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Heaviside Condition Applied to Lossy Transmission Lines Terminated by RLC-Circuits

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Abstract:- Here we consider lossy transmission lines terminated by a circuit consisting of linear and nonlinear RCL-elements. Using the Kirchhoff's laws we derive boundary conditions and formulate the mixed problem for hyperbolic system describing the lossy transmission line. Then we reduce the mixed problem to an initial value problem on the boundary. To obtain a distortionless propagation we change variables and formulate a mixed problem for the hyperbolic system with respect to the new variables. The nonlinear characteristics of the RLC-elements generate nonlinearity in the equations of neutral type on the boundary. Since we are not able to eliminate some transitional currents and voltages we have to consider a system of 6 equations for 6 unknown functions. Under Heaviside conditions we show that natural solutions are distortionless ones. By means of fixed point technique we prove existence-uniqueness of an oscillatory solution.

***Keywords*:-** *fixed point method, Heaviside condition, hyperbolic system, lossy transmission line, oscillatory solution, RLC-circui.*

I. INTRODUCTION

The main purpose of the present paper is to analyse a lossy transmission line terminated by a particular circuit consisting of linear and nonlinear *RCL*-loads shown in Fig. 1. The case of lossless transmission line is considered in the recent paper [1] using the methods from [2]. Various problems concerning transmission lines can be found in [3]-[19]. Here we obtain conditions for existence-uniqueness of a generalized oscillatory solution. In Section 2 we formulate a mixed problem for the lossy transmission line and derive the boundary conditions using the Kirchhoff's law. The nonlinear characteristics of *RLC*- circuit generate nonlinear boundary conditions. The main difficulty here is caused by the fact that some additional currents and voltages cannot be eliminated and we succeed to reduce the problem to 6 equations for 6 unknown functions. In Section 3 we transform the hyperbolic transmission line system in a diagonal form under the Heaviside condition and formulate the initial and boundary conditions with respect to the new variables. We show that oscillatory (not periodic) solutions are specific for such problems. In Section 4 we give an operator presentation of the oscillatory problem. In Section 5 we prove an existence-uniqueness theorem for the oscillatory solution by fixed point method. In Section 6 using numerical example we demonstrate how to apply our method to particular problems.

II. DERIVATION OF THE BOUNDARY CONDITIONS FOR LOSSY TRANSMISSION LINE SYSTEM AND FORMULATION OF THE MIXED PROBLEM

We proceed from the lossy transmission line system of equations

$$\frac{\partial u(x,t)}{\partial x}+L\frac{\partial i(x,t)}{\partial t}+Ri(x,t)=0,$$
$$\frac{\partial i(x,t)}{\partial x}+C\frac{\partial u(x,t)}{\partial t}+Gu(x,t)=0$$

Where L is per unit-length inductance, C – per unit-length capacitance, R – per unit-length resistance and G – per unit-length conductance, Λ is the length of the transmission line, $v=1/\sqrt{LC}$ is the speed of propagation and $T=\Lambda/(1/\sqrt{LC})\ =\Lambda\sqrt{LC}$ is the time delay. This system is of hyperbolic type and we formulate the mixed problem for (1): to find a solution $(u(x,t),i(x,t))$ For $(x,t)\in\Pi=\{(x,t)\in R^2:0\le x\le\Lambda,\ \ t\ge 0\}$, Satisfying the initial conditions

$$u(x,0)=u_0(x),\ i(x,0)=i_0(x)\,\text{for}\ x\in[0,\Lambda].$$

where $u_0(x),\ i_0(x)$ are prescribed functions. To derive the boundary conditions we proceed from Fig. 1. The main difficulty is caused by the circuit configuration shown on Fig. 1. Using the Kirchhoff's laws we find relations between currents and voltages. The problem is to choose the unknown functions and the number of equations in order to obtain compatible system. We assume that R_1 and L_{11} are linear loads, that is, $u_{R_1}(t)=R_1 i_{R_1}(t)$,

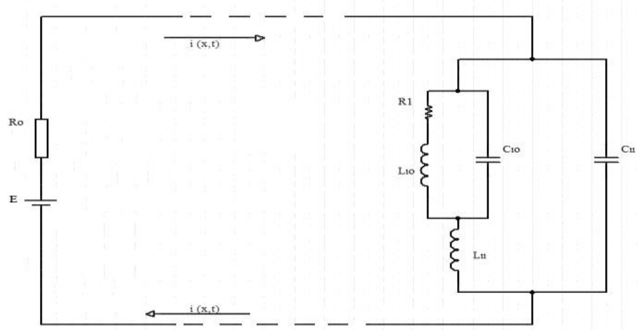

Fig 1:- Lossy transmission line under Heaviside condition terminated by RLC- circuit

$$u_{L_{11}}(t)=\frac{dL_{11}(i_{L_{11}})}{dt}=\frac{dL_{11}(i_{L_{01}})}{di_{L_{11}}}\frac{di_{L_{11}}(t)}{dt}=L_{11}\frac{di_{L_{11}}(t)}{dt},$$

while $C_{10}=C_{10}(u_{C_{10}})$, $C_{11}(u_{C_{11}})$ and $L_{10}=L_{10}(i_{L_{10}})$

are nonlinear ones which means

$$i_{C_{10}}(t)=\frac{dC_{10}(u_{C_{10}})}{du_{C_{10}}}\frac{du_{C_{10}}(t)}{dt},$$

$$i_{C_{11}}(t)=\frac{dC_{11}(u_{C_{11}})}{dt}=\frac{dC_{11}(u_{C_{11}})}{du_{C_{11}}}\frac{du_{C_{11}}(t)}{dt},$$

$$u_{L_{10}}(t)=\frac{dL_{10}(i_{L_{00}})}{di_{L_{10}}}\frac{di_{L_{10}}(t)}{dt}.$$

We notice that $u_{C_{11}}(t)=u(\Lambda,t),\ i_{R_1}(t)=i_{L_{10}}(t)$ and $i_{C_{10}}(t)=i_{L_{11}}(t)$. Proceeding as in [1], [2] the Kirchhoff's Current Law gives:

$$i_{L_{10}}+i_{C_{10}}+i_{C_{11}}=i(\Lambda,t)\ \Leftrightarrow\ i_{L_{10}}+i_{C_{10}}+C_{11}\frac{du_{C_{11}}(t)}{dt}=i(\Lambda,t)$$

$$\Leftrightarrow\ i_{L_{10}}+i_{C_{10}}+C_{11}\frac{du(\Lambda,t)}{dt}=i(\Lambda,t)\ .$$

Similarly, the Kirchhoff's Voltage Law gives

$$u_{R_1}+u_{L_{10}}+u_{L_{11}}=u(\Lambda,t),$$

$$u_{R_1}+u_{L_{10}}=u_{C_{10}}\ \Leftrightarrow\ R_1 i_{L_{10}}+\frac{dL_{10}}{di_{L_{10}}}\frac{di_{L_{10}}}{dt}=u_{C_{10}},$$

$$u_{L_{11}}+u_{C_{10}}=u(\Lambda,t)\ \Leftrightarrow\ L_{11}\frac{di_{L_{11}}}{dt}+u_{C_{10}}=u(\Lambda,t)\ .$$

Finding appropriate relationships between the unknown functions we obtain the system

$$C_{11}\frac{du(\Lambda,t)}{dt}=i(\Lambda,t)-i_{L_{10}}(t)-i_{C_{10}}(t),$$

$$\frac{dL_{10}(i_{L_{10}})}{di_{L_{10}}}\frac{di_{L_{10}}(t)}{dt}=C_{10}^{-1}(i_{C_{10}}(t))-R_1 i_{L_{10}}(t),$$

$$L_{11}\frac{di_{L_{11}}(t)}{dt}=u(\Lambda,t)-C_{10}^{-1}(i_{C_{10}}(t)),$$

$$u_{R_1}+u_{L_{10}}+u_{L_{11}}=u(\Lambda,t).$$

Having considered $i_{C_{10}}(t)=i_{L_{11}}(t)$ we obtain the right-hand side boundary conditions:

$$C_{11}\frac{du(\Lambda,t)}{dt}=i(\Lambda,t)-i_{L_{10}}(t)-i_{C_{10}}(t),$$

$$\frac{dL_{10}(i_{L_{10}})}{di_{L_{10}}}\frac{di_{L_{10}}(t)}{dt}=C_{10}^{-1}(i_{C_{10}}(t))-R_1 i_{L_{10}}(t),$$

(3)

$$L_{11}\frac{di_{C_{10}}(t)}{dt}=u(\Lambda,t)-C_{10}^{-1}(i_{C_{10}}(t))\ .$$

Similar reasoning leads to boundary conditions for the left-hand side of the line:

$$E(t)-u(0,t)-R_0 i(0,t)=0,\ t\ge 0\,.$$

Therefore the boundary conditions are (3) and (4).

Now we are able to formulate the following mixed problem for (1): to find a solution $u(x,t),i(x,t)$ of (1) in Π, satisfying the initial conditions (2) and boundary conditions (3), (4).

III. TRANSFORMATION OF THE HYPERBOLIC SYSTEM IN A DIAGONAL FORM

Let us write down (1) in a matrix form:

$$\frac{\partial U(x,t)}{\partial t}+A\frac{\partial U(x,t)}{\partial x}+A_1 U=\bar{0}$$

where $U=\begin{bmatrix}u(x,t)\\ i(x,t)\end{bmatrix}$, $\frac{\partial U}{\partial t}=\begin{bmatrix}\frac{\partial u(x,t)}{\partial t}\\ \frac{\partial i(x,t)}{\partial t}\end{bmatrix}$, $\frac{\partial U}{\partial x}=\begin{bmatrix}\frac{\partial u(x,t)}{\partial x}\\ \frac{\partial i(x,t)}{\partial x}\end{bmatrix}$,

$$A=\begin{bmatrix}0 & 1/C\\ 1/L & 0\end{bmatrix},\ A_1=\begin{bmatrix}G/C & 0\\ 0 & R/L\end{bmatrix}.$$

To transform A into a diagonal form we form a matrix by its eigen-vectors

$H=\begin{bmatrix}\sqrt{C} & \sqrt{L}\\ -\sqrt{C} & \sqrt{L}\end{bmatrix}$ (cf. [2]). Its inverse one is

$H^{-1}=\begin{bmatrix}1/2\sqrt{C} & -1/2\sqrt{C}\\ 1/2\sqrt{L} & 1/2\sqrt{L}\end{bmatrix}$ and

$$HAH^{-1}=A^{can}=\begin{bmatrix}1/\sqrt{LC} & 0\\ 0 & -1/\sqrt{LC}\end{bmatrix}.$$

New variables $Z=\begin{bmatrix}V(x,t)\\ I(x,t)\end{bmatrix}$ are introduced by the

formulas $Z=HU,\ U=H^{-1}Z$ or

$$V(x,t)=\sqrt{C}\ u(x,t)+\sqrt{L}\ i(x,t)$$
$$I(x,t)=-\sqrt{C}\,u(x,t)+\sqrt{L}\ i(x,t)$$

and

$$u(x,t)=\left(V(x,t)-I(x,t)\right)/2\sqrt{C}$$
$$i(x,t)=\left(V(x,t)+I(x,t)\right)/2\sqrt{L}\,.$$

Substituting $U=H^{-1}Z$ in $\frac{\partial U}{\partial t}+A\frac{\partial U}{\partial x}+A_1U=\bar{0}$

we obtain $\frac{\partial\left(H^{-1}Z\right)}{\partial t}+A\frac{\partial\left(H^{-1}Z\right)}{\partial x}+A_1\left(H^{-1}Z\right)=\bar{0}\,.$

Then we multiply the matrix equation $H^{-1}\frac{\partial Z}{\partial t}+AH^{-1}\frac{\partial Z}{\partial x}+A_1H^{-1}Z=\bar{0}$ by H from the left:

$$\frac{\partial Z}{\partial t}+A^{can}\frac{\partial Z}{\partial x}+\left(HA_1H^{-1}\right)Z=\overline{0}.$$

The Heaviside condition $G/C=R/L$ implies

$$HA_1H^{-1}=\frac{1}{2}\begin{bmatrix} G/C+R/L & -G/C+R/L \\ -G/C+R/L & G/C+R/L\end{bmatrix}=\begin{bmatrix} R/L & 0 \\ 0 & R/L\end{bmatrix}$$

.

Then (5) becomes

$$\frac{\partial V(x,t)}{\partial t}+\frac{1}{\sqrt{LC}}\frac{\partial V(x,t)}{\partial x}+\frac{R}{L}V(x,t)=0$$
$$\frac{\partial I(x,t)}{\partial t}-\frac{1}{\sqrt{LC}}\frac{\partial I(x,t)}{\partial x}+\frac{R}{L}I(x,t)=0.$$

A new substitution

$$V(x,t)=e^{-Rt/L}W(x,t),\ I(x,t)=e^{-Rt/L}J(x,t)$$

leads to the system

$$\frac{\partial W(x,t)}{\partial t}+\frac{1}{\sqrt{LC}}\frac{\partial W(x,t)}{\partial x}=0$$
$$\frac{\partial J(x,t)}{\partial t}-\frac{1}{\sqrt{LC}}\frac{\partial J(x,t)}{\partial x}=0.$$

Finally the transformation formulas are

$$W(x,t)=e^{Rt/L}\sqrt{C}\,u(x,t)+e^{Rt/L}\sqrt{L}\ \ i(x,t)$$
$$J(x,t)=-e^{Rt/L}\sqrt{C}\,u(x,t)+e^{Rt/L}\sqrt{L}\ \ i(x,t)$$

and

$$u(x,t)=\left(e^{-Rt/L}W(x,t)-e^{-Rt/L}J(x,t)\right)/(2\sqrt{C})$$
$$i(x,t)=\left(e^{-Rt/L}W(x,t)+e^{-Rt/L}J(x,t)\right)/(2\sqrt{L}).$$

(7)

Now we are able to formulate a mixed problem in the new variables: to find a solution of (6) satisfying initial conditions

$$W(x,0)=\sqrt{C}\ u_0(x)+\sqrt{L}\ \ i_0(x)\equiv W_0(x),\ x\in[0,\Lambda]$$
$$J(x,0)=-\sqrt{C}\,u_0(x)+\sqrt{L}\ \ i_0(x)\equiv J_0(x),\ x\in[0,\Lambda]$$

and boundary conditions obtained after substituting in (3), (4) the voltage and current from (7):

$$E(t)-\left(e^{-Rt/L}W(0,t)+e^{-Rt/L}J(0,t)\right)/(2\sqrt{C})$$
$$-R_0\left(e^{-Rt/L}W(0,t)+e^{-Rt/L}J(0,t)\right)/\left(2\sqrt{L}\right)=0,\ t\ge 0$$

(8)

$$\left(C_{11}/2\sqrt{C}\right)\frac{d}{dt}\left(e^{-Rt/L}W(\Lambda,t)-e^{-Rt/L}J(\Lambda,t)\right)$$
$$=\left(e^{-Rt/L}W(\Lambda,t)+e^{-Rt/L}J(\Lambda,t)\right)/(2\sqrt{L})-i_{L_{10}}(t)-i_{C_{10}}(t).$$

$$\frac{dL_{10}(i_{L_{10}})}{di_{L_{10}}}\frac{di_{L_{10}}(t)}{dt}=C_{10}^{-1}(i_{C_{10}}(t))-R_1 i_{L_{10}}(t),$$

(9)

$$L_{11}\frac{di_{C_{10}}(t)}{dt}=\left(e^{-Rt/L}W(\Lambda,t)-e^{-Rt/L}J(\Lambda,t)\right)/(2\sqrt{C})-C_{10}^{-1}(i_{C_{10}}(t)).$$

Integrating (6) along the characteristics we have $W(0,t-T)=W(\Lambda,t),\ J(0,t)=J(\Lambda,t-T)$.

We assume that the unknown functions are $W(0,t)\equiv W(t),\ J(\Lambda,t)\equiv J(t),\ i_{L_{10}}(t),\ i_{C_{10}}(t)$ and recalling the denotation $Z_0=\sqrt{L/C}$ we have to solve the following system consisting of differential equations with constant delays:

$$W(t)=\left(2E(t)\sqrt{L(0,t)}+\left(Z_0-R_0\right)J(t-T)\right)/(Z_0+R_0),$$
$$\frac{dJ(t)}{dt}=\frac{dW(t-T)}{dt}-\left(\frac{R}{L}+\frac{1}{C_{11}Z_0}\right)W(t-T)$$
$$+\left(\frac{R}{L}-\frac{1}{C_{11}Z_0}\right)J(t)+e^{Rt/L}\frac{2\sqrt{C}}{C_{11}}\left(i_{L_{10}}(t)+i_{C_{10}}(t)\right),$$
$$\frac{di_{L_{10}}(t)}{dt}=\frac{C_{10}^{-1}(i_{C_{10}}(t))-R_1 i_{L_{10}}(t)}{dL_{10}(i_{L_{10}})/di_{L_{10}}}, \qquad (10)$$
$$\frac{di_{C_{10}}(t)}{dt}=e^{-Rt/L}\frac{W(t-T)-J(t)}{2L_{11}\sqrt{C}}-\frac{1}{L_{11}}C_{10}^{-1}(i_{C_{10}}(t)).$$

So we have obtained a particular case of neutral system of differential equations with retarded arguments.

It is proved in [2] that the above initial value problem is equivalent to the mixed problem for the hyperbolic transmission line system (6). The needed initial functions we can obtain by transition of the initial functions along the characteristics of the hyperbolic system from $[0,\Lambda]$ to $[0,T]$. This means that T becomes an initial point of the initial interval $[0,T]$. The exponential multiplayer in (10) shows that we cannot look for periodic solutions. So we have to solve the following problem: to find an oscillatory solution of (10) on $[T,\infty)$, where $W_0(t),\ J_0(t)$ are prescribed initial oscillatory functions defined on $[0,T]$.

We assume that the nonlinear characteristics are of the polynomial type:

$$L_{10}(i)=\sum_{n=1}^{m}l_n i^n,\ \frac{dL_{10}(i)}{di}=\sum_{n=1}^{m}nl_n i^{n-1},\ \frac{d^2L_{10}(i)}{di^2}=\sum_{n=2}^{m}(n+1)nl_n i^{n-2}.$$

To divide the expression $dL_{10}(i_{L_{10}})/di_{L_{10}}$ we assume

Assumptions (L): There is a constant i_0 such that

$$\left|i_{L_{10}}\right|\le i_0\Rightarrow dL_{10}(i)/di_{L_{10}}=\sum_{n=1}^{m}nl_n\left(i_{L_{10}}\right)^{n-1}\Rightarrow \hat{L}_{10}>0.$$

The nonlinear capacitance is $C_{10}(u)=c_0/\sqrt[h]{1-(u/\Phi)}$ where $c_0>0,\Phi>0,\ h\in[2,3]$ are prescribed constants. (cf. [10]).

Assumptions (C): $|u| \le \varphi_0 < \Phi$.

Since $dC_{10}(u)/du = c_0 / h\Phi\left(\sqrt[h]{1-(u/\Phi)}\right)^{1+h} > 0$,

$$C_{10}(u):[-\varphi_0,\varphi_0] \to \left[c_0/\sqrt[h]{1+(\varphi_0/\Phi)}, c_0/\sqrt[h]{1-(\varphi_0/\Phi)} \right]$$

then there exists

$$C_{10}^{-1}(u):\left[\frac{c_0}{\sqrt[h]{1+(\phi_0/\Phi)}}, \frac{c_0}{\sqrt[h]{1-(\phi_0/\Phi)}} \right] \to [-\phi_0,\phi_0],$$

$$\left|C_{10}^{-1}(.)\right| \le \phi_0 .$$

The minimal value of $\frac{dC_{10}(u)}{du}$ is

$$\min\left\{ \frac{dC_{10}(u)}{du} : |u| \le \phi_0 \right\} = \frac{dC_{10}(-\phi_0)}{du} = \frac{c_0}{h\Phi\left(\sqrt[h]{1+(u/\Phi)}\right)^{1+h}} = \hat{C}_{10} > 0$$

because

$$\frac{d^2C_{10}(u)}{du^2} = -\frac{1+h}{h}\frac{c_0}{h\Phi}\left(1-(u/\Phi)\right)^{-((1+h)/h)-1}(-1/\Phi)$$

$$= \frac{1+h}{h^2}\frac{c_0}{\Phi^2}\left(1-(u/\Phi)\right)^{-(1+2h)/h} > 0 ,$$

$$\left|\frac{dC_{10}(u)}{du}\right| \le \frac{c_0}{h\Phi}\left(1-\frac{\varphi_0}{\Phi}\right)^{-(1+h)/h} \equiv M ,$$

$$\left|\frac{d^2C_{10}(u)}{du^2}\right| \le \frac{1+h}{h^2}\frac{c_0}{\Phi^2}\left(1-\frac{\varphi_0}{\Phi}\right)^{-\frac{1+2h}{h}} = H,$$

$$\left|\frac{\partial C_{10}^{-1}(i_{C10})}{\partial i_{C10}}\right| \frac{1}{\hat{L}_{10}} \le \frac{h\Phi}{c_0\hat{L}_{10}}\left(1+\frac{\varphi_0}{\Phi}\right)^{(1+h)/h}.$$

- *Operator presentation of the oscillatory problem*

Let us put $t_0 \equiv T$ and $W_0(t),\ J_0(t)$ are prescribed initial oscillatory functions on the interval $[0,T]$.

Let the set of zeros of the initial functions ($W_0(\tau_k) = J_0(\tau_k) = 0$) be $0 = \tau_0 < \tau_1 < ... < \tau_k < \tau_{k+1} < ... < \tau_n = T$.

Let $S = \{t_k\}_{k=0}^{\infty}$ be a strictly increasing sequence of real numbers defined in the following way:

$t_0 = T + \tau_0,\ t_1 = T + \tau_1, t_2 = T + \tau_2,\ ....\ ,t_n = T + \tau_n = T + T = 2T$,

$t_{n+1} = T + t_1, t_{n+2} = T + t_2,\ ....\ ,t_{2n} = T + t_n = 3T$, ….

Obviously

1) $\lim_{k\to\infty} t_k = \infty$;

2) Let $A = A_1 \cup A_2 = \{\tau_0,\tau_1,...,\tau_k,\tau_{k+1},...,\tau_n\} \cup \{t_0,t_1,...,t_k,...,t_n,...\}$. Then for every $t_k \in A_2$ there is t_s such that $t_k - T = t_s \in A_1 \cup A_2$, provided $t_k - T \ge \tau_0$;

3) $0 < \min\{t_{k+1} - t_k : k = 0,1,2,...,n\} \le \max\{t_{k+1} - t_k : k = 0,1,2,...,n\} = T_0 < \infty$.

Denote by $C_{W_0}^A[0,\infty)$ the set of all continuous functions coinciding with $W_0(t)(J_0(t))$ on $[0,T]$ and vanishing at the points of $A = A_1 \cup A_2$. Consider the operators

$S_W(W): C_W^A[0,\infty) \to C_W^A[0,\infty)$ and

$S_J(J): C_J^A[0,\infty) \to C_J^A[0,\infty)$

acting by the formulas

$$S_W(W)(t) = \begin{cases} W(t-T),\ t \ge T \\ W_0(t),\ t\in[0,T] \end{cases}$$ and

$$S_J(J)(t) = \begin{cases} J(t-T),\ t \ge T \\ J_0(t),\ t\in[0,T] \end{cases}.$$

Introduce the sets for the unknown functions $W(t),\ J(t),\ i_{L_{10}}(t),\ i_{C_{10}}(t)$:

$$M_W = \left\{ W(.) \in C_W^A[0,\infty) : |W(t)| \le W_0 e^{\mu(t-t_k)}, t\in[t_k,t_{k+1}] \right\},$$

$$M_J = \left\{ J(.) \in C_J^A[0,\infty) : |J(t)| \le J_0 e^{\mu(t-t_k)}, t\in[t_k,t_{k+1}] \right\},$$

$$M_{L_{10}} = \left\{ i_{L_{10}}(.) \in C[t_0,\infty) : i_{L_{10}}(t_k) = 0; e^{Rt/L}\left|i_{L_{10}}(t)\right| \le J_{L_1} e^{\mu(t-t_k)}, t\in[t_k,t_{k+1}] \right\},$$

$$M_{C_{10}} = \left\{ i_{C_{10}}(.) \in C[t_0,\infty) : i_{C_{10}}(t_k) = 0; e^{Rt/L}\left|i_{C_{10}}(t)\right| \le J_{C_1} e^{\mu(t-t_k)}, t\in[t_k,t_{k+1}] \right\}$$

$(k = 0,1,2,...)$ where $W_0, J_0, J_{L_1}, J_{C_1}, \mu$ are positive constants and $\mu T_0 = \mu_0 = \text{const} > 0$.

Introduce the following families of pseudo-metrics

$$\rho_k(W,\bar{W}) = \max\left\{ \left|W(t) - \bar{W}(t)\right| e^{-\mu(t-t_k)} : t\in[t_k,t_{k+1}] \right\},$$

$$\rho_k(J,\bar{J}) = \max\left\{ \left|J(t) - \bar{J}(t)\right| e^{-\mu(t-t_k)} : t\in[t_k,t_{k+1}] \right\},$$

$$\rho_k(i_{L10},\bar{i}_{L10}) = \max\left\{ \left|i_{L10}(t) - \bar{i}_{L10}(t)\right| e^{-\mu(t-t_k)} : t\in[t_k,t_{k+1}] \right\},$$

$$\rho_k(i_{C10},\bar{i}_{C10}) = \max\left\{ \left|i_{C10}(t) - \bar{i}_{C10}(t)\right| e^{-\mu(t-t_k)} : t\in[t_k,t_{k+1}] \right\}.$$

The set $M_W \times M_J \times M_{L_{10}} \times M_{C_{10}}$ turns out into a complete uniform space with respect to the countable saturated family of pseudo-metrics (cf. [1])

$$\rho_k\left((W,J,J_{L_{10}},J_{C_{10}}),(\bar{W},\bar{J},\bar{J}_{L_{10}},\bar{J}_{C_{10}})\right)$$
$$= \max\left\{ \rho_k(W,\bar{W}), \rho_k(J,\bar{J}), \rho_k(J_{L_{10}},\bar{J}_{L_{10}}), \rho_k(J_{C_{10}},\bar{J}_{C_{10}}) \right\} (k = 0,1,2,...).$$

An operator

$$B = (B_W, B_J, B_{L_{10}}, B_{C_{10}}) : M_W \times M_J \times M_{L_{10}} \times M_{C_{10}}$$
$$\to M_W \times M_J \times M_{L_{10}} \times M_{C_{10}}$$

is called contractive if

$$\rho_k\left((B_W,B_J,B_{L_{10}},B_{C_{10}}),(\bar{B}_W,\bar{B}_J,\bar{B}_{L_{10}},\bar{B}_{C_{10}})\right)$$
$$\le l_0 \rho_{j(k)}\left((W,J,J_{L_{10}},J_{C_{10}}),(\bar{W},\bar{J},\bar{J}_{L_{10}},\bar{J}_{C_{10}})\right)$$

where $l_0 < 1$ and $j : A \times M_W \times M_J \times M_{L_{10}} \times M_{C_{10}} \to A$. Here the index set is $A = \{0,1,2,...\}$. The map j is defined as follows: if $t_k - T = t_s$ then

$$j(k) = \begin{cases} s, \text{if } \max\left\{ \rho_s\left(W,\tilde{W}\right), \rho_k\left(J,\tilde{J}\right), \rho\rho_k\left(i_{L_{10}},\tilde{i}_{L_{10}}\right), \rho_k\left(i_{C_{10}},\tilde{i}_{C_{10}}\right) \right\} = \rho_s\left(W,\tilde{W}\right) \\ k, \text{if } \max\left\{ \rho_s\left(W,\tilde{W}\right), \rho_k\left(J,\tilde{J}\right), \rho_k\left(i_{L_{10}},\tilde{i}_{L_{10}}\right), \rho_k\left(i_{C_{10}},\tilde{i}_{C_{10}}\right) \right) \\ \quad = \rho_k\left(J,\tilde{J}\right) \vee \rho_k\left(i_{L_{10}},\tilde{i}_{L_{10}}\right) \vee \rho_k\left(i_{C_{10}},\tilde{i}_{C_{10}}\right). \end{cases}$$

Define the operator

$B_W = B_W(W,J,J_{L10},J_{C10})$, $B_J(W,J,J_{L10},J_{C10})$,
$B_{L10} = B_{L10}(W,J,J_{L10},J_{C10})$,
$B_{C10} = B_{C10}(W,J,J_{L10},J_{C10})$
by the formulas

$$B_W^{(k)}(W,J)(t) := \begin{cases} \dfrac{2E(t)\sqrt{L}}{Z_0+R_0} + \dfrac{Z_0-R_0}{Z_0+R_0} S_T(J)(t), & t\in[t_k,t_{k+1}] \\ W_0(t), & t\in[0,T] \end{cases}$$

$$B_J^{(k)}(W,J,i_{L_{10}},i_{C_{10}})(t) := \begin{cases} \int_{t_k}^{t} \bar I(W,J,i_{L_{10}},i_{C_{10}})(s)ds \\ -\dfrac{t-t_k}{t_{k+1}-t_k}\int_{t_k}^{t_{k+1}} \bar I(W,J,i_{L_{10}},i_{C_{10}})(s)ds, & t\in[t_k,t_{k+1}] \\ J_0(t), & t\in[0,T] \end{cases}$$

$$B_{L10}^{(k)}(i_{L10},i_{C10})(t) := \int_{t_k}^{t} \bar I_{L10}(i_{L10},i_{C10})(s)ds - \frac{t-t_k}{t_{k+1}-t_k}\int_{t_k}^{t_{k+1}} \bar I_{L10}(i_{L10},i_{C10})(s)ds,\ t\in[t_k,t_{k+1}]$$

$$B_{C10}^{(k)}(W,J,i_{C10})(t) := \int_{t_k}^{t} \bar I_{C10}(W,J,i_{C10})(s)ds - \frac{t-t_k}{t_{k+1}-t_k}\int_{t_k}^{t_{k+1}} \bar I_{C10}(W,J,i_{C10})(s)ds,\ t\in[t_k,t_{k+1}]$$

where

$$\bar U_k(W,J) = \left[2E(t)\sqrt{L(0,t)} + (Z_0-R_0)S_T(J)(t)\right]/(Z_0+R_0)$$

$$\bar I_k(W,J,i_{L_{10}},i_{C_{10}}) = \frac{dS_T(W)(t)}{dt} - \left(\frac{R}{L}+\frac{\sqrt{C}}{C_{11}\sqrt{L}}\right)S_T(W)(t) + \left(\frac{R}{L}-\frac{\sqrt{C}}{C_{11}\sqrt{L}}\right)J(t) + e^{Rt/L}\frac{2\sqrt{C}}{C_{11}}\left(i_{L_{10}}(t)+i_{C_{10}}(t)\right);$$

$$\bar I_{L10,k}(i_{L10},i_{C10}) = \frac{C_{10}^{-1}(i_{C10}(t)) - R_1 i_{L10}(t)}{dL_{10}(i_{L10})/di_{L10}};$$

$$\bar I_{C_{10},k}(W,J,i_{C_{10}}) = e^{-Rt/L}\frac{S_T(W)(t)-J(t)}{2L_{11}\sqrt{C}} - \frac{1}{L_{11}}C_{10}^{-1}(i_{C_{10}}(t))$$

We assume

(IN): $|W_0(t)| \le W_0 e^{\mu(t-\tau_k)}, |J_0(t)| \le J_0 e^{\mu(t-\tau_k)}, t\in[0,T]$;

Assumptions (U): $e^{\mu T_0}(W_0+J_0)/2 \le \varphi_0$.

It follows

$$|u(0,t)| \le (|W(t)|+|J(t-T)|)/2 \le \left(W_0 e^{\mu(t-t_k)} + J_0 e^{\mu(t-T-t_k)}\right)/2 \le e^{\mu T_0}\left(W_0 + J_0 e^{-\mu T}\right)/2 \le \phi_0;$$

$$|u(\Lambda,t)| \le (|W(t-T)|+|J(t)|)/2 \le \left(W_0 e^{\mu(t-T-t_k)} + J_0 e^{\mu(t-t_k)}\right)/2 \le e^{\mu T_0}\left(W_0 e^{-\mu T} + J_0\right)/2 \le \phi_0.$$

Now we formulate the main problem: to find an oscillatory solution (W,J,i_{L10},i_{C10}) of the system (10) coinciding with prescribed oscillatory initial functions $W_0(t),J_0(t)$ on the interval $[0,T]$

$$W(t)=W_0(t),\ \frac{dW(t)}{dt}=\frac{dW_0(t)}{dt}, t\in[0,T];$$

$$J(t)=J_0(t),\ \frac{dJ(t)}{dt}=\frac{dJ_0(t)}{dt}, t\in[0,T];$$

$$W_0(T)=0,\ J_0(T)=0, i_{L10}(T)=0,\ i_{C10}(T)=0.$$

Lemma 1. If $(W,J,i_{L10},i_{C10}) \in M_W\times M_J\times M_{L10}\times M_{C10}$ then $\left(B_W(.), B_J(.), B_{i_{L10}}(.), B_{i_{C10}}(.)\right) \in M_W\times M_J\times M_{L10}\times M_{C10}$.

Proof: We first prove that functions defined by the formulas (11) are continuous ones. Indeed, the continuity of the first component is obvious. For the second one we have

$$B_J^{(k)}(W,J)(t_{k+1}) = \int_{t_k}^{t_{k+1}} \bar I_k(W,J)(s)ds - \frac{t_{k+1}-t_k}{t_{k+1}-t_k}\int_{t_k}^{t_{k+1}} \bar I_k(W,J)(s)ds = 0,$$

$$B_J^{(k)}(W,J)(t_{k+1}) = \int_{t_k}^{t_{k+1}} \bar I_k(W,J)(s)ds - \frac{t_{k+1}-t_{k+1}}{t_{k+1}-t_k}\int_{t_{k+1}}^{t_{k+2}} \bar I_k(W,J)(s)ds = 0, \quad (11)$$

$$B_J^{(0)}(W,J)(t_0) = \int_{t_0}^{t_0} \bar I_k(W,J)(s)ds - \frac{t_0-t_0}{t_1-t_0}\int_{t_0}^{t_1} \bar I_k(W,J)(s)ds = 0,$$

$$B_J^{(0)}(W,J)(t_0) = J_0(t_0) = 0.$$

For the another component we proceed in a similar way.

Lemma 1 is thus proved.

Further on we call a generalized solution of the oscillatory problem (10) the solution of the equations

$$W = B_W(W,J),\ J = B_J(W,J,i_{L_{10}},i_{C_{10}}),$$
$$i_{L_{10}} = B_{i_{L_{10}}}(i_{L_{10}},i_{C_{10}}),\ i_{C_{10}} = B_{i_{C_{10}}}(W,J,i_{C_{10}}),$$

that is, the fixed point of the operator *B*. In this manner we avoid the conformity condition [2].

I. Existence-uniqueness of an oscillatory solution

Theorem 1. Let the conditions **(U), (L), (C)** be fulfilled and also the conditions:

$|W_0(t)| \le W_0 e^{\mu(t-t_k)}; |J_0(t)| \le J_0 e^{\mu(t-t_k)}, t\in[\tau_k;\tau_{k+1}]$ $(k=0,1,2,\ldots,n-1)$;

$$e^{\mu T_0}\frac{W_0+J_0}{2} \le \phi_0;\quad \frac{2\sqrt{L}}{Z_0+R_0}W_0 + \frac{|Z_0-R_0|}{Z_0+R_0}J_0 e^{-\mu T} \le W_0;$$

$$W_0 e^{-\mu T} + 2(e^{\mu T_0}-1)/\mu \cdot \left(\left(\frac{R}{L}+\frac{\sqrt{C}}{C_{11}\sqrt{L}}\right)\left(W_0 e^{-\mu T}+J_0\right) + \frac{2\sqrt{C}\left(J_{L_1}+J_{C_1}\right)}{C_{11}}\right) \le J_0;$$

$$2\frac{\phi_0 + R_1 J_{L_1}}{\hat L_{10}}\frac{e^{\mu T_0}-1}{\mu} \le J_{L_1};\quad K_W = \frac{|Z_0-R_0|}{Z_0+R_0}e^{-\mu T} < 1;$$

$$2\left(\frac{W_0 e^{-\mu T}+J_0}{2L_{11}\sqrt{C}} + \frac{\phi_0}{L_{11}}\right)\frac{e^{\mu T_0}-1}{\mu} \le J_{C_1};$$

$$K_J = e^{-\mu T} + 2\frac{e^{\mu T_0}-1}{\mu}\left[\left(\frac{R}{L}+\frac{\sqrt{C}}{C_{11}\sqrt{L}}\right)\left(e^{-\mu T}+1\right) + \frac{4\sqrt{C}}{C_{11}}\right] < 1$$

$$K_{L_1}=2\frac{e^{\mu T_0}-1}{\mu\hat{L}_{10}}\left(R_1+\Phi c_0^{h-2}\left(\sqrt[h]{1+(\phi_0/\Phi)}\right)^2\right)<1;$$

$$K_{C_1}=2\frac{e^{\mu T_0}-1}{\mu L_{11}}\left(\frac{e^{-\mu T}+1}{2\sqrt{C}}+\Phi c_0^{h-2}\left(\sqrt[h]{1+(\phi_0/\Phi)}\right)^2\right)<1.$$

Then there exists a unique oscillatory solution of (10).

Proof: We show that B maps $M_W\times M_J\times M_{L_1}\times M_{C_1}$ into itself. Indeed, for $t\in[t_k,t_{k+1}]$ we have

$$\left|B_W^{(k)}(W,J)(t)\right|\le\frac{2|E(t)|\sqrt{L}}{Z_0+R_0}+\frac{|Z_0-R_0|}{Z_0+R_0}S_T(J)(t)$$

$$\le\frac{2\sqrt{L}}{Z_0+R_0}W_0e^{\mu(t-t_k)}+\frac{|Z_0-R_0|}{Z_0+R_0}J_0e^{-\mu T}e^{\mu(t-t_k)}\le W_0e^{\mu(t-t_k)};$$

$$\left|B_J^{(k)}(W,J,i_{L_{10}},i_{C_{10}})(t)\right|$$

$$\le\left|\int_{t_k}^t\bar{I}(W,J,i_{L_{10}},i_{C_{10}})(s)ds\right|+\left|\frac{t-t_k}{t_{k+1}-t_k}\right|\left|\int_{t_k}^{t_{k+1}}\bar{I}(W,J,i_{L_{10}},i_{C_{10}})(s)ds\right|$$

$$\le\left|\int_{t_k}^t\frac{dW(s-T)}{ds}ds\right|+\frac{2\sqrt{C}}{C_{11}}\left|\int_{t_k}^t e^{Rs/L}\left(i_{L_{10}}(s)+i_{C_{10}}(s)\right)ds\right|$$

$$+\left(\frac{R}{L}+\frac{\sqrt{C}}{C_{11}\sqrt{L}}\right)\left[\left|\int_{t_k}^t W(s-T)ds\right|+\left|\int_{t_k}^t J(s)ds\right|\right]$$

$$+\left|\int_{t_k}^{t_{k+1}}\frac{dW(s-T)}{ds}ds\right|+\frac{2\sqrt{C}\left(J_{L_1}+J_{C_1}\right)}{C_{11}}\left|\int_{t_k}^{t_{k+1}}e^{\mu(s-t_k)}ds\right|$$

$$+\left(\frac{R}{L}+\frac{\sqrt{C}}{C_{11}\sqrt{L}}\right)\left[\left|\int_{t_k}^{t_{k+1}}W(s-T)ds\right|+\left|\int_{t_k}^{t_{k+1}}J(s)ds\right|\right]$$

$$\le|W(t-T)|+\frac{2\sqrt{C}\left(J_{L_1}+J_{C_1}\right)}{C_{11}}\left[\frac{e^{\mu(t-t_k)}-1}{\mu}+\frac{e^{\mu(t_{k+1}-t_k)}-1}{\mu}\right]$$

$$+\left(\frac{R}{L}+\frac{\sqrt{C}}{C_{11}\sqrt{L}}\right)\left(W_0e^{-\mu T}+J_0\right)\left[\frac{e^{\mu(t-t_k)}-1}{\mu}+\frac{e^{\mu(t_{k+1}-t_k)}-1}{\mu}\right]$$

$$\le e^{\mu(t-t_k)}W_0e^{-\mu T}+\frac{e^{\mu(t-t_k)}-1}{\mu}\left[\left(\frac{R}{L}+\frac{\sqrt{C}}{C_{11}\sqrt{L}}\right)\left(W_0e^{-\mu T}+J_0\right)+\frac{2\sqrt{C}\left(J_{L_1}+J_{C_1}\right)}{C_{11}}\right]$$

$$+e^{\mu(t-t_k)}\frac{e^{\mu T_0}-1}{\mu}\left[\left(\frac{R}{L}+\frac{\sqrt{C}}{C_{11}\sqrt{L}}\right)\left(W_0e^{-\mu T}+J_0\right)+\frac{2\sqrt{C}\left(J_{L_1}+J_{C_1}\right)}{C_{11}}\right]$$

$$\le e^{\mu(t-t_k)}\left[W_0e^{-\mu T}+2\frac{e^{\mu T_0}-1}{\mu}\left(\left(\frac{R}{L}+\frac{\sqrt{C}}{C_{11}\sqrt{L}}\right)\left(W_0e^{-\mu T}+J_0\right)+\frac{2\sqrt{C}\left(J_{L_1}+J_{C_1}\right)}{C_{11}}\right)\right]$$

$$\le J_0e^{\mu(t-t_k)};$$

$$\left|B_{L_{10}}^{(k)}(i_{L_{10}},i_{C_{10}})(t)\right|$$

$$\le\left|\int_{t_k}^t\bar{I}_{L_{10}}(i_{L_{10}},i_{C_{10}})(s)ds\right|+\left|\frac{t-t_k}{t_{k+1}-t_k}\right|\left|\int_{t_k}^{t_{k+1}}\bar{I}_{L_{10}}(i_{L_{10}},i_{C_{10}})(s)ds\right|$$

$$\le\left|\int_{t_k}^t\frac{C_{10}^{-1}(i_{C_{10}}(s))-R_1i_{L_{10}}(s)}{dL_{10}(i_{L_{10}})/di_{L_{10}}}ds\right|+\left|\int_{t_k}^{t_{k+1}}\frac{C_{10}^{-1}(i_{C_{10}}(s))-R_1i_{L_{10}}(s)}{dL_{10}(i_{L_{10}})/di_{L_{10}}}ds\right|$$

$$\le\frac{\varphi_0+R_1J_{L_1}}{\hat{L}_{10}}\left(\left|\int_{t_k}^t e^{\mu(s-t_k)}ds\right|+\left|\int_{t_k}^{t_{k+1}}e^{\mu(s-t_k)}ds\right|\right)$$

$$\le\frac{\varphi_0+R_1J_{L_1}}{\hat{L}_{10}}\left(\frac{e^{\mu(t-t_k)}-1}{\mu}+\frac{e^{\mu(t_{k+1}-t_k)}-1}{\mu}\right)$$

$$\le e^{\mu(t-t_k)}2(e^{\mu T_0}-1)(\varphi_0+R_1J_{L_1})/(\mu\hat{L}_{10})\le J_{L_1}e^{\mu(t-t_k)};$$

$$\left|B_{C_{10}}^{(k)}(W,J,i_{C_{10}})(t)\right|$$

$$\le\left|\int_{t_k}^t\bar{I}_{C_{10}}(W,J,i_{C_{10}})(s)ds\right|+\left|\frac{t-t_k}{t_{k+1}-t_k}\right|\left|\int_{t_k}^{t_{k+1}}\bar{I}_{C_{10}}(W,J,i_{C_{10}})(s)ds\right|$$

$$\le\left|\int_{t_k}^t\left(e^{-Rs/L}\frac{W(s-T)-J(s)}{2L_{11}\sqrt{C}}-\frac{1}{L_{11}}C_{10}^{-1}(i_{C_{10}}(s))\right)ds\right|$$

$$+\left|\int_{t_k}^{t_{k+1}}\left(e^{-Rs/L}\frac{W(s-T)-J(s)}{2L_{11}\sqrt{C}}-\frac{1}{L_{11}}C_{10}^{-1}(i_{C_{10}}(s))\right)ds\right|$$

$$\le\left(\frac{W_0e^{-\mu T}+J_0}{2L_{11}\sqrt{C}}+\frac{\varphi_0}{L_{11}}\right)\left(\frac{e^{\mu(t-t_k)}-1}{\mu}+\frac{e^{\mu(t_{k+1}-t_k)}-1}{\mu}\right)$$

$$\le e^{\mu(t-t_k)}2\frac{e^{\mu T_0}-1}{\mu L_{11}}\left(\frac{W_0e^{-\mu T}+J_0}{2\sqrt{C}}+\varphi_0\right)\le e^{\mu(t-t_k)}J_{C_1}.$$

It remains to show that the operator B is a contractive one. Indeed

$$\left|B_W^{(k)}(W,J)(t)-B_W^{(k)}(\tilde{W},\tilde{J})(t)\right|$$

$$\le|Z_0-R_0|\left|J(t-T)-\tilde{J}(t-T)\right|/(Z_0+R_0)$$

$$\le e^{\mu(t-t_k)}e^{-\mu T}|Z_0-R_0|\rho_{k-T}(J,\tilde{J})/(Z_0+R_0)$$

which implies

$$\rho_k(B_W^{(k)}(W,J),B_W^{(k)}(\tilde{W},\tilde{J}))$$

$$\le e^{-\mu T}|Z_0-R_0|\rho_{k-T}(J,\tilde{J})/(Z_0+R_0)\equiv K_W\rho_{k-T}(J,\tilde{J}).$$

It follows

$$\rho_k\left(B_W^{(k)}(W,J),B_W^{(k)}(\tilde{W},\tilde{J})\right)\le K_W\rho_{j(k)}\left((W,J,i_{L_{10}},i_{C_{10}}),(\tilde{W},\tilde{J},\tilde{i}_{L_{10}},\tilde{i}_{C_{10}})\right)$$

Further on we have

$$\left|B_J^{(k)}(W,J,i_{L_{10}},i_{C_{10}})(t)-B_J^{(k)}(\tilde{W},\tilde{J},\tilde{i}_{L_{10}},\tilde{i}_{C_{10}})(t)\right|\le$$

$$\le\left|\int_{t_k}^t\left(\frac{dW(s-T)}{ds}-\frac{dW(s-T)}{ds}\right)ds\right|$$

$$+\left(\frac{R}{L}+\frac{\sqrt{C}}{C_{11}\sqrt{L}}\right)\rho_{k-T}\left(W,\tilde{W}\right)e^{-\mu T}\int_{t_k}^t e^{\mu(s-t_k)}ds$$

$$+\left(\frac{R}{L}+\frac{\sqrt{C}}{C_{11}\sqrt{L}}\right)\rho_k\left(J,\tilde{J}\right)\int_{t_k}^t e^{\mu(s-t_k)}ds$$

$$+\frac{2\sqrt{C}}{C_{11}}\left(\rho_k\left(i_{L_{10}},\tilde{i}_{L_{10}}\right)+\rho_k\left(i_{C_{10}},\tilde{i}_{C_{10}}\right)\right)\int_{t_k}^t e^{\mu(s-t_k)}ds$$

$$+\left(\frac{R}{L}+\frac{\sqrt{C}}{C_{11}\sqrt{L}}\right)\int_{t_k}^{t_{k+1}}e^{\mu(s-t_k)}ds\left[\rho_{k-T}\left(W,\tilde{W}\right)e^{-\mu T}+\rho_k\left(J,\tilde{J}\right)\right]$$

$$+(1/C_{11})\left[2\sqrt{C}\left(\rho_k\left(i_{L10},\tilde{i}_{L10}\right)+\rho_k\left(i_{C10},\tilde{i}_{C10}\right)\right)\right]\int_{t_k}^{t_{k+1}}e^{\mu(s-t_k)}ds$$

$$\le\left|W(t-T)-\tilde{W}(t-T)\right|$$

$$+\left(\frac{R}{L}+\frac{\sqrt{C}}{C_{11}\sqrt{L}}\right)\left(\rho_{k-T}\left(W,\tilde{W}\right)e^{-\mu T}+\rho_k\left(J,\tilde{J}\right)\right)\frac{e^{\mu(t-t_k)}-1}{\mu}$$

$$+(1/C_{11})\left[2\sqrt{C}\left(\rho_k\left(i_{L10},\tilde{i}_{L10}\right)+\rho_k\left(i_{C10},\tilde{i}_{C10}\right)\right)\right]\left[\left(e^{\mu(t-t_k)}-1\right)/\mu\right]$$

$$+\left(\frac{R}{L}+\frac{\sqrt{C}}{C_{11}\sqrt{L}}\right)\left(\rho_{k-T}\left(W,\tilde{W}\right)e^{-\mu T}+\rho_k\left(J,\tilde{J}\right)\right)\frac{e^{\mu(t_{k+1}-t_k)}-1}{\mu}$$

$$+\left(2\sqrt{C}/C_{11}\right)\left(\rho_k\left(i_{L10},\tilde{i}_{L10}\right)+\rho_k\left(i_{C10},\tilde{i}_{C10}\right)\right)\left[\left(e^{\mu(t_{k+1}-t_k)}-1\right)/\mu\right]$$

$$\le e^{\mu(t-t_k)}\rho_{k-T}\left(W,\tilde{W}\right)e^{-\mu T}$$

$$+\left(\frac{R}{L}+\frac{\sqrt{C}}{C_{11}\sqrt{L}}\right)\left(\rho_{k-T}\left(W,\tilde{W}\right)e^{-\mu T}+\rho_k\left(J,\tilde{J}\right)\right)2\frac{e^{\mu T_0}-1}{\mu}$$

$$+\left(2\sqrt{C}/C_{11}\right)\left[\rho_k\left(i_{L10},\tilde{i}_{L10}\right)+\rho_k\left(i_{C10},\tilde{i}_{C10}\right)\right]\left[2\left(e^{\mu T_0}-1\right)/\mu\right]$$

$$\le e^{\mu(t-t_k)}\rho_{k-T}\left(W,\tilde{W}\right)e^{-\mu T}$$

$$+\left(\frac{R}{L}+\frac{1}{C_{11}Z_0}\right)\left(\rho_{k-T}\left(W,\tilde{W}\right)e^{-\mu T}+\rho_k\left(J,\tilde{J}\right)\right)2\frac{e^{\mu T_0}-1}{\mu}$$

$$+\left(2\sqrt{C}/C_{11}\right)\left[\rho_k\left(i_{L10},\tilde{i}_{L10}\right)+\rho_k\left(i_{C10},\tilde{i}_{C10}\right)\right]\left[2\left(e^{\mu T_0}-1\right)/\mu\right]$$

$$\le e^{\mu(t-t_k)}\left[e^{-\mu T}+2\frac{e^{\mu T_0}-1}{\mu}\left(\left(\frac{R}{L}+\frac{1}{C_{11}Z_0}\right)\left(e^{-\mu T}+1\right)+\frac{4\sqrt{C}}{C_{11}}\right)\right]$$

$$.\max\left\{\rho_{k-T}\left(W,\tilde{W}\right),\rho_k\left(J,\tilde{J}\right),\rho_k\left(i_{L10},\tilde{i}_{L10}\right),\rho_k\left(i_{C10},\tilde{i}_{C10}\right)\right\}.$$

It follows

$$\rho_k\left(B_J^{(k)}(W,J),B_J^{(k)}(\tilde{W},\tilde{J})\right)$$

$$\le K_J\rho_{j(k)}\left((W,J,i_{L10},i_{C10}),(\tilde{W},\tilde{J},\tilde{i}_{L10},\tilde{i}_{C10})\right),$$

where

$$K_J=e^{-\mu T}+\frac{2e^{\mu T_0}-2}{\mu}\left[\left(\frac{R}{L}+\frac{1}{C_{11}Z_0}\right)\left(e^{-\mu T}+1\right)+\frac{4\sqrt{C}}{C_{11}}\right].$$

In a similar way we obtain

$$\left|B_{L10}^{(k)}(i_{L10},i_{C10})(t)-B_{L10}^{(k)}(\tilde{i}_{L10},\tilde{i}_{C10})(t)\right|$$

$$\le\int_{t_k}^{t}\left|\overline{I}_{L10}(i_{L10},i_{C10})(s)-\overline{I}_{L10}(\tilde{i}_{L10},\tilde{i}_{C10})(s)\right|ds$$

$$+\left|\frac{t-t_k}{t_{k+1}-t_k}\right|\left|\int_{t_k}^{t_{k+1}}\left(\overline{I}_{L10}(i_{L10},i_{C_1})(s)-\overline{I}_{L10}(\tilde{i}_{L10},\tilde{i}_{C10})(s)\right)ds\right|$$

$$\le\int_{t_k}^{t}\left|\frac{C_{10}^{-1}(i_{C10}(s))-R_1i_{L10}(s)}{dL_{10}(i_{L10})/di_{L10}}-\frac{C_{10}^{-1}(\tilde{i}_{C10}(s))-R_1\tilde{i}_{L10}(s)}{dL_{10}(\tilde{i}_{L10})/di_{L10}}\right|ds$$

$$+\left|\int_{t_k}^{t_{k+1}}\left(\frac{C_{10}^{-1}(i_{C10}(s))-R_1i_{L10}(s)}{dL_{10}(i_{L10})/di_{L10}}-\frac{C_{10}^{-1}(\tilde{i}_{C10}(s))-R_1\tilde{i}_{L10}(s)}{dL_{10}(\tilde{i}_{L10})/di_{L10}}\right)ds\right|$$

$$\le\rho_k\left((W,J,i_{L10},i_{C10}),(\tilde{W},\tilde{J},\tilde{i}_{L10},\tilde{i}_{C10})\right)e^{\mu(t-t_k)}$$

$$.\left[\left(2e^{\mu T_0}-2\right)/\hat{L}_{10}\mu\right]\left[R_1+\Phi c_0^{h-2}\left(\sqrt[h]{1+(\varphi_0/\Phi)}\right)^2\right]$$

$$\equiv e^{\mu(t-t_k)}K_{L_1}\rho_k\left((W,J,i_{L10},i_{C10}),(\tilde{W},\tilde{J},\tilde{i}_{L10},\tilde{i}_{C10})\right),$$

where

$$K_{L_1}=\left[\left(2e^{\mu T_0}-2\right)/\hat{L}_{10}\mu\right]\left[R_1+\Phi c_0^{h-2}\left(\sqrt[h]{1+(\varphi_0/\Phi)}\right)^2\right].$$

It follows

$$\rho_k\left(B_{L10}^{(k)}(i_{L10},i_{C10}),B_{L10}^{(k)}(\tilde{i}_{L10},\tilde{i}_{C10})\right)$$

$$\le K_{L_1}\rho_k\left((W,J,i_{L10},i_{C10}),(\tilde{W},\tilde{J},\tilde{i}_{L10},\tilde{i}_{C10})\right).$$

Finally

$$\left|B_{C10}^{(k)}(W,J,i_{C10})(t)-B_{C10}^{(k)}(\tilde{W},\tilde{J},\tilde{i}_{C10})(t)\right|$$

$$\le\int_{t_k}^{t}\left|\overline{I}_{C10}(W,J,i_{C10})(s)-\overline{I}_{C10}(\tilde{W},\tilde{J},\tilde{i}_{C10})(s)\right|ds$$

$$+\left|\frac{t-t_k}{t_{k+1}-t_k}\right|\int_{t_k}^{t_{k+1}}\left(\overline{I}_{C10}(W,J,i_{C10})(s)-\overline{I}_{C10}(\tilde{W},\tilde{J},\tilde{i}_{C10})(s)\right)ds$$

$$\le\int_{t_k}^{t}\left|e^{-Rs/L}\frac{W(s-T)-\tilde{W}(s-T)-J(s)+\tilde{J}(s)}{2L_{11}\sqrt{C}}+\frac{C_{10}^{-1}(i_{C10}(s))-C_{10}^{-1}(\tilde{i}_{C10}(s))}{L_{11}}\right|ds$$

$$+\left|\int_{t_k}^{t_{k+1}}\left(e^{-\frac{R}{L}s}\frac{W(s-T)-\tilde{W}(s-T)-J(s)+\tilde{J}(s)}{2L_{11}\sqrt{C}}+\frac{C_{10}^{-1}(i_{C10}(s))-C_{10}^{-1}(\tilde{i}_{C10}(s))}{L_{11}}\right)ds\right|$$

$$\le\left(\frac{\rho_{k-T}\left(W,\tilde{W}\right)e^{-\mu T}+\rho_k\left(J,\tilde{J}\right)}{2L_{11}\sqrt{C}}+\frac{\rho_k(i_{C10},\tilde{i}_{C10})\Phi c_0^{h-2}\left(\sqrt[h]{1+(\varphi_0/\Phi)}\right)^2}{L_{11}}\right)$$

$$.\left(\int_{t_k}^{t}e^{\mu(s-t_k)}ds+\int_{t_k}^{t_{k+1}}e^{\mu(s-t_k)}ds\right)$$

$$\le e^{\mu(t-t_k)}\max\left\{\rho_{k-T}\left(W,\tilde{W}\right),\rho_k\left(J,\tilde{J}\right),\rho_k\left(i_{C10},\tilde{i}_{C10}\right)\right\}$$

$$.\left[2\left(e^{\mu T_0}-1\right)/\mu\right]\left(\frac{e^{-\mu T}+1}{2L_{11}\sqrt{C}}+\frac{\Phi c_0^{h-2}\left(\sqrt[h]{1+(\varphi_0/\Phi)}\right)^2}{L_{11}}\right)$$

$$\le e^{\mu(t-t_k)}K_{C_1}\rho_{j(k)}\left((W,J,i_{L10},i_{C10}),(\tilde{W},\tilde{J},\tilde{i}_{L10},\tilde{i}_{C10})\right),$$

where

$$K_{C_1}=2\frac{e^{\mu T_0}-1}{\mu L_{11}}\left(\frac{e^{-\mu T}+1}{2\sqrt{C}}+\Phi c_0^{h-2}\left(\sqrt[h]{1+(\phi_0/\Phi)}\right)^2\right).$$

It follows

$$\rho_k\left(B_{C10}^{(k)}(W,J,i_{C10}),B_{C10}^{(k)}(\tilde{W},\tilde{J},\tilde{i}_{C10})\right)$$

$$\le K_{C_1}\rho_k\left((W,J,i_{L10},i_{C10}),(\tilde{W},\tilde{J},\tilde{i}_{L10},\tilde{i}_{C10})\right).$$

Therefore

$$\rho_k\left(B(W,J,i_{L10},i_{C10}),B(\tilde{W},\tilde{J},\tilde{i}_{L10},\tilde{i}_{C10})\right)$$

$$\le K\rho_k\left((W,J,i_{L10},i_{C10}),(\tilde{W},\tilde{J},\tilde{i}_{L10},\tilde{i}_{C10})\right)$$

where $K=\max\{K_W,K_J,K_{L_1},K_{C_1}\}<1$. In view of fixed point theorem from [2] and a remark in [18], the operator B has a unique fixed point which is an oscillatory solution of (10).

Theorem 1 is thus proved.

IV. NUMERICAL EXAMPLE

Here we collect all inequalities guaranteeing an existence-uniqueness result.

For a transmission line with length $\Lambda = 10m$, $L = 0{,}45\,\mu H/m,\ C = 80\ pF/m,\ R = 0{,}0875\,\Omega$ we have $Z_0 = \sqrt{L/C} = \sqrt{0{,}45.10^{-6}/80.10^{-12}} = 47{,}5\Omega$; $\sqrt{LC} = 6.10^{-9}$; $T = \Lambda\sqrt{LC} = 6.10^{-8}\sec$; $R/L = 0{,}0875/0{,}45.10^{-6} = 1{,}94.10^5$. Choose $\mu = 10^{12}$ and $T_0 = 10^{-14}$, then $\mu T_0 = \mu_0 = 0{,}01$ and $\mu T = 10^{12}.6.10^{-9} = 6.10^3 = 6000$. We have also $C_{10}(u) = c_0/\sqrt{1-(u/\Phi)} = c_0/\sqrt{1-(u/\Phi)}$, where $h = 2$; $R_0 = R_1 = 35{,}5\,\Omega$; $c_0 = 5.10^{-11}F$ and $\Phi = 0{,}52\,V \Rightarrow \phi_0 = 0{,}51 < 0{,}52 = \Phi$; $C_{11} = 50.10^{-12}F$; $L_{11} = 3.10^{-6}H$. We choose $L_{10}(i) = 3i - (1/12)i^3$. Then

$$dL_{10}(i_{L10})/di_{L10} = 3 - 0{,}25 i_{L10}^{\ 2}.$$

For $i_0 = 1$ we obtain $3 - 0{,}25 i_{L_{10}}^{\ 2} > 3 - 0{,}25 = 2{,}75$ and then $\sum_{n=2}^{3}(n+1)nl_n|i_0|^{n-2} = 0{,}5$; $\hat{L}_{10} = 2{,}75$. If we choose $h = 2$ and $W_0 = J_0 = J_{L_1} = J_{C_1} = \phi_0$, $e^{-\mu T} = e^{-6000} \approx 0$, then the above inequalities become

$$e^{0{,}01}\frac{1}{2} \le 0{,}51 \Rightarrow 0{,}505 \le 0{,}51;\qquad \frac{2\sqrt{0{,}45.10^{-6}}}{83} \le 1;$$

$$2\frac{e^{0{,}01}-1}{10^{12}}\left(1{,}94.10^5 + \frac{1}{50.10^{-12}.47{,}5} + \frac{4\sqrt{80.10^{-12}}}{50.10^{-12}}\right) \le 1;$$

$$2\frac{1+35{,}5}{2{,}75}\frac{e^{0{,}01}-1}{10^{12}} \le 1;\quad 2\left(\frac{1}{2\sqrt{80.10^{-12}}} + 1\right)\frac{e^{0{,}01}-1}{10^{12}.3.10^{-6}} \le 1;$$

$$K_W = \frac{12}{83}e^{-6000} \approx 0 < 1;$$

$$K_J = 2\frac{e^{0{,}01}-1}{10^{12}}\left[1{,}94.10^5 + \frac{1}{50.10^{-12}.47{,}5} + \frac{4\sqrt{80.10^{-12}}}{50.10^{-12}}\right] < 1$$

;

$$K_{L_1} = 2\frac{e^{0{,}01}-1}{10^{12}.2{,}75}(35{,}5 + 0{,}52(1+0{,}98)) < 1;$$

$$K_{C_1} = 2\frac{e^{0{,}01}-1}{10^{12}.3.10^{-6}}\left(\frac{1}{2.10^{-6}\sqrt{80}} + 0{,}52(1+0{,}98)\right).$$

V. CONCLUSION.

The solution can be approximated by an iterative sequence of successive approximations.

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