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Fuzzy Model for Solving Stag Hunt Game with Two Decision – Maker

S. A. Sahathana Thasneem
Research Scholar in Mathematics
Sri G.V.G Visalakshi College for Women
Udumalpet

B.Kalaiselvi
Associate Professor in Mathematics
Sri G.V.G Visalakshi College for Women
Udumalpet

Abstract:- In game theory the players deals with a decision – making situation with conflicting objectives. In the case of disagreement and doubtfulness the fuzzy approach is more effective and conducive. The generally known stag hunt game which portrays the combat between safety and social cooperation is solved using a fuzzy approach.

Keywords:- *fuzzy set, decision making , membership function , stag hunt.*

I. INTRODUCTION

A game is defined by a common situation in which two or more contestants are involved in decision making movement in expectation of certain result over a duration of time which is considered in The theory of Games and Economic Behavior[9] by von Neumann and Mongenstern and its concepts discussed in Fun and Games, A text on game theory[2] done by K.Binmore. A set of moves a player will follow in a given game are described as strategies which must be complete, defining an action in every possibility and each player has an non trivial set of strategies and he chooses among them. While facing the conflicting objectives the players are likely to maximize his own outcome and maximize / minimize his opponents payoff. He may either cooperate or non cooperate with the opponent. In some situations the game may have uncertain values to the objective functions. The vagueness is due to the unknown decisions of the opponents[4]. Also the real life situations may contain imprecision that cannot be expressed efficiently in crisp numbers[7][8]. Using the concept of fuzzy game is the way to deal with these situations related to the payoff functions. In contrariety to conventional logic prof L.A. Zadeh initiated a fuzzy logic[10] and the Applications of fuzzy sets[11] by H.J. Zimmermann. Instead of using the classical Boolean {0,1} values, the degree of an element in certain group is mapped by fuzzy sets between [0,1]. In a conflicting situation the intellectual and social characteristics of the opponent are taken into consideration. Since such things are complicated in classical game theory methods such as minmax, maxmin and dominance. A 2×2 strategic games was solved by using a fuzzy procedure[6] by Song and Kandel and the prisoner's dilemma game was solved with this approach[3] by Mubarak. S. a triangular membership function and a conditional membership function are used to represent the player's degree of satisfaction.

II. STAG HUNT

Jean – Jacques Rousseau described a situation in which two persons go out for hunting. Each can independently choose to hunt a stag or hunt a rabbit. Each player must take an action without knowing the preference of the other. If a person hunts a stag, they must have the association with the other in order to succeed. An individual put up a rabbit himself, but a rabbit is valueless than a stag. This is useful correspondence for social cooperation, such as international agreements and climate change. Another names for it or its variants include "assurance game", "coordination game" and "trust dilemma". Every player must choose an action without conscious about the choice of the other. The stag hunt vary from the prisoner's dilemma in that there are two pure strategy nash equilibria : while both players cooperate and both of the players defect. In the stag hunt, in spite of the fact that both players cooperating is pareto efficient, the only pure Nash equilibrium is when both players choose to defect. The general form of stag hunt game is

		P_2's strategies	
		Stag	Rabbit
P_1's Strategies	Stag	(Q,Q)	(R,S)
	Rabbit	(S,R)	(T,T)

Table 1

with the condition $Q > S \geq T > R$ where Q, R, S and T are the payoffs for player 1 and player 2. Given in the example

		P_2's strategies	
		Stag	Rabbit
P_1's Strategies	Stag	(2,2)	(0,1)
	Rabbit	(1,0)	(1,1)

Table 2

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III. PRELIMINARIE

The fuzzy set theory is used to frame a model that will overcome the short comings of classical game model.Let

$$S_i = \left\{\frac{p_i}{p_{ij}} \geq 0 \ \sum_{k=1}^{n_i} p_{1j} = 1\right\} \quad (1)$$

be the strategy set of player i. T_{ij} is the function represents the j[th] target for the player I which measures the fulfillment of the j[th] goal for the player. Each target function is a function of (P_1, P_2,……. P_j,….P_L) where L is the players set, Each other player's reaction affects the player's outcome. Player1 construct a conditional fuzzy set A_{1j} in S_1, corresponds to the goal T_{1j}. On the state that the strategies of other players are $p_j \in S_j,\ 2 \leq j \leq I$. The membership function of A_{1j} is denoted as

$$\mu_{A_{1j}} = \mu_{1j}\left(\frac{T_{1j}(P_1, P_2)}{P_2}\right) \quad (2)$$

which is used to maximize player1's j[th] goal.

player1's first target is to maximize his payoff,

The Membership function is

$$\mu_{1j}\big(T_{1j}(P_1, P_2)/P_2\big) = \frac{T_{1j}(P_1, P_2) - min}{max - min} \quad (3)$$

max,min are maximum and minimum estimate of the objective function.The objective function has a finite range. If the goal is to minimize a value, the membership function is,

$$\mu_{1j}\big(T_{1j}(P_1, P_2)/P_2\big) = \frac{T_{1j}(P_1, P_2) - max}{min - max} \quad (4)$$

If $T_{1j}(P_1, P_2) = max$, then $\mu_{1j}\big(T_{1j}(P_1, P_2)/P_2\big) = 0$,

If $T_{1j}(P_1, P_2) = min$, then $\mu_{1j}\big(T_{1j}(P_1, P_2)/P_2\big) = 1$

The fuzzy set A_1 has the possible strategies $S_2 \times S_3 \times \ldots\ldots\ldots\ldots S_I\ for\ P_2$. If there are two players in the game the membership function of A_1 is

$$\mu_1(P_2) = \begin{cases} 2P_2 & 0 \leq P_2 \leq 0.5 \\ -2P_2 + 2 & 0.5 \leq P_2 \leq 1 \end{cases}$$

To find the optimum strategy for player 1, constructing an unconditional fuzzy set A'_{1j} the membership function of A'_{1j} is

$$\mu'_{1j}(P_1) = \max_{P_2}\big(\mu_{1j}\big(T_{1j}(P_1, P_2)/P_2\big).\mu_1(P_2)\big) \quad (5)$$

$\mu_{1j}.\mu_1$ is the chance of two events at the same time. The weight vector is used to aggregate the two objective membership functions for the player.

$$W = (W_1, W_2, \ldots\ldots\ldots\ldots, W_n) \text{ and } \sum_{j=1}^{n} W_j = 1$$

W_1 is assigned for his first goal and W_2 is assigned for his second goal.

The objective function is ,

$$\mu(P_1) = W_1.\mu'_{11}(P_1) + W_2.\mu'_{12}(P_1) + \cdots + W_n.\mu'_{1n}(P_1)$$

For player 1 with two targets

$$\mu(P_1) = W_1.\mu'_{11}(P_1) + W_2.\mu'_{12}(P_1) \quad (6)$$

The optimal strategy for player1 is P_1^* is defined as

$$\mu(P_1^*) = \max_{P_1}\left(\sum_{j=1}^{n} W_j.\mu'_{1j}(P_1)\right)$$

here for player 1's two goals

$$\mu(P_1^*) = \max_{P_1}\left(\sum_{j=1}^{2} W_j.\mu'_{1j}(P_1)\right) \quad (7)$$

IV. FUZZY MODEL TO STAG HUNT'S GENERAL FORM

Let the payoff table of the game be

		P₂'s Strategies	
		C	N
P₁'s Strategies	C	(q,q)	(r,s)
	N	(s,r)	(t,t)

Table 3

The four parameters q ,r ,s ,t should satisfy $q > s \geq t > r >$. The strategy combination for player 1 is (P_1, 1-P_1) and for player 2 is (P_2 ,1-P_2).

Assuming the triangular membership function for player 1 as

$$\mu_1(P_2) = \begin{cases} 2P_2, & if\ 0 \leq P_2 \leq 1 \\ -2P_2 + 2, & if\ 0.5 \leq P_2 \leq 1 \end{cases}$$

In the first case the player 1 will maximize his own payoff and minimize player2's payoff, from

$$T_{11}(P_1, P_2) = P_2(q.P_1 - r.P_1 + s - sP_1 - t + tP_1) + P_1.r + t - tP_1$$

The conditional membership function is,

$$\mu_{11}(T_{11}(P_1, P_2)/P_2)) = \frac{T_{11}(P_1, P_2) - r}{q - r}$$

If $0 \leq P_2 \leq 0.5$, then

$$f(P_1, P_2) = \mu_{11}(P_1, P_2).\mu_1(P_2) = \mu_{11}(T_{11}(P_1, P_2)/P_2).2P_2$$

$$f(P_1, P_2) = \frac{2}{q - r}\left(P_2{}^2(qP_1 - rP_1 + s - sP_1 - t + tP_1) + P_2\begin{pmatrix} P_1 r + t \\ -tP_1 - r \end{pmatrix}\right)$$

$$\frac{\partial f(P_1, P_2)}{\partial P_2} = \frac{2}{q - r}\begin{pmatrix} 2P_1P_2(q - r) + 2P_2(1 - P_1) \\ (s - t) + (1 - P_1)(t - r) \end{pmatrix}$$

Here $1-P_1 \geq 0$ and $q > s \geq t > r$. So the above term is ≥ 0. i.e) $f(P_1,P_2)$ is monotonically increasing in the interval [0,0.5].

If $0.5 \leq P_2 \leq 1$ then,

$$f(P_1,P_2) = \mu_{11}(P_1,P_2).\mu_1(P_2) = \frac{2}{q-r}.A.(1-2P_2) - B, \quad \text{where}$$

$A = (q-r)P_1 + (1-P_1)(s-t) \geq 0$ and

$B = (r-t)(P_1-1) \geq 0$.

Thus $f(P_1,P_2)$ reaches it's maximum value at $P_2 = 0.5$.

$$\mu'_{11}(P_1) = \frac{1}{2(s-r)}(P_1(q+r-s-t) + (s+t-2r)$$

The second target of the prisoner is to minimize the rate of player 2

$$T_{12}(P_1,P_2) = P_1P_2(q-s-r+t) + P_1(s-t) + P_2(r-t) + t$$

The conditional membership function is

$$\mu_{12}(T_{12}(P_1,P_2)/P_2) = \frac{T_{12}(P_1,P_2)-q}{r-q} = \frac{1}{r-q}(P_1P_2(q-s-r+t) + P_1(s-t) + P_2(r-t) + t - q)$$

If $0 \leq P_2 \leq 0.5$, then

$$f(P_1,P_2) = \mu_{12}(P_1,P_2).2P_2$$

$$\frac{\partial f(P_1,P_2)}{\partial P_2} = \frac{2}{r-q}\begin{pmatrix}2P_1P_2(q-s-r+t)+P_1(s \\ -t)+2P_2(r-t)+(t-q)\end{pmatrix} \geq 0$$

If $0.5 \leq P_2 \leq 1$, then

$\mu_1(P_2) = 2 - 2P_2$ and

$$f(P_1,P_2) = \frac{1}{r-q}\begin{pmatrix}P_1P_2(q-s-r+t)+P_1(s \\ -t)+P_2(r-t)+t-q\end{pmatrix}.(2-2P_2)$$

$$\frac{\partial f(P_1,P_2)}{\partial P_2} = \frac{2}{r-q}\left((1-2P_2)A - B\right)$$

where

$A = (q-s)P_1 + (r-t)(1-P_1) \leq 0$ $B = sP_1 - tP_1 + t - q \leq 0$

Thus $f(P_1,P_2)$ is monotonically decreasing in the interval [0.5,1].

$$\mu'_{12}(P_1) = \max_{P_2}(\mu_{12}(T_{12}(P_1,P_2)/P_2).\mu_1(P_2)) = \frac{1}{4(q-r)}(P_1(r+t-q-s) + 2q - r - t)$$

If the weight vector is $W = (W_1,W_2)^T$ then

$$\mu(P_1) = \frac{W_1}{2(q-r)}\begin{pmatrix}P_1(q+r-s-t) \\ +(s+t-2r)\end{pmatrix} + \frac{W_2}{2(q-r)}\begin{pmatrix}P_1(r+t-q-s) \\ +2q-r-t\end{pmatrix}$$

$$\frac{\partial\mu(P_1)}{\partial P_1} = \frac{W_1}{2(q-r)}(q+r-s-t) + \frac{W_2}{2(q-r)}(r+t-q-s) \leq 0$$

Hence (0,1) is the optimal strategy combination .

If player1 cooperates with player 2 his second target is to maximize the value of player2

$$T_{12}(P_1,P_2) = P_1P_2(q-s-r+t) + P_1(s-t) + P_2(r-t) + t$$

The conditional membership function is given by

$$\mu_{12}(T_{12}(P_1,P_2)/P_2) = \frac{1}{q-r}(AP_2 + B)$$

where

$A = (q-s)P_1 + (1-P_1)(r-t \leq 0$

$B = sP_1 - tP_1 + t \geq 0$

If $0 \leq P_2 \leq 0.5$ then,

$$f(P_1,P_2) = \mu_{12}(P_1,P_2).\mu_1(P_2) = \mu_{12}(P_1,P_2).2P_2 = \frac{1}{q-r}(AP_2 + B).2P_2$$

$$f(P_1,P_2) = \frac{1}{q-r}\left(AP_2{}^2 + BP_2\right)$$

$$\frac{\partial f(P_1,P_2)}{\partial P_2} = \frac{2}{q-r}(2AP_2 + B)$$

If $A \leq 0$ and $B \geq 0$ then it's unable to decide $f(P_1,P_2)$ is monotonically increasing or decreasing on the interval [0,0.5] .

$$f(P_1,P_2) = \frac{2A}{(q-r)}\left(\left(P_2 + \frac{B}{2A}\right)^2 - \frac{B^2}{4A^2}\right) \text{ where}$$

$$\frac{B}{-2A} = \frac{sP_1 - tP_2 + t - r}{2(sP_2 + rP_2 - qP_2 - tP_2 + t - r)} \geq 0$$

Hence the maximum value attained at $P_2 = 0.5$.

If $0.5 \leq P_2 \leq 1$ and $\mu_1(P_2) = 2 - 2P_2$

$$f(P_1,P_2) = \mu_{12}(P_1,P_2).\mu_1(P_2) = \frac{1}{q-r}(AP_2 + B).(2-2P_2)$$

$$f(P_1,P_2) = \frac{2}{s-r}(AP_2 + B - AP_2{}^2 - BP_2)$$

$$\frac{\partial f(P_1,P_2)}{\partial P_2} = \frac{2}{s-r}(A - 2AP_2 - B)$$

If $A \leq 0$ and $B \geq 0$, then it's unable to decide $f(P_1,P_2)$ is monotonically increasing or decreasing on the interval [0.5,1]

$$f(P_1,P_2) = \frac{-2A}{q-r}\left(\left(P_2 - \frac{A-B}{2A}\right)^2 - \frac{B}{A} - \left(\frac{A-B}{2A}\right)^2\right)$$

Hence the maximum value happen at $P_2 = 0.5$.

$$\mu'_{12}(P_1) = \max_{P_2}(\mu_{12}(P_1,P_2)/P_2).\mu_1(P_2)$$

$$\mu'_{12}(P_1) = \frac{2}{q-r}\left(AP_2{}^2 + BP_2\right) = \frac{1}{2(q-r)}\left((q+s-r-t)P_1 + t - r\right)$$

If the weight vector is $W = (W_1,W_2)^T$,

$$\mu(P_1) = W_1.\mu'_{11}(P_1) + W_2.\mu'_{12}(P_1)$$
$$= \frac{W_1}{2(q-r)}(P_1(q+r-s-t)+s+t-2r)$$
$$+\frac{W_2}{2(q-r)}(P_1(q+s-r-t)+t-r)$$
$$\frac{\partial\mu(P_1)}{\partial P_1} = \frac{1}{2(q-r)}\big(W_1(q+r-s-t) + W_2(q+s-r-t)\big)\ .$$

V. APPLICATION TO STAG HUNT

The payoff table has both ordinal payoff and relative preference for the outcome.

		P₂'s strategies	
		Stag	Rabbit
P₁'s Strategies	Stag	[4,4][1] (3,3)[2]	[0,3] (1,2)
	Rabbit	[3,0] (2,1)	[3,3] (2,2)

Table 4

In this payoff table 1 denotes the Cardinal payoffs and 2 denotes the Relative preferences. Player 1's first goal is to maximize his own payoff and minimizing Player2's payoff.
The strategy set is

$$S_1 = \{(P_1, 1-P_1)/\ \ 0 \le P_1 \le 1\}$$
$$S_2 = \{(P_2, 1-P_2)/0 \le P_2 \le 1\}$$

The expected payoff value is given by

$$T_{11}(P_1,P_2) = P_1.4.P_2 + P_1.0.(1-P_2) + (1-P_1).3.P_2 + (1-P_1).3.(1-P_2)$$
$$= -3P_1 + 4P_1P_2 + 3\ .$$

The conditional membership function is,

$$\mu_{11}(T_{11}(P_1,P_2)/P_2)) = \frac{-3P_1+4P_1P_2+3-0}{4-0}$$
$$= \frac{1}{4}(-3P_1+4P_1P_2+3)\ .$$
$$f(P_1,P_2) = \mu_{11}(P_1,P_2).\mu_1(P_2)$$

If $0 \le P_2 \le 0.5$, then

$$f(P_1,P_2) = \mu_{11}(T_{11}(P_1,P_2)/P_2).2P_2$$
$$= \frac{1}{4}(-3P_1+4P_1P_2+3).2P_2$$
$$= \frac{1}{4}\big(-6P_1P_2 + 8P_1P_2{}^2 + 6P_2\big)$$
$$\frac{\partial f(P_1,P_2)}{\partial P_2} = \frac{1}{4}(-6P_1+8P_1P_2+6)$$

which is monotonically increasing and reaches maximum value at $P_2 = 0.5$
If $0.5 \le P_2 \le 1$, then

$$f(P_1,P_2) = \mu_{11}(T_{11}(P_1,P_2)/P_2).(2-2P_2)$$
$$= \frac{1}{4}(-3P_1+4P_1P_2+3).(2-2P_2)$$
$$f(P_1,P_2) = \frac{1}{4}\big(-6P_1+14P_1P_2+6-8P_1P_2{}^2-6P_2\big)$$
$$\frac{\partial f(P_1,P_2)}{\partial P_2} = \frac{1}{4}(14P_1-6-16P_1P_2)$$

which is monotonically decreasing and attains maximum at $P_2 = 0.5$.

$$\mu'_{11}(P_1) = \frac{1}{4}(-P_1+3)$$

The second goal is to minimize the payoff of P_2

$$T_{12}(P_1,P_2) = P_1.4.P_2 + P_1.3.(1-P_2) + (1-P_1).0P_2 + (1-P_1).3.(1-P_2)$$
$$= 4P_1P_2 - 3P_2 + 3\ .$$

conditional membership function is

$$\mu_{12}(T_{12}(P_1,P_2)/P_2)) = \frac{4P_1P_2-3P_2+3}{0-4}$$
$$= \frac{1}{4}(3P_2-4P_1P_2+1)$$

If $0 \le P_2 \le 0.5$, then

$$f(P_1,P_2) = \mu_{12}(T_{12}(P_1,P_2)/P_2).2P_2$$
$$= \frac{1}{4}(3P_2-4P_1P_2+1).2P_2$$
$$= \frac{1}{4}\big(6P_2{}^2 - 8P_1P_2{}^2 + 2P_2\big)$$
$$\frac{\partial f(P_1,P_2)}{\partial P_2} = \frac{1}{4}(12P_2-16P_1P_2+2)$$

Thus $f(P_1,P_2)$ is monotonically increasing if P_2 increases on the interval [0,0.5] and reach maximum when P_2=0.5
If $0.5 \le P_2 \le 1$, then

$$f(P_1,P_2) = T_{12}(\mu_{12}(P_1,P_2)/P_2).(2-2P_2)$$
$$= \frac{1}{4}\big(4P_2 - 8P_1P_2 + 2 - 6P_2{}^2 + 8P_1P_2{}^2\big)$$
$$\frac{\partial f(P_1,P_2)}{\partial P_2} = \frac{1}{4}(4-8P_1-12P_2+16P_1P_2)$$

Here $f(P_1,P_2)$ is monotonically decreasing in the interval [0.5,1].

$$\mu'_{12}(P_1) = \frac{1}{4}\left(\frac{5}{2}-4P_1\right)$$

The aggregate membership function of P_1 is

$$\mu(P_1) = W_1\left(\frac{1}{4}(3-P_1)\right) + W_2\left(\frac{1}{4}\left(\frac{5}{2}-4P_1\right)\right)$$
$$\mu(P_1) = \frac{W_1}{4}(3-P_1) + \frac{W_2}{4}\left(\frac{5}{2}-4P_1\right)$$

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$$\frac{\partial\mu(P_1)}{\partial P_1} = \frac{-1}{4}W_1 - \frac{6}{10}W_2 \le 0$$

Thus $\mu_1(P_1)$ has the maximum value when $P_1 = 0$. The optimal strategy combination is (0,1). If P_1 trusts P_2 , then his second target is maximizing $P_2's$ payoff,

$$\mu_{12}(T_{12}(P_1,P_2)/P_2)) = \frac{T_{12}(P_1,P_2) - min}{max - min} = \frac{4P_1P_2 - 3P_2 + 3}{4}$$

If $0 \le P_2 \le 0.5$, then

$$f(P_1,P_2) = \mu_{12}(T_{12}(P_1,P_2)/P_2).2P_2 = \frac{1}{4}(8P_1{P_2}^2 - 6{P_2}^2 + 6P_2)$$

$$\frac{\partial f(P_1,P_2)}{\partial P_2} = \frac{1}{4}(16P_1P_2 - 12P_2 + 6) \ge 0$$

Thus $f(P_1,P_2)$ is monotonically increasing if P_2 increases on the interval [0,0.5] and reach maximum when P_2=0.5

If $0.5 \le P_2 \le 1$, then

$$f(P_1,P_2) = \mu_{12}(T_{12}(P_1,P_2)/P_2).(2 - 2P_2) = \frac{1}{4}(4P_1P_2 - 3P_2 + 3).(2 - 2P_2) = \frac{1}{4}\left(8P_1P_2 - 12P_2 + 6 - 8P_1{P_2}^2 + 6{P_2}^2\right)$$

$$\frac{\partial f(P_1,P_2)}{\partial P_2} = \frac{1}{4}(8P_1 - 12 - 16P_1P_2 + 12P_2) \le 0$$

Here $f(P_1,P_2)$ is monotonically decreasing in the interval [0.5,1] . This implies

$$\mu'_{12}(P_1) = = \frac{1}{4}\left(2P_1 + \frac{3}{2}\right)$$

The aggregate membership function

$$\mu(P_1) = W_1\left(\frac{1}{4}(3 - P_1)\right) + W_2\left(\frac{1}{4}\left(2P_1 + \frac{3}{2}\right)\right)$$

$$\mu(P_1) = \frac{W_1}{4}(3 - P_1) + \frac{W_2}{4}\left(2P_1 + \frac{3}{2}\right)$$

$$\frac{\partial\mu(P_1)}{\partial P_1} = -\frac{1}{4}W_1 + \frac{2}{4}W_2 \ge 0$$

The optimal strategy depends on the weight vectors W_1andW_2.If $W_1 \le 2W_2$,then $\mu(P_1)$ is monotonically increasing function and the optimal strategy is (1,0). If $W_1 > 2W_2$,then the optimal strategy is (0,1) .

In the view of player 2,

His first goal is to maximize his own payoff and minimize Player1's payoff .The strategy set is ,

$$S_1 = \{(P_1, 1 - P_1)/ \ \ 0 \le P_1 \le 1\}$$
$$S_2 = \{(P_2, 1 - P_2)/ \ \ 0 \le P_2 \le 1\}$$

The membership function is

$$\mu_2(P_1) = \begin{cases} 2P_1 & 0 \le P_1 \le 0.5 \\ 2 - 2P_1 & 0.5 \le P_1 \le 1 \end{cases}$$

Now $P_2's$ first goal is to maximize his own payoff and second is to minimize $P_1's$ payoff, that is to maximize the expected payoff $T_{21}(P_1,P_2)$,

$$T_{21}(P_1,P_2) = P_1.4.P_2 + P_1.3.(1 - P_2) + (1 - P_1).0.P_2 + (1 - P_1).3.(1 - P_2) = 4P_1P_2 - 3P_2 + 3 .$$

The conditional membership function is ,

$$\mu_{21}(T_{21}(P_1,P_2)/P_1)) = \frac{T_{21}(P_1,P_2) - min}{max - min} = \frac{1}{4}(4P_1P_2 - 3P_2 + 3)$$

.If $0 \le P_1 \le 0.5$, then

$$f(P_1,P_2) = \mu_{21}(T_{21}(P_1,P_2)/P_2).2P_1 = \frac{1}{4}\left(8{P_1}^2P_2 - 6P_1P_2 + 6P_1\right)$$

$$\frac{\partial f(P_1,P_2)}{\partial P_1} = \frac{1}{4}(16P_1P_2 - 6P_2 + 6)$$

Thus $f(P_1,P_2)$ is monotonically increasing on the interval [0,0.5] and the maximum value reaches at $P_1 = 0.5$.

If $0.5 \le P_1 \le 1$, then

$$f(P_1,P_2) = \mu_{21}(T_{21}(P_1,P_2)/P_2).(2 - 2P_1) = \frac{1}{4}(14P_1P_2 - 6P_2 + 6 - 8{P_1}^2P_2 - 6P_1)$$

$$\frac{\partial f(P_1,P_2)}{\partial P_1} = \frac{1}{4}(14P_2 - 16P_1P_2 - 6)$$

Thus $f(P_1,P_2)$ is monotonically decreasing on the interval [0.5,1] and it will reach it's maximum value at $P_1 = 0.5$

$$\mu'_{21}(P_2) = \frac{1}{4}(-P_2 + 3)$$

The second target of P_2 is minimize $P_1's$ payoff,

$$T_{22}(P_1,P_2) = P_1.4.P_2 + P_1.0.(1 - P_2) + (1 - P_1).3.P_2 + (1 - P_1).3.(1 - P_2) = -3P_1 + 4P_1P_2 + 3$$

$$\mu_{22}(T_{22}(P_1,P_2)/P_1)) = \frac{T_{22}(P_1,P_2) - min}{max - min} = \frac{3P_1 - 4P_1P_2 + 1}{4} .$$

If $0 \le P_1 \le 0.5$, then

$$f(P_1,P_2) = \mu_{22}(T_{22}(P_1,P_2)/P_1).2P_1 = \frac{1}{4}(3P_1 - 4P_1P_2 + 1).2P_1$$

$$\frac{\partial f(P_1,P_2)}{\partial P_1} = \frac{1}{4}(12P_1 - 16P_1P_2 + 2) \ge 0$$

Thus $f(P_1,P_2)$ is monotonically increasing on the interval [0,0.5] and the maximum value reaches at $P_1 = 0.5$.

If $0.5 \le P_1 \le 1$, then

$$f(P_1,P_2) = \mu_{22}(T_{22}(P_1,P_2)/P_1).(2 - 2P_1)$$

$$f(P_1,P_2) = \frac{1}{4}(3P_1 - 4P_1P_2 + 1).(2 - 2P_1) = \frac{1}{4}\left(4P_1 - 8P_1P_2 - 6{P_1}^2 + 8{P_1}^2P_2\right)$$

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$$\frac{\partial f(P_1,P_2)}{\partial P_1} = \frac{1}{4}(4 - 8P_2 - 12P_1 + 16P_1P_2)$$

Here $f(P_1,P_2)$ is monotonically decreasing on the interval [0.5,1] and it will reach it's maximum value at $P_1 = 0.5$.

$$\mu'_{22}(P_2) = \frac{1}{4}\left(\frac{5}{2} - 2P_2\right)$$

If the weight vector is $W = (W_1, W_2)^T$, then the aggregate membership function for P_2

$$\mu(P_2) = W_1\left(\frac{1}{4}(-P_2 + 3)\right) + W_2\left(\frac{1}{4}\left(\frac{5}{2} - 2P_2\right)\right)$$
$$= \frac{W_1}{4}(-P_2 + 3) + \frac{W_2}{4}\left(\frac{5}{2} - 2P_2\right)$$
$$\frac{\partial \mu(P_2)}{\partial P_2} = \frac{-1}{4}W_1 - \frac{2}{4}W_2$$
$$= -\frac{1}{4}W_1 - \frac{2}{4}W_2$$

Which is positive and takes maximum value at $P_2 = 0$ .The optimal strategy is (0,1) .If the second target of P_2 is to maximize the payoff of P_1 , then

$$T_{22}(P_1,P_2) = -3P_1 + 4P_1P_2 + 3$$
$$\mu_{22}(T_{22}(P_1,P_2)/P_1) = \frac{-3P_1 + 4P_1P_2 + 3 - 0}{4 - 0}$$
$$= \frac{4P_1P_2 + 3 - 3P_1}{4}$$

If $0 \le P_1 \le 0.5$, then

$$f(P_1,P_2) = \frac{1}{4}(4P_1P_2 + 3 - 3P_1).2P_1$$
$$= \frac{1}{4}\left(8{P_1}^2P_2 - 6{P_1}^2 + 6P_1\right)$$
$$\frac{\partial f(P_1,P_2)}{\partial P_1} = \frac{1}{4}(16P_1P_2 - 12P_1 + 6) \ \ge 0$$

Thus $f(P_1,P_2)$ is monotonically increasing on the interval [0,0.5] and the maximum value reaches at $P_1 = 0.5$.

If $0.5 \le P_1 \le 1$, then

$$f(P_1,P_2) = \frac{1}{4}(4P_1P_2 + 3 - 3P_1).(2 - 2P_1)$$
$$= \frac{1}{4}\left(8P_1P_2 - 12P_1 + 6 - 8{P_1}^2P_2 + 6{P_1}^2\right)$$
$$\frac{\partial f(P_1,P_2)}{\partial P_1} = \frac{1}{4}(8P_2 - 16P_1P_2 + 12P_1) \ \le 0$$

Here $f(P_1,P_2)$ is monotonically decreasing on the interval [0.5,1] and it will reach it's maximum value at $P_1 = 0.5$

$$\mu'_{22}(P_2) = \frac{1}{4}\left(2P_2 + \frac{3}{2}\right)$$

The aggregate membership function for P_2

$$\mu(P_2) = W_1\left(\frac{1}{4}(-P_2 + 3)\right) + W_2\left(\frac{1}{4}\left(2P_2 + \frac{3}{2}\right)\right)$$
$$= \frac{W_1}{4}(-P_2 + 3) + \frac{W_2}{4}\left(2P_2 + \frac{3}{2}\right)$$
$$\frac{\partial \mu(P_2)}{\partial P_2} = -\frac{W_1}{4} + \frac{2W_2}{4}$$

The optimal strategy depends on the weight vector , which represents the attitude of the player about the structure of his goals. If $W_1 \le 2W_2$, then $\mu(P_2)$ is monotonically increasing function with respect to P_2 and takes maximum value when $P_1 = 1$. The optimal strategy combination is (1,0). If $W_1 > 2W_2$, then the optimal strategy is (0,1). The complete solution is given in the payoff table 5

			Player 2		
			Non - cooperative	Cooperative	
				$W_1 \le 2W_2$	$W_1 > 2W_2$
Player 1	Non cooperative		(2,2)	(2,1)	(2,2)
	Cooperative	$W_1 \le 2W_2$	(1,2)	(3,3)	(1,2)
		$W_1 > 2W_2$	(2,2)	(2,1)	(2,2)

Table 5:- Solution to the Stag Hunt Game

VI. CONCLUSION

In this paper we prescribe a fuzzy model to speak to the contention circumstance in games with strategies .The philosophical intentions or good qualities of players in the diversion are illustrated by fuzzy set model, which are typically dubious and indeterminate by and by. The weight vector data that speaks to the player's emotional qualities, the ideal technique can be effortlessly decided.

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