, there are several procedures. As an indication, let’s take the two examples of deployable procedures to calculate 35×14:

- Procedure 1 based on the additive decomposition of one of the factors and distributivity of the multiplication with respect to the addition:

  $(35 \times 10) + (35 \times 4) = 350 + 140 = 490.$
- Procedure 2 consisting of multiplicative decomposition or factorization and associativity:
  $35 \times 14 = 35 \times (2 \times 7) = (35 \times 2) \times 7 = 70 \times 7 = 490.$

On the other hand, in the case where the two factors are prime integers, for example 37×13, applying ladite procedure 1 (additive decomposition of one of the factors and distributivity of the multiplication with respect to the addition), we have: $(37 \times 10) + (37 \times 2) + (37 \times 1) = 370+74+37 = 300+70+70+30+4+7 = 481$, whereas, ladite procedure 2 (factorization and associativity of multiplication) proves to be impossible. Therefore, other methods must be used to address a situation analogous to the latter without being exclusive of this case.

That is why, convinced of the ease of adding in relation to mentally multiplying, we will use a procedure based on schematization and the choice of an auxiliary reference factor while facilitating the product calculation; thus, the choice of reference must be done among the multiples of 10. Indeed, to carry out the computation of x.10n, it is enough to add n zeros to the right of x. Our study relates to the learning of mental computation, including the mental multiplication of two factors, according to the schematic technique using circles and ellipses connected by arrows: it is a schema-based strategy. Moreover, this choice of schema learning is justified by the fact that a figure is psychologically more quickly and durably perceptible by the visual effect than a sequence of sentences in well-arranged words. This is a new, thoughtful calculation procedure whose intermediate procedures are represented using diagrams and the decomposition of the initial product into several simple additions. Schematization can be an important methodological aid to help students overcome their difficulties in mentally retaining intermediate procedures [37]. But schematization is not an obvious and natural activity for them. Then came the question: what are the aids to the schematization of the mental multiplication algorithm with reference to two whole factors proposed to students?

II. SCHEMATIZATION OF THE MENTAL MULTIPLICATION

In order to trigger procedural and associative memory, both generators of long-term memory, our approach will be to use a schema of a logical fractionation of the approach leading to the product value of the two. Whole factors, with effective implication of the learner by realizing the principle of “do to understand” [38]. The oral usually plays a determining role in the automation of the procedure, it is necessary to teach a technique of mental multiplication by asking any learner to verbalize said procedure. Our procedure of mental multiplication of two factors A and B according to the schematic technique based on a reference uses circles and ellipses connected by arrows. Three choices are possible depending on the distance between these two factors:

- If $A < B$ and A is close to B, the procedure based on a reference should be chosen.
- If $A < B$ and A and B are very distant, the double-referenced procedure should be chosen.

- If A=B, the mental multiplication of two factors A and B is a square A^2 and the procedure based on a reference with A = B should be chosen.

A. Mental multiplication by simple reference

The mental multiplication of two factors according to the schematic technique based on a reference is thus obtained from the algorithm deduced from the following algebraic identities that we can, or that we should rather ask the students to develop $(A - R)(B - R)$ and deduce AB :

For all $R \in \mathbb{R}$,

$AB = R[A + (B - R)] + (A - R)(B - R);$ (1)

that is: $AB = R[B + (A - R)] + (A - R)(B - R).$ (2)

By putting $a = A - R$ et $b = B - R$, we have got:

$AB = R(A + b) + ab$ or $AB = R(B + a) + ab.$ (3)

The idea is to reduce oneself to operations manipulating smaller numbers, mathematics constituting the science that seeks simple things as illustrated by the use of so-called reference functions of the second and the terminal for the study of a real function (the said associated function), the integration by parts, the decomposition into simple elements of a rational fraction, the integration by change of variable in an integral computation and simplification of a fraction etc...: thus, the choice of the reference R must be guided by this wish to have $|A-R| < A$ and $|B-R| < B$ in order to find themselves in the situation of the usual multiplication table and demoralized, their product being easy to memorize in an intermediate phase of said procedure. Thus, the student will be able to perceive the utility, even the necessity, of choosing the reference R not far from the two factors, or from one of the factors. This algorithm can therefore be schematized in the way shown in Figure 1 and Figure 2. Inspired by the importance of a pictorial representation that must precede any action of abstraction and also by the utility or the effectiveness of such a drawing or graph to solve some difficult discrete or combinatorial problems, which gave birth to the famous theory of graphs, this strategy based on a schematization of the procedure is thus well justified on the psychopedagogical level.

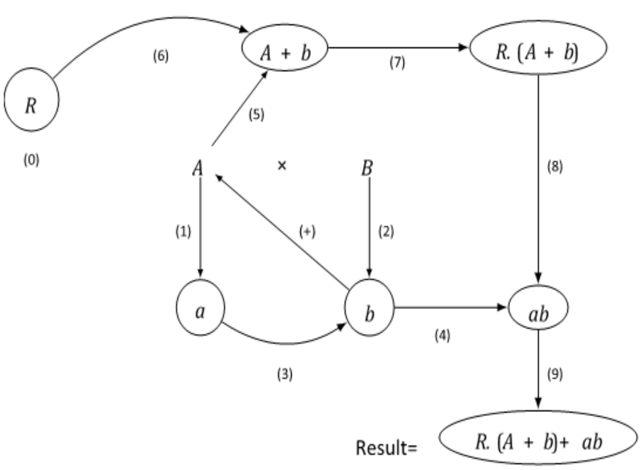

Fig 1:- Mental Multiplication by a simple reference

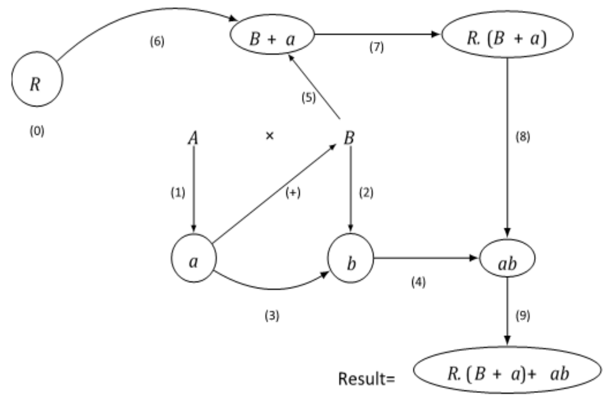

Fig 2:- Another mental multiplication procedure with a simple reference

The first step (step 0) is to judiciously choose the reference R among the multiples of the ten powers closest to one of two factors A and B. The following steps are indicated by the numbers in parentheses near the arrows of this diagram. These are easy steps to calculate and remember: for example, step 4 uses the multiplication table memory and gives a small number that is easy to memorize. The Figure 3 shows the 37 × 29 calculation example.

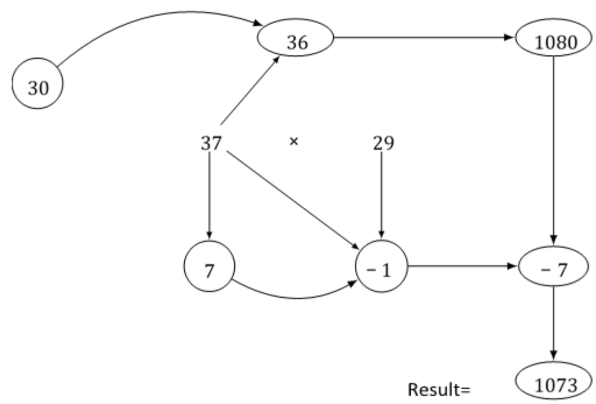

Fig 3:- Multiplication 37 × 29 by a simple reference

Note that the memorization-knowledge of the multiplication table from 1 to 5 suffices, because that of 6 to 9, in case of forgetfulness, can easily be found by this schematization taking 10 as a reference:

$$6 \le R \le 9 \Leftrightarrow -4 \le R - 10 \le -1 \Leftrightarrow 1 \le |R - 10| \le 4.$$

On the other hand, in the case of $R > \sup(A, B)$, in order to avoid negative numbers, as it is easy to verify that, for all

$R \in \mathbb{R}$, $AB = R[A - (R - B)] + (R - A)(R - B),$ (4)

We can also propose, starting from the identity, by posing a'= R-A and b' = R-B: AB = R (A-b) + a'b', which we can schematize as in Figure 4. At the primary level, for example, this is mandatory, but not in the secondary.

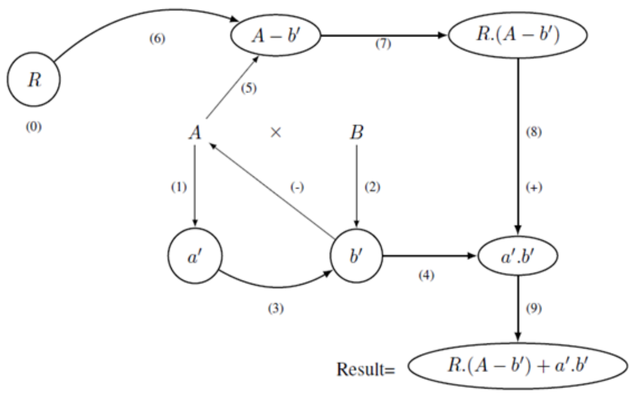

Fig 4:- Other mental multiplication by a simple reference in the case where R > sup (A, B)

➢ *Note*

Note that if the two factors A and B are very distant from each other, that is to say if A << B, then this scheme is not more convenient. With a product of two factors very distant from each other, we will necessarily have a remnant of great value, so difficult to remember. So, one of the remains will necessarily be a big figure that will not be simple to multiply by a factor, and moreover the product of this fat remains by a factor will give a large number necessarily difficult to memorize. For example, to calculate 17×97, one would be embarrassed for the optimal choice of the reference. This choice must be made among 10 or 20 with respect to 17 and among 90 or 100 compared to 97. Let us see this example, represented by Figure 5, by choosing 100 as reference. To avoid these difficulties caused by the large difference between the two factors and between one of the factors and the reference among the four optimal ones (10; 20; 90; 100), we propose the schematic technique of the mental multiplication by double reference described below.

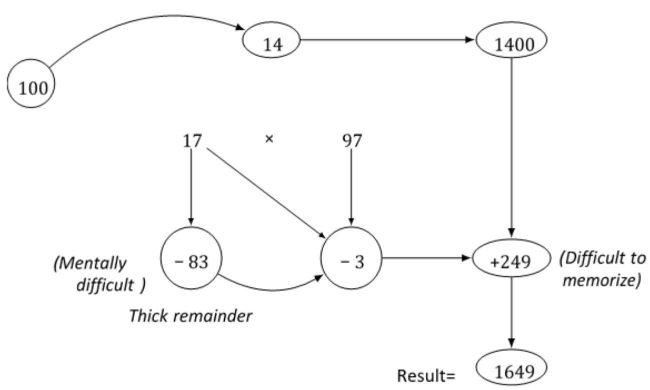

Fig 5:- Mental multiplication 17 × 97 by a reference

B. Mental multiplication by double reference

Mental multiplication by double reference is obtained by exploiting the following algebraic equality that can be entrusted to students in order to involve them in their own learning, through the development of (A − R)(B − λR) and deduce AB. So for all R ∈ ℝ, and for all λ ∈ ℝ, if A << B,

$$AB = R[\lambda A + (B - \lambda R)] + (A - R)(B - \lambda R) \quad (5)$$

$$\text{or } AB = R[\lambda(A - R) + B] + (A - R)(B - \lambda R). \quad (6)$$

We will choose R multiple of 10 and λ such that R is close to A and λR close to B. By putting a = A − R and b = B − λR, we have got: if A << B,

$$AB = R(\lambda A + b) + a.b \text{ or } AB = R(\lambda a + B) + a.b. \quad (7)$$

These algorithms can be schematized in the manner represented by Figure 6 and Figure 7. Take for example the product 17×97. Take R = 10 and λ = 10 and we have the following diagram represented by Figure 8.

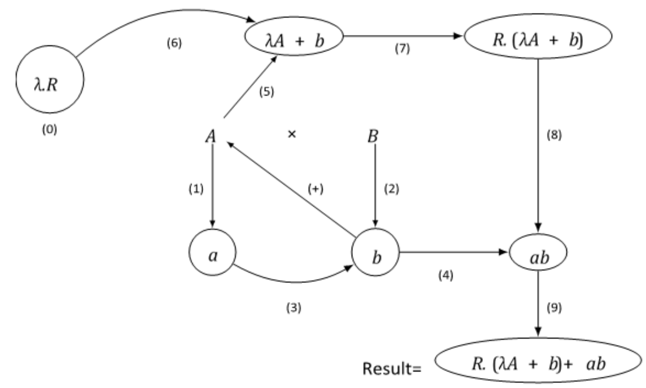

Fig 6:- Algorithm of mental multiplication by double reference

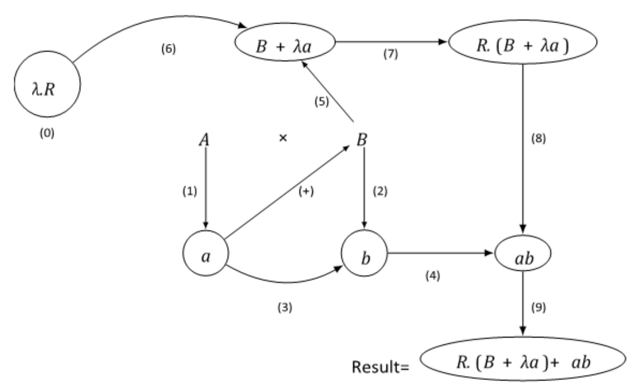

Fig 7:- Another dual-reference mental multiplication algorithm

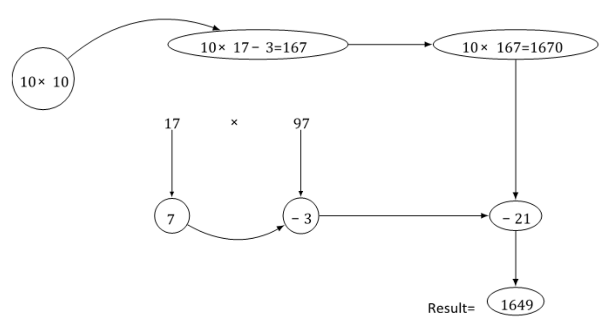

Fig 8:- Multiplication algorithm of 17 × 97 by double reference

C. The special case of the mental square

The procedure of calculating a square A^2 can also be obtained from the procedure of mental multiplication of two factors according to the schematic technique based on a reference with A = B. Thus, we have: for all $R \in \mathbb{R}$,

$$A^2 = R[A + (A - R)] + (A - R)^2 . \quad (8)$$

By putting a = A − R, we have got:

$$A^2 = R(A + a) + a^2 \quad (9)$$

This algorithm can be schematized in the same way as in Figure 1 by replacing B with A. If we replace R by A − a in equation (1), we obtain the method already proposed by Benjamin and Shermer [39],

$$A^2 = (A - a)(A + a) + a^2. \quad (10)$$

The addition and multiplication tables are indispensable. Without them, mental calculation cannot exist; in any case, it is quite certain that, deprived of their assistance, the child will never calculate quickly. Finally, care must be taken to memorize half of the numbers from 1 to 100 and double the numbers from 1 to 50.

III. METHODS AND MATERIALS

A. Materials

Our test consists of 10 items (denoted respectively Item_13 × 17, Item_19 × 17, Item_23 × 29, Item_97 × 127, Item_47 × 53, Item_97 × 98, Item_97 × 107, Item_13 × 79, Item_989 × 997, Item_103 × 797) (see Appendix) of the mental multiplication of two factors according to the schematic technique based on one or two references. This test was offered to 218 people, including 94 students in the 1st S class, 82 students in the final D class and 42 students of Technical and Commercial Banking (Bac +2 years) in the IST (Institut Suprieur de la Technologie) and we use the model. Two-parameter IRM (Item Response Model) using the eirt program to plot ICC (Item Characteristic Curve) and TCC (Characteristic Curve of the Test) and make estimates of IRM item parameters. The eirt program is a module for Excel. In particular, this eirt program is restricted to the case where the latent variable (the skill) is one-dimensional and of normal distribution, and the number of possible answers to the items (the options) is finite. Note that this evaluation test was preceded by three training sessions with oral exercise.

B. Methods Of Research

Our experimentation consists of four steps: the teaching of the procedures described in the previous section, training exercises with verbalization, an evaluation test and the analysis of evaluation results through the learners' productions. Now, in the training-learning of the procedure, it is permissible to:

- operate by making the learners verbalize individually in turn against their peers;
- do repetitive exercises in writing and oral under stopwatch;
- evaluate with individualized stopwatch, that is to say give instructions, to each learner, to mention its start time and its end time.

C. Overview of an Item Response Model (IRM)

Born in the 1960s, the Item Response Model [40, 41, 42] deals with the processes that underlie an individual's response to a question or item. The IRM is based on two basic assumptions:

- The performance of a respondent to an item can be predicted or explained by a set of factors called traits, latent traits or abilities;
- The relation between the item score and the trait score measured by this item can be described by an increasing monotonic function, commonly referred to as the Item Characteristic Curve (ICC).

In fact, the ICC makes it possible to estimate the psychometric qualities of the items. IRM are a class of probabilistic models. They model the probability that an individual gives a certain answer to an item, based on student and item parameters. In a very general way, the IRM can be presented in the following way:

$$P(Y = k|\theta,\xi) = F(\theta,\xi,k).$$

The probability that an individual gives the answer k to the item Y depends on characteristics θ concerning the individual and characteristics ξ concerning the item Y. There are several models of IRM whose mathematical forms, the characteristic curves of the item and the number of parameters to estimate may vary from model to model. We present some of these most used models in the sciences of education [43, 44, 45, 46, 47].

The Rasch model (One parameter response model) is one of the simplest models of the IRM since each item is characterized by a unique parameter called the difficulty parameter of the item. This model is written as follows (see equation (11)):

$$P_{ij} = P\left(Y_i^j = 1/\theta_i, b_j\right) = \frac{1}{1 + e^{-\left(\theta_i - b_j\right)}} \quad \text{………(11)}$$

Where P_{ij} is the probability that the individual i will succeed item j and

$$Y_i^j = \begin{cases} 1 \text{ if the individual } i \text{ succes item } j \\ 0 \text{ if the individual } i \text{ fails item } j \end{cases}$$

It is a sigmoid function of the skill level θ_i of the individual i and the difficulty level b_j of the item j.

The second model, proposed by Birnbaum, is the two-parameter model obtained by introducing a second parameter, called discrimination. This model is defined by the expression (see equation (12)

$$P_{ij} = P\left(Y_i^j = 1 / \theta_i, a_j, b_j\right) = \frac{1}{1 + e^{-1,7 a_j(\theta_i - b_j)}} \quad \text{..........(12)}$$

Where the discrimination parameter a_j ($a_j > 0$) represents the slope at the point of inflection of the characteristic curve of item j which varies from one item to another and the constant 1.7 is introduced to approximate the sigmoid function of the distribution function of the reduced normal centered law [42].

The IRM allows to estimate, for a given trait, the probability of choosing a response option to a given item. Thus, the probability of responding to an item option varies depending on the scores obtained on the trait being measured. Figure 9 illustrates the concept of item characteristic curve. The probability of succeeding the item (on the ordinate) depends on the skill level (on the abscissa). The horizontal axis represents all the factors called traits, latent traits or cleverness, while the vertical axis represents the performance of the successes to the item. An item characteristic curve is monotonically increasing when an individual's score on a given item increases according to their total score. Thus, at one item, two individuals with different skill levels will give different answers to this item. The point on the skill scale below the point of inflection (where the curve ceases to become concave and begins to become convex) generally refers to the difficulty of the item. In addition, the slope of the curve at the point of inflection expresses the discriminatory power of an item according to the level of skill. Whatever the model used to define the characteristic curve of an item (with one, two or three parameters), the parameter called difficulty is always present. Under the IRM, it is conventionally defined as the value of theta which corresponds to a probability of success exactly equal to 0.5.

It is precisely this value of theta that we call the "difficulty parameter bj" of the item. If the curve in Figure 9 moves to the right, the difficulty of the item would be greater. In this example 9, the difficulty parameters of the items represented by the three curves are respectively from left to right: b_1 (value read on the horizontal axis for a probability of success equal to 0.5), b_2 and b_3, with $b_1 < b_2 < b_3$.

The item described by the first curve is rather easy, the one described by the second is of medium difficulty while the last is the most difficult. It should also be noted that the measure of the latent trait in the subjects and the difficulty of the items are expressed on the same scale, ranging from −3 et +3. In order to determine the difficulty of an item j at a given location of the scale θ, it is necessary to proceed to a visual examination of the ICC and to interpret the difficulty of the item according to the value of Pij(θ), the probability of success of the item, at this point on the scale: the value of parameter bj gives us here only a general indication of the difficulty of item j.

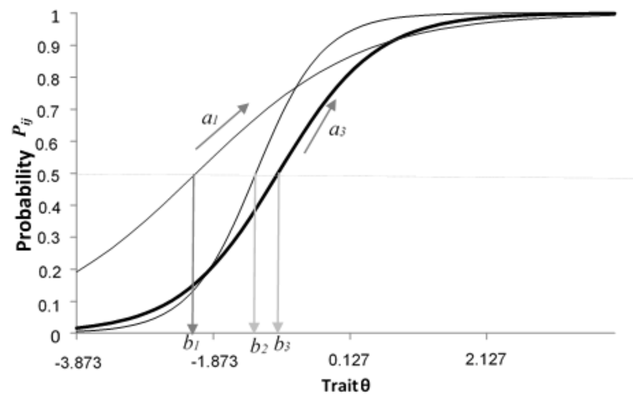

Fig 9:- Example of the three characteristic curves of the three items

A second important feature of the item is its discriminating power, that is, its ability to differentiate individuals according to the degree to which they possess the latent trait. In IRM, the value of "discrimination parameter" aj is in fact proportional to the slope of the geometric tangent passing through the point of inflection of the curve where this slope is maximum. This slope varies theoretically between 0 (when the angle formed with the horizontal axis is equal to 0°) and +∞ (angle equal to 90°). The slope can therefore be more or less inclined: the steeper the slope, the more the item is discriminating and vice versa. In practice, it may happen that this parameter is negative sign for the item that is better done by the less good students than by the best (that is to say, less good and better depending on the position they occupy on the scale of theta scores).

In the example presented in Figure 9, the respective discrimination parameters aj of the characteristic curves of three items are $a_1 = 0.9$ for the curve with the lowest slope, of $a_3 = 1.3$ (intermediate case) and of $a_2 = 2.5$ for the curve with the steepest slope. For a highly discriminating item, the probability of success will be very low at a certain skill level and very high beyond this level. Thus, a small difference in skill level can lead to very different probabilities of success. For its part, a non-discriminating item may lead to small differences in the probability of success for a significant skill level gap. Discrimination of an item is its ability to differentiate subjects who achieve a high level of achievement from the overall test with subjects with a lower level of achievement. We are talking about the discriminating power of an item. A good item here is an item that clearly distinguishes subjects on their level of overall success.

The models that have just been presented thus make it possible to construct the characteristic curves for all the items that appear in a test. From these curves, the "characteristic curve of the test" (CCT) can be defined. It is obtained simply by adding, for each value of theta, the probabilities relating to

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the different items. The y-axis is a scale whose values range from 0 to n, where n is the total number of items. The shape of the curve can be used to determine: (a) what is the difficulty of the test and (b) for which values of theta (i.e on which portion of the skill scale) the instrument discriminates the better the individuals. Finally, the characteristic curve of the test also makes it possible to perform a kind of scale conversion, from which each individual result (initially expressed by a theta score between −3.873 and +4) can be interpreted as a percentage: more exactly, as the expected percentage of successful items in the universe of items from which those who compose the test originate (scale of true scores in the usual sense that this term assumes in theory of measurement). To obtain this percentage, divide by n and multiply by 100 the sum of the probabilities obtained for a given value of θ (calculation of an ordinary percentage).

IV. RESULTS AND DISCUSSION

IRM have many practical advantages in calculating student numeracy skills in an item test. They make it possible to model "the probability that a student gives a certain answer to an item, according to parameters concerning the student and the item". The ten characteristic curves of the items (see Figure 10) are drawn to determine the difficulties of the items and their powers of discrimination with respect to the cleverness of the items. By observing these ten ICCs of the test, the 10 items are arranged in order of difficulty, left on the right, following the horizontal line for a probability of success P_{ij} = 0.5: the Item_97×107 described by the first curve is therefore rather easy; the Item_13×17, Item_19×17, Item_23×29, Item_97× 98 described respectively by the second, third, fourth and fifth are also easy; the Item_47×53, Item_989×997 and Item_97×127 described respectively by the sixth, seventh and eighth are of medium difficulty ; while the Item_103×797 described by the last curve is the most difficult (see Figure 10). This means that the mental multiplications of two factors according to the schematic technique based on a reference are easy for the pupils and the others based on two references have a medium difficulty and therefore deserve that have devote more time of exercise of drive to be automated.

The discrimination parameter can thus be interpreted in terms of the amount of information carried by the item. In general, the slopes of these ten test ICCs are so steep that the items are more dis-criminating. In addition, according to Table 1, which illustrates the estimation of item parameters, discrimination parameters range from 0.686 to 3.047. Vrignaud [48] considered that the discrimination parameter of an item is very satisfactory if it is greater than 0.40; that it is satisfactory between 0.20 and 0.40; that it is weak between 0.10 and 0.20; that it is insufficient below 0.10. Thus, the test of mental multiplications of two factors according to the schematic technique based on one or two references can differentiate the strong pupils in mental calculation compared to the weak pupils. So it's a good test.

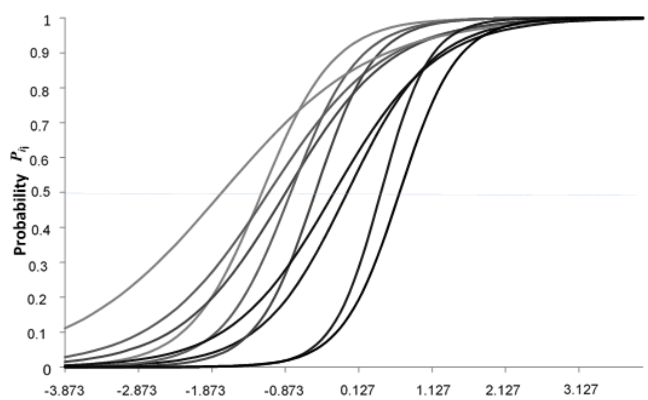

Fig 10:- Characteristic curves of the ten items

According to the TCC (see Figure 11), the test appears to be of medium difficulty since at the value 5 (= n /2 with n = 10) on the vertical axis corresponds a value of theta between −0.873 and 0.127 on the horizontal axis.

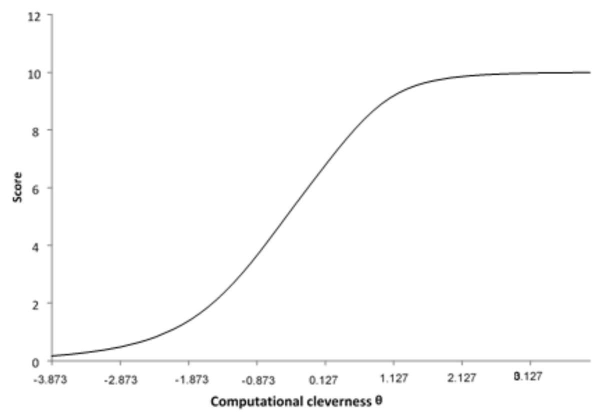

Fig 11:- Characteristic curve of the test

When we re-introduce this test of these ten items using procedures that mobilize the (additive or multiplicative) decomposition of factors, associativity and distributivity, or the mental simulation procedure of the standard written algorithm, we have found that these ten items of mental multiplication prove to be very difficult, despite the fact that these learners sampled have good mathematical skills. For example, using a non-schematic strategy, the item_47×53 is performed as follows:

$$\begin{aligned} 47 \times 53 &= (40+7) \times (50+3) \\ &= (40 \times 50)+(40 \times 3)+(7 \times 50)+(7 \times 3) \\ &= 2000+120+350+21 = 2000 + 470 + 21 \\ &= 2000 + 491 = 2491. \end{aligned}$$

While using the schematization of mental multiplication by a simple reference (schematic strategy), a learner made this item_47×53 as follows (Fig. 12):

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Item	Discrimination parameters (a_j)	Difficulty parameters (b_j)
Item_13×17	2.029	-1.191
Item_19×17	1.276	-1.097
Item_23×29	1.395	-0.867
Item_97×127	1.717	-0.027
Item_47×53	2.505	-0.463
Item_97×98	2.167	-0.798
Item_97×107	0.974	-1.734
Item_13×79	3.047	0.432
Item_989×997	1.437	-0.208
Item_103×797	2.558	0.686

Table 1. Estimates of discrimination parameters (aj) and difficulty parameters (bj)

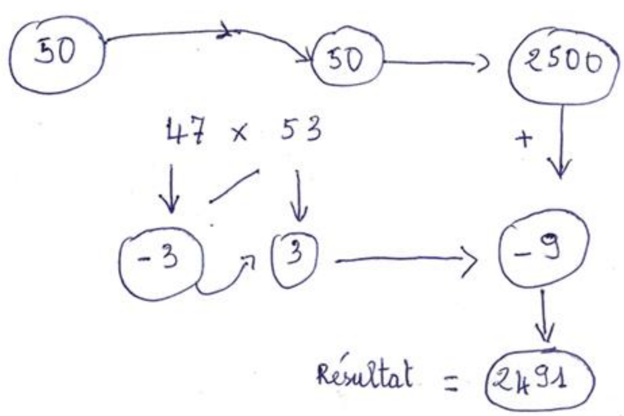

Fig 12:- Extract from the multiplication 47 × 53 by a reference of a learner

We can say that for a mental multiplication of two prime factors, a schematized strategy (schematization of the mental multiplication by a simple reference or by double reference) is easy that strategies not schematized. Therefore, it is advisable to teach and practice this schematic strategy for this kind of multiplication (that is, a mental multiplication of two prime factors), even to calculate a square. Moreover, the teaching of this schematized strategy can accustom learners to learning a strategy based on a schema for solving mathematical problems. The schema-based strategy is important because it allows students with low memory capacity to organize information using semantic relationships and thus acquire the correct word-solving skills [49].

V. CONCLUSION

This study made it possible, thanks to the schematization of the algorithm, to facilitate the techniques of the mental multiplication of two factors based on one or two references and to find a new procedure for mentally calculating the square of a number. Pedagogical experiments following the evaluations in different groups of pupils and students were carried out. The analysis of learners' productions is done according to the IRM model implemented in the eirt program. This has shown us that this schematic procedure is so interesting and easy that one should be sure to practice it from the college. Indeed, this procedure involves negative numbers that are unsuitable for primary school students, unless one restricts oneself to integers between ten and twenty. In future work, we will consider developing procedures adapted to primary school students without involving negative numbers and proposing an opportunity to integrate the processing time in IRM. Finally, since the ability to mental calculation is recognized as non-innate, we believe that the teaching of this technique of mental multiplication based on a diagram and a judicious choice of reference can contribute effectively to the training in mental thinking thought and automated with all its important issues recalled in the introduction of paper.

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APPENDIX

➢ *Test (Table 2)*

Mentally perform the multiplications of two integer factors according to the schematic technique based on a judiciously chosen reference.

Item_13×17	13 × 17
Item_19×17	19 × 17
Item_23×29	23 × 29
Item_97×127	97 × 127
Item_47×53	47 × 53
Item_97×98	97 × 98
Item_97×107	97 × 107
Item_13×79	13 × 79
Item_989×997	989 × 997
Item_103×797	103 × 797

Table 2. 10 items of mental multiplications