A Method for Building Mathematical Models of the Slab

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Abstract:- The control of the temperature of the slab, as the control of the temperature field in the slab when only measuring the temperature in the furnace, is a highly applicable problem in many industries. In order to control the temperature of the slab, it is necessary to know the temperature distribution of the slab or the mathematical model of the slab. There are several ways to calculate the temperature field of the slab, however, these methods are often very complex and are not conducive to the design of the controller

The article presents a mathematical model of slab based on the transfer function model. Based on the slab transfer function model, it is easy to understand the temperature distribution in the slab and the design of the slab temperature controller. The results of the research have been verified through simulation and have shown the possibility of being able to apply in practice.

Keywords:- *Slab, Transfer Function Model, Mathematical Model, Temperature Field.*

I. INTRODUCTION

Heating equipment is equipment which is widely used in industry, medical and civil. In the industry, it is often used in heat treatment, melting ferrous and non-ferrous metals. In this study, the authors were interested in the heat-treatment of slabs. One requirement of heat treatment is that the temperature of the furnace must be controlled according to temperature of slab. There are two ways to control the temperature of the slab:

- Direct measurement method of the temperature of the slab: This method, if implemented, is high-precision control. However, in the reality, it is only possible to measure the surface temperature of the slab, it is impossible to measure the thermal distribution within the slab because it is impossible to place a temperature sensor within slab.

- Indirect measurement method of the temperature of the slab: This method calculates the temperature of the slab according to the heat transfer equations, and takes that as the basis for the control. Based on the temperature calculation model of the slab, it is possible to calculate the temperature of the surface of the slab and the heat distribution in the slab from the furnace temperature. This method requires relatively complex heat transfer equations, depending on the size and shape of the slab and must be tested to determine the actual parameters of the slab model. However, with the widespread use of computers today, this method can be implemented.

On the control side, the heating process (firing) of the metal ingots in the furnace is a process with distributed parameters, ie, the control objects is not only described by ordinary differential equations but also described by differential equation. For heat metal, we can easily see the temperature distribution in the slab change according to the thickness of the slab. Depending on the distribution of the control action (e.g. heat source) the temperature field in the slab may be change with the length and width of the slab. To calculate the temperature field of the slab, we can use the numerical method [1]; modeling method [3]; finite element method [4]. However, commonly used controller design methods that commonly used now are based on the transfer function of the object to calculate the controller. Thus, if we build a mathematical model of the slab in the form of transfer function, the design of the slab temperature controller will be very convenient. Therefore, in this paper, the authors will build the transfer function model of the slab.

II. BUILDING TRANSFER FUNCTION MODEL OF THE SLAB

Consider a one-sided burning furnace for the thin slab as shown in Fig. 1

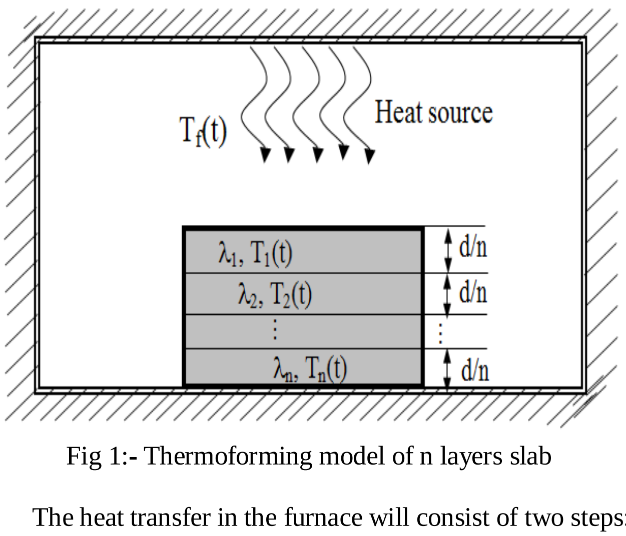

Fig 1:- Thermoforming model of n layers slab

The heat transfer in the furnace will consist of two steps:

- *Step 1:* The heat energy of the furnace is transferred to the outer surface of the slab by radiation or convection heat transfer, in which case the radiant heat transfer is predominant, the convective heat transfer is secondary.

- *Step 2:* Heat conduction from the outside to the inside of the slab.

Considering the slab with the following parameters:

Thermal conductivity: $\lambda\left(\frac{W}{mK}\right)$

Heat transfer coefficient: $\alpha\left(\frac{W}{m^2}\right)$

Length: $a(m)$

Width: $b(m)$

Thickness: $d(m)$

Specific weight: $\rho\left(\frac{Kg}{m^3}\right)$

Specific heat: $c\left(\frac{l}{kg.K}\right)$

Surface area of the slab is : $A = a.b\left(m^2\right)$

Assuming the volume of the furnace is small, the temperature in the furnace is the same and the heat transfer through the head and sides of the slab is ignored, it is possible to divide the slab into n layers as shown in Fig 1. According to Fig 1, we can consider that the temperature in the furnace is the input of the thermal conduction in the slab and the temperature of the bottom layer is the output of the thermal conduction in the slab. The choice of n depends on how much "thickness" of the slab and accuracy required.

A. Building mathematical model of thin slab

The slab are thin when the coefficient BIO is <0.25; [2], in this case we consider slabs as having one layer (*n = 1*). The mathematical model of thin slab is constructed as follows:

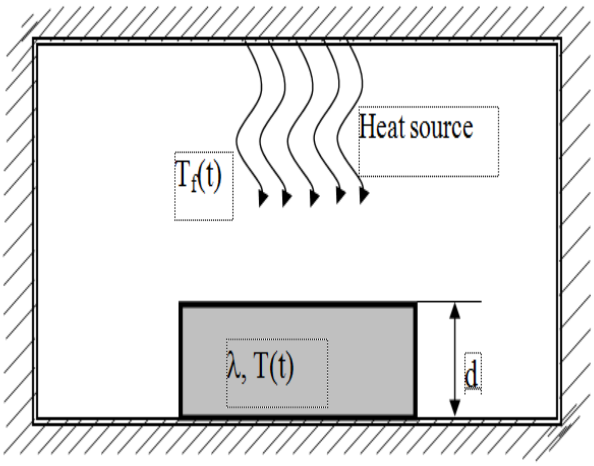

Fig 2:- Thermoforming model of thin slab

The heat flow into the slab is described below

$$Q = A\alpha\left(T_f - T\right) = \frac{T_f - T}{R}$$

With : $R = \frac{1}{A\alpha}$

Because there is no heat flowing out of the slab, the amount of heat entering the slab is

$$Q = cm\frac{dT}{dt} = C\frac{dT}{dt} \quad \text{with} \quad C = cm$$

From there, we have the equation of heat balance as follows

$$\frac{T_f - T}{R} = C\frac{dT}{dt} \qquad (1)$$

Laplace transform of the equation of heat balance (1), we obtain

$$T_f - T = RCTs \Rightarrow T_f = (1 + RCs)T$$

Set $\tau = RC$, we will have

$$T_f(s) = (1 + \tau s)T(s)$$

From the above results, we can describe the slab in the form of the transfer function as follows

$$W(s) = \frac{T(s)}{T_f(s)} = \frac{1}{1 + \tau s} \qquad (2)$$

B. Building mathematical model of thick slab

Consider the thick slab divided into two layers as follows

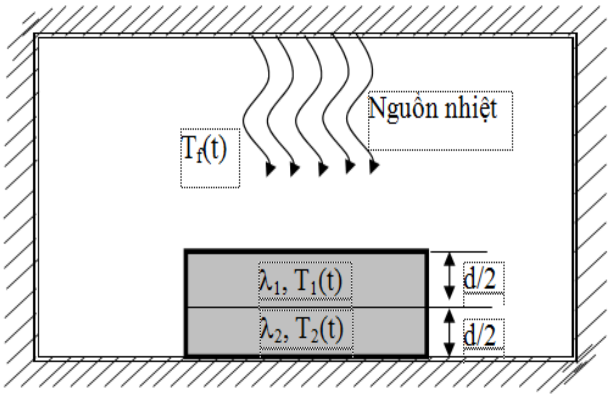

Fig 3:- Thermoforming model of two layer slab

Heat flow into layer 1 is described by the following equation

$$Q_1 = A\alpha\left(T_f - T_1\right) = \frac{T_f - T_1}{R_1}$$

with $R_1 = \frac{1}{A\alpha}$

As the heat flowing to layer 1 is also the heat flow into layer 2 so we have

$$Q_2 = \frac{\lambda_1 A}{d/2}(T_1 - T_2) = \frac{(T_1 - T_2)}{R_2}$$

with $R_2 = \frac{d/2}{\lambda_1 A}$

The amount of heat entering layer 1 is described by the following equation

$$Q_3 = cm_1 \frac{dT_1}{dt} = C_1 \frac{dT_1}{dt}$$

with $C_1 = cm_1$

The amount of heat entering layer 2 is described by the following equation

$$Q_4 = cm_2 \frac{dT_2}{dt} = C_2 \frac{dT_2}{dt}$$

with $C_2 = cm_2$

From here we have the heat balance equation of the layer 2 as follows

$$C_2 \frac{dT_2}{dt} = \frac{(T_1 - T_2)}{R_2} \quad (3)$$

The Laplace transform of equation (3), we obtain the following results:

$$C_2 R_2 T_2 s = T_1 - T_2 \Rightarrow T_1(s) = (1 + C_2 R_2 s) T_2(s) \quad (4)$$

From (4), we have the transfer function of layer 2

$$W_2(s) = \frac{T_2(s)}{T_1(s)} = \frac{1}{1 + R_2 C_2 s} \quad (5)$$

The heat balance equation of layer 1 is as follows

$$C_1 \frac{dT_1}{dt} = \frac{T_f - T_1}{R_1} - \frac{(T_1 - T_2)}{R_2} \quad (6)$$

The laplace transform of equation (6), we obtain the following results:

$$\left(s - \frac{W_2(s)}{R_2 C_1} + \frac{1}{R_1 C_1} + \frac{1}{R_2 C_1}\right) T_1(s) = \frac{T_f(s)}{R_1 C_1} \quad (7)$$

From (7), we have a transfer function of layer 1 as follows

$$W_1(s) = \frac{T_f(s)}{T_1(s)} = \frac{1}{R_2 C_1}\left(\frac{1}{s - \frac{W_2(s)}{R_2 C_1} + \frac{1}{R_1 C_1} + \frac{1}{R_2 C_1}}\right) \quad (8)$$

Reducing the transfer function (8), we have the following results

$$W_1(s) = \frac{1}{1 + R_1 C_1 s + \frac{R_1}{R_2}(1 - W_2(s))} \quad (9)$$

From building the transfer function of two layers slab, we can build the transfer function of the n layers slab as follows.

$$W_n(s) = \frac{1}{R_n C_n s + 1}$$

$$W_{n-1}(s) = \frac{1}{1 + R_{n-1} C_{n-1} s + \frac{R_{n-1}}{R_n}(1 - W_n(s))}$$

…

$$W_2(s) = \frac{1}{1 + R_2 C_2 s + \frac{R_2}{R_3}(1 - W_3(s))}$$

$$W_1(s) = \frac{1}{1 + R_1 C_1 s + \frac{R_1}{R_2}(1 - W_2(s))}$$

With $R_1 = 1/A\alpha; R_2 = \frac{d/n}{\lambda_1 A};_2 R_3 = \frac{d/n}{\lambda_2 A}; \ldots; R_n = \frac{d/n}{\lambda_{n-1} A}$

The value n is chosen according to the "thickness" of the slab and the accuracy requirement when describing the thermal conductivity in the slab.

III. SIMULATE THE TEMPERATURE OF THE SLAB BY THE TRANSFER FUNCTION

To test the accuracy and correctness of the transfer function model of the slab, the authors selected the slab with the following parameters:

Thermal conductivity: λ = 55.8 w/m.K
Heat transfer coefficient: ρ = 7800kg/m^3
Specific weight: c = 460 j/kg.K
Specific heat: α = 335 w/m^2
Length: a = 0.6 m
Width: b = 0.3 m
Thickness: d = 0.06 m
Surface area of the slab is : A = a*b = 0.6*0.3 = 0.18 m^2
In this study, we divide the slab into 3 layers. We have the thickness of each layer is $d/3 = 0.02 \ (m^2)$

The volume of each layer is $V_1 = V_2 = V_3 = 0.6 * 0.3 * 0.02 = 0.0036 \ (m^3)$

The mass of each layer is $m_1 = m_2 = m_3 = V_1 * \rho = 0.0036 * 7800 = 28.08$ kg

$C_1=C_2=C_3=m_1*c=28.08*460=12916.8$

$$R_1=\frac{1}{A\alpha}=\frac{1}{0.18*335}=0.0165$$

$$R_2=R_3=\frac{l}{\lambda A}=\frac{d/3}{\lambda A}=\frac{0.02}{55.8*0.18}=0.00199$$

The transfer function of each layer of the slab is as follows

$$W_3(s)=\frac{1}{R_3C_3s+1}=\frac{1}{25.72s+1}$$

$$W_2(s)=\frac{1}{1+R_2C_2s+\frac{R_2}{R_3}(1-\mathrm{W}_3(\mathrm{s}))}=\frac{25.72s+1}{661.5s^2+77.16s+1}$$

$$W_1(s)=\frac{1}{1+R_1C_1s+\frac{R_1}{R_2}(1-\mathrm{W}_2(\mathrm{s}))}$$

$$=\frac{661.5s^2+77.16s+1}{1.417.10^5s^3+2.27.10^4.4s^2+719.8s+1}$$

Performing Matlab - Simulink simulation of transfer function of the slab as follows

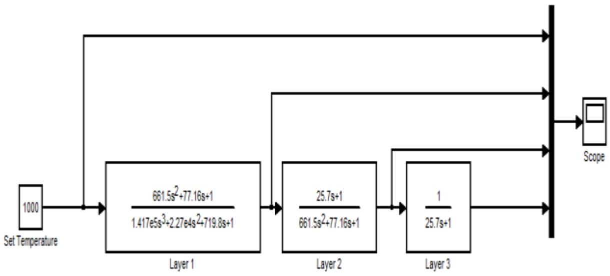

Fig 4:- The transfer function model of slab in Matlab – Simulink

Assuming the furnace temperature (set temperature) is kept constant Tf = 1000oC, we obtain the simulation result as follows

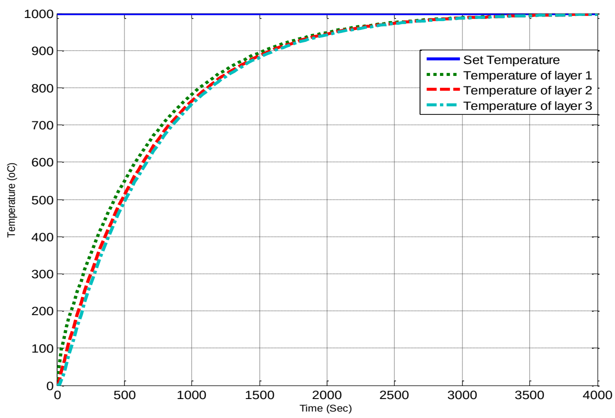

Fig 5:- The temperature of the layers of the slab

As shown in Fig. 5, after 2000s, the temperature of the three layers of the slab reached almost the same level.

Comparing the transfer function model of the slab with the differential model of the slab, we obtained the following results.

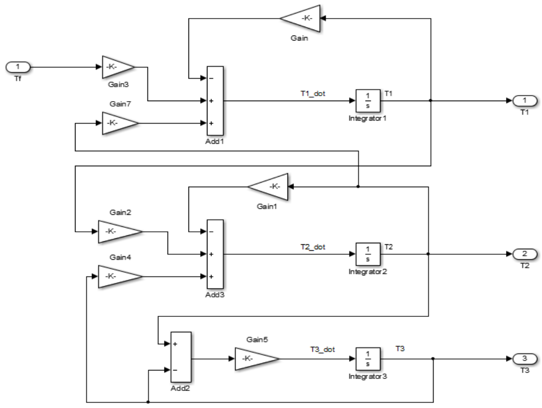

Fig 6:- The differential model of the slab in Matlab – Simulink

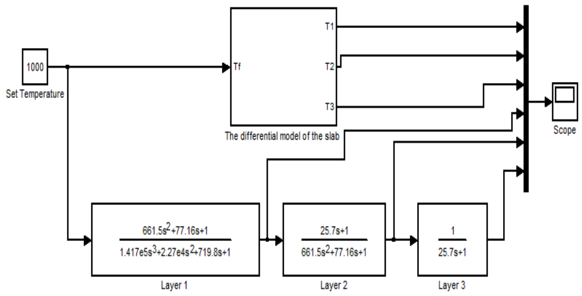

Fig 7:- The transfer function model and the differential model of the slab in Matlab – Simulink

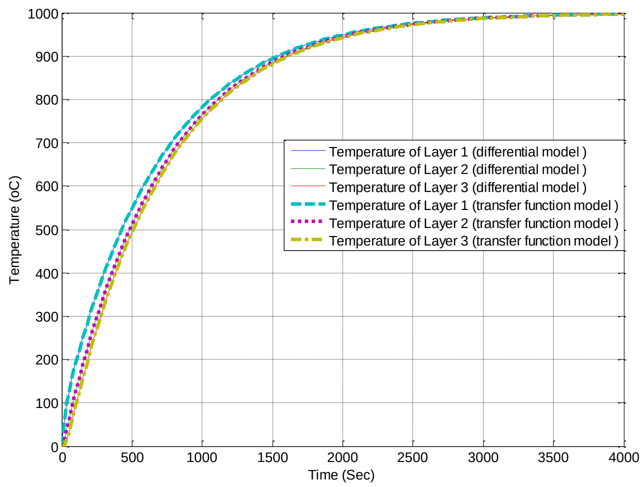

Fig 8:- The temperature of the layers of the slab

As shown in Fig. 8, the response of the transfer function of the slab coincides completely with the response of the differential model of the slab.

Thus, the transfer function of the slab is completely accurate. The transfer function of the slab have allowed the design of the plate temperature controller to be easier.

IV. CONCLUSION

The paper presents the method of constructing the mathematical model of the slab by the transfer function method. Based on the transfer function of the slab, it is easy to investigate the temperature variation in the slab and to make it easier to design the slab temperature controller. The simulated results show the correctness of the mathematical form of the transfer function of the slab. To test the practical applicability of this study we need to experiment on the real model that the results of this study will have very high practical significance.

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