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One Raised Product Prime Labeling of some Tree Graphs

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Abstract:- A graph in which vertices are labeled with first p natural numbers, where p is the number of vertices of the graph and the edges are labeled with one plus product of the labels of the incident vertices is said to admit one raised product prime labeling, if the greatest incidence number of each vertex of degree one is one.

Keywords:- *Graph Labeling, Product, Prime Labeling, Prime Graph, Tree.*

I. INTRODUCTION

In this paper, we consider only finite, connected and circuit less graphs. Number of vertices of the graph is p and number of edges of the graph is q. Labeling of a graph is an allotment of integers to the vertices or edges or both. Product labeling of a graph is the labeling of the edges with product of the end vertex values. One raised product labeling is the labeling of the edges with one plus product of the end vertex values.A labeled graph is said to admit prime labeling, if the greatest common incidence number of each vertex of degree one is one.

Some basic notations and definitions are taken from [1],[2],[3] and [4] . Some basic concepts are taken from Frank Harary [1]. In this paper we investigated one raised product prime labeling of some tree graphs

➢ *Definition: 1.1*

Let G be a graph with p vertices and q edges. The greatest common incidence number (***gcin***) of a vertex of degree greater than or equal to 2, is the greatest common divisor (gcd) of the labels of the incident edges.

II. Main Results

➢ *Definition 2.1*

Let G = (V,E) be a graph with p vertices and q edges . Define a bijection f : V → {1,2,…, p} by $f(v_i) = i$, $1 \le i \le p$ and define a 1-1 mapping f^*_{orpp} : E → set of natural numbers N by $f^*_{orpp}(uv) = f(u)f(v) + 1$ .The induced function f^*_{orpp} is said to be one raised product prime labeling, if for each vertex of degree at least 2, the ***gcin*** of the labels of the incident edges is 1.

➢ *Definition 2.2*

A graph which admits one raised product prime labeling is called one raised product prime graph.

- **Theorem 2.1** Comb graph is one raised product prime graph, if (n-1)≢0(mod3).

Proof:
Let G be the graph and let $v_1, v_2, \ldots, v_{2n}$ are the vertices of G.
Here |V(G)| = 2n and |E(G)| = 2n-1.
Define a function f : V → {1,2,…,2n} by
$f(v_i) = i$, $1 \le i \le 2n$.
Edge labeling f^*_{orpp} is defined as follows

$f^*_{orpp}(v_{2i-1}\, v_{2i+1}) = 4i^2$, $1 \le i \le n-1$.
$f^*_{orpp}(v_{2i-1}\, v_{2i}) = 4i^2-2i+1$, $1 \le i \le n$.
Clearly f^*_{orpp} is 1-1.

gcin of (v_{2i-1}) = gcd of $\{f^*_{orpp}(v_{2i-1}\, v_{2i+1}), f^*_{orpp}(v_{2i-1}\, v_{2i})\}$
= gcd of $\{4i^2, 4i^2-2i+1\}$
= gcd of $\{2i, 2i(2i-1)+1\}$
= 1, $1 \le i \le n-1$.

gcin of (v_{2n-1}) = gcd of $\{f^*_{orpp}(v_{2n-3}\, v_{2n-1}), f^*_{orpp}(v_{2n-1}\, v_{2n})\}$
= gcd of $\{(2n-2)^2, 4n^2-2n+1\}$
= gcd of $\{2n-2, 4n^2-2n+1\}$
= gcd of $\{n-1, 4n^2-2n+1\}$
= gcd of $\{n-1, 2n+1\}$
= gcd of $\{3, n-1\}$
= 1.

Hence $P_n \Theta K_1$, is one raised product prime graph.

Example 2.1 G = $P_5 \Theta K_1$

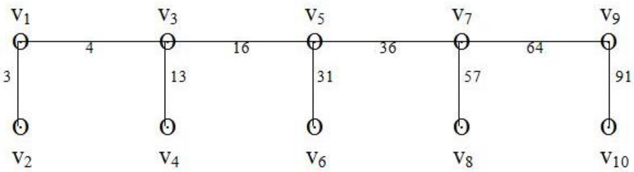

Fig 1

- **Theorem 2.2** Centipede graph is one raised product prime graph.

Proof:
Let G be the graph and let $v_1, v_2, \ldots, v_{3n}$ are the vertices of G.
Here |V(G)| = 3n and |E(G)| = 3n-1.
Define a function f : V → {1,2,…,3n} by
$f(v_i) = i$, $1 \le i \le 3n$.
Edge labeling f^*_{orpp} is defined as follows

$f^*_{orpp}(v_{3i-2}\, v_{3i-1}) = 9i^2-9i+3$, $1 \le i \le n$.
$f^*_{orpp}(v_{3i-1}\, v_{3i}) = 9i^2-3i+1$, $1 \le i \le n$..
$f^*_{orpp}(v_{3i-1}\, v_{3i+2}) = 9i^2+3i-1$, $1 \le i \le n-1$.
Clearly f^*_{orpp} is 1-1.

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gcin of (v_{3i-1}) $= \gcd$ of $\{f^*_{orpp}(v_{3i-2}\ v_{3i-1}), f^*_{orpp}(v_{3i-1}\ v_{3i})\}$
$= \gcd$ of $\{9i^2-9i+3, 9i^2-3i+1\}$
$= \gcd$ of $\{6i-2, 9i^2-9i+3\}$
$= \gcd$ of $\{3i-1, (3i-1)(3i-2)+1\}$
$= 1, \quad 1 \le i \le n.$

Hence G, is one raised product prime graph.

Example 2.2 G = $P_4 \odot 2K_1$

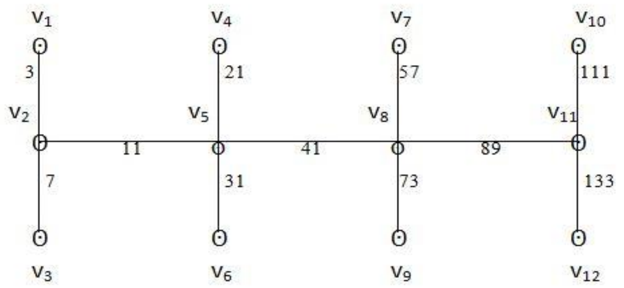

Fig 2

- **Theorem 2.3** Twig graph is one raised product prime graph.

Proof:
Let G be the graph and let $v_1,v_2,\ldots,v_{3n-4}$ are the vertices of G.
Here $|V(G)| = 3n-4$ and $|E(G)| = 3n-5$.
Define a function $f : V \rightarrow \{1,2,\ldots,3n-4\}$ by
$f(v_i) = i, \ 1 \le i \le 3n-4.$

For the vertex labeling f, the induced edge labeling f^*_{orpp} is defined as follows

$f^*_{orpp}(v_1\ v_3) = 4.$
$f^*_{orpp}(v_{3i-1}\ v_{3i}) = 9i^2-3i+1, \quad 1 \le i \le n-2.$
$f^*_{orpp}(v_{3i}\ v_{3i+1}) = 9i^2+3i+1, \quad 1 \le i \le n-2.$
$f^*_{orpp}(v_{3i}\ v_{3i+3}) = 9i^2+9i+1, \quad 1 \le i \le n-3.$
$f^*_{orpp}(v_{3n-4}\ v_{3n-5}) = 9n^2-27n+21.$

Clearly f^*_{orpp} is an 1-1.

gcin of (v_{3i}) $= \gcd$ of $\{f^*_{orpp}(v_{3i-1}\ v_{3i}), f^*_{orpp}(v_{3i}\ v_{3i+1})\}$
$= \gcd$ of $\{9i^2-3i+1, 9i^2+3i+1\}$
$= \gcd$ of $\{6i, 9i^2-3i+1\}$
$= \gcd$ of $\{3i, 3i(3i-1)+1\}$
$= 1, \quad 1 \le i \le n-2.$

Hence G is one raised product prime graph.

Example 2.3 G be the twig graph of path P_4.

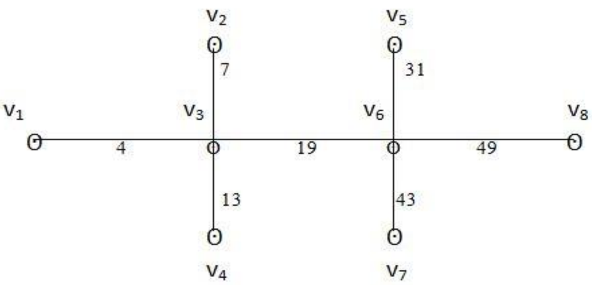

Fig 3

- **Theorem 2.4** Coconut tree graph is one raised product prime graph.

Proof:
Let G = CT(m,n) be the graph and let $v_1,v_2,\ldots,v_{m+n}$ are the vertices of G.
Here $|V(G)| = m+n$ and $|E(G)| = m+n-1$.
Define a function $f : V \rightarrow \{1,2,\ldots,m+n\}$ by
$f(v_i) = i, \ 1 \le i \le m+n.$

Edge labeling f^*_{orpp} is defined as follows

$f^*_{orpp}(v_i\ v_{i+1}) = i^2+i+1, \quad 1 \le i \le m-1.$
$f^*_{orpp}(v_m\ v_{m+i}) = m^2+mi+1, \quad 1 \le i \le n.$

Clearly f^*_{orpp} is an 1-1.

gcin of (v_{i+1}) $= \gcd$ of $\{f^*_{orpp}(v_i\ v_{i+1}), f^*_{orpp}(v_{i+1}\ v_{i+2})\}$
$= \gcd$ of $\{i^2+i+1, i^2+3i+3\}$
$= \gcd$ of $\{2i+2, i(i+1)+1\}$
$= \gcd$ of $\{i+1, i(i+1)+1\}$
$= 1, \quad 1 \le i \le m-2.$

gcin of (v_m) $= \gcd$ of $\{f^*_{orpp}(v_{m-1}\ v_m), f^*_{orpp}(v_m\ v_{m+1})\}$
$= \gcd$ of $\{m^2-m+1, m^2+m+1\}$
$= \gcd$ of $\{2m, m^2-m+1\}$
$= \gcd$ of $\{m, m(m-1)+1\}$
$= 1.$

Hence G is one raised product prime graph.

Example 2.4 G = CT(4,3)

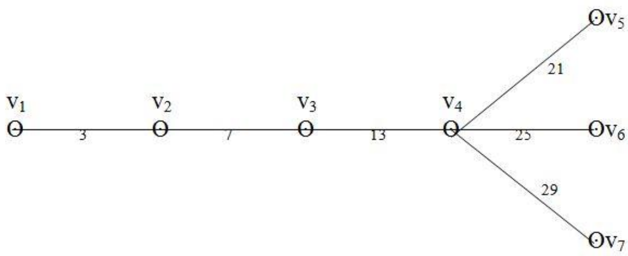

Fig 4

- **Theorem 2.5** Double coconut tree graph is one raised product prime graph.

Proof:
Let G = DCT(n,m,n) be the graph and let $v_1,v_2,\ldots,v_{m+2n}$ are the vertices of G.
Here $|V(G)| = m+2n$ and $|E(G)| = m+2n-1$.
Define a function $f : V \rightarrow \{1,2,\ldots,m+2n\}$ by
$f(v_i) = i, \ 1 \le i \le m+2n.$

Edge labeling f^*_{orpp} is defined as follows

$f^*_{orpp}(v_i\ v_{n+1}) = in+i+1, \quad 1 \le i \le n.$
$f^*_{orpp}(v_{n+i}\ v_{n+i+1}) = (n+i)^2+n+i+1, \quad 1 \le i \le m-1.$
$f^*_{orpp}(v_{m+n}\ v_{m+n+i}) = (m+n)^2+i(m+n)+1, \quad 1 \le i \le n.$

Clearly f^*_{orpp} is 1-1.

gcin of (v_{n+i}) $= \gcd$ of $\{f^*_{orpp}(v_{n+i-1}\ v_{n+i}), f^*_{orpp}(v_{n+i}\ v_{n+i+1})\}$
$= \gcd$ of $\{(n+i)^2-(n+i)+1,$

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$(n+i)^2+(n+i)+1\}$
$= \gcd \text{ of } \{2(n+i), (n+i)^2-(n+i)+1\}$
$= \gcd \text{ of } \{n+i, (n+i)(n+i-1)+1\}$
$= 1, \quad 1 \le i \le m.$

Hence G is one raised product prime graph.

Example 2.5 G = DCT(3,4,3)

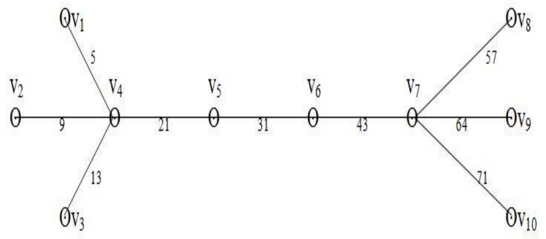

Fig 5

- **Theorem 2.6** H graph of path P_n is one raised product prime graph, if n ≡ 0(mod4) and n is even,

Proof:
Let G = H(P_n) be the graph and let $v_1,v_2,\ldots,v_{2n}$ are the vertices of G.
Here |V(G)| = 2n and |E(G)| = 2n-1.
Define a function f : V → {1,2,…,2n} by
$f(v_i) = i, \; 1 \le i \le 2n.$
Edge labeling f_{orpp}^* is defined as follows

$f_{orpp}^*(v_i\, v_{i+1}) = i^2+i+1, \quad 1 \le i \le n-1.$
$f_{orpp}^*(v_{n+i}\, v_{n+i+1}) = (n+i)^2+n+i+1, \quad 1 \le i \le n-1.$
$f_{orpp}^*(v_{\frac{n+2}{2}}\, v_{\frac{3n+2}{2}}) = \frac{(n+2)(3n+2)}{4} + 1.$

Clearly f_{orpp}^* is 1-1.

gcin of (v_{i+1}) $= \gcd \text{ of } \{f_{orpp}^*(v_i\, v_{i+1}), f_{orpp}^*(v_{i+1}\, v_{i+2})\}$
$= \gcd \text{ of } \{i^2+i+1, i^2+3i+3\}$
$= \gcd \text{ of } \{2i+2, i(i+1)+1\}$
$= \gcd \text{ of } \{i+1, i(i+1)+1\}$
$= 1, \quad 1 \le i \le n-2.$

gcin of (v_{n+i+1}) $= \gcd \text{ of } \{f_{orpp}^*(v_{n+i}\, v_{n+i+1}), f_{orpp}^*(v_{n+i+1}\, v_{n+i+2})\}$
$= \gcd \text{ of } \{(n+i)^2+(n+i)+1, (n+i)^2+3(n+i)+3\}$
$= \gcd \text{ of } \{2(n+i)+2, (n+i)^2+(n+i)+1\}$
$= \gcd \text{ of } \{n+i+1, (n+i)(n+i+1)+1\}$
$= 1, \quad 1 \le i \le n-2.$

Hence G is one raised product prime graph.

Example 2.6 G = H(P_4)

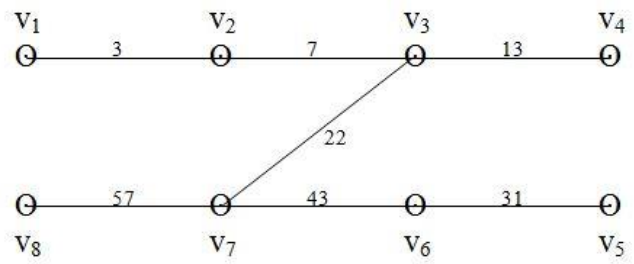

Fig 6

- **Theorem 2.7** Hurdle graph Hd_n is one raised product prime graph.

Proof :
Let G = Hd_n and let $v_1,v_2,\ldots,v_{2n-2}$ are the vertices of G.
Here |V(G)| = 2n-2 and |E(G)| = 2n-3.
Define a function f : V → {1,2,…,2n-2} by
$f(v_i) = i, \; 1 \le i \le 2n-2.$
Edge labeling f_{orpp}^* is defined as follows

$f_{orpp}^*(v_1\, v_2) = 3.$
$f_{orpp}^*(v_{2i}\, v_{2i+1}) = 4i^2+2i+1, \quad 1 \le i \le n-2.$
$f_{orpp}^*(v_{2i}\, v_{2i+2}) = 4i^2+4i+1, \quad 1 \le i \le n-2.$

Clearly f_{orpp}^* is 1-1.

gcin of (v_{2i}) $= \gcd \text{ of } \{f_{orpp}^*(v_{2i}\, v_{2i+1}), f_{orpp}^*(v_{2i}\, v_{2i+2})\}$
$= \gcd \text{ of } \{4i^2+2i+1, 4i^2+4i+1\}$
$= \gcd \text{ of } \{2i, 4i^2+2i+1\}$
$= \gcd \text{ of } \{2i, 2i(2i+1)+1\}$
$= 1, \quad 1 \le i \le n-2.$

Hence G is one raised product prime graph.

Example 2.7 G = Hd_5.

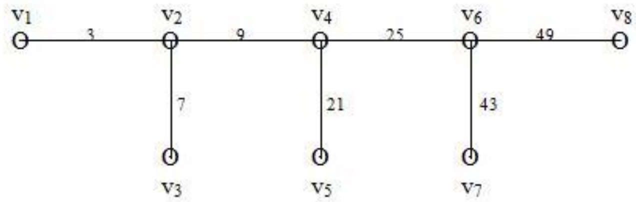

Fig 7

- **Theorem 2.8** Let G be the graph obtained by joining pendant edges alternately to the vertices of path P_n(n > 3). G is denoted by the symbol P_n Θ A(K_1). G is one raised product prime graph, if n is odd and pendant edges start from the first vertex.

Proof :
Let G be the graph and let $v_1,v_2,\ldots,v_{\frac{3n+1}{2}}$ are the vertices of G.
Here |V(G)| = $\frac{3n+1}{2}$ and |E(G)| = $\frac{3n-1}{2}$.
Define a function f : V → $\{1,2,\ldots,\frac{3n+1}{2}\}$ by
$f(v_i) = i, \; i = 1,2,\ldots,\frac{3n+1}{2}.$
For the vertex labeling f, the induced edge labeling f_{orpp}^* is defined as follows

$f_{orpp}^*(v_1\, v_2) = 3.$

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$f^*_{orpp}(v_2\ v_3)$ = 7.

$f^*_{orpp}(v_{3i}\ v_{3i+1})$ = $9i^2+3i+1$, $i = 1,2,\ldots,\frac{n-1}{2}$.

$f^*_{orpp}(v_{3i+1}\ v_{3i+2})$ = $9i^2+9i+3$, $i = 1,2,\ldots,\frac{n-1}{2}$.

$f^*_{orpp}(v_{3i+1}\ v_{3i+3})$ = $9i^2+12i+4$, $i = 1,2,\ldots,\frac{n-3}{2}$.

Clearly f^*_{orpp} is 1-1.

gcin of (v_2) = gcd of $\{f^*_{orpp}(v_1\ v_2), f^*_{orpp}(v_2\ v_3)\}$
= gcd of {3,7}
= 1.

gcin of (v_3) = gcd of $\{f^*_{orpp}(v_2\ v_3), f^*_{orpp}(v_3\ v_4)\}$
= gcd of {7,13}
= 1.

gcin of (v_{3i+1}) = gcd of $\{f^*_{orpp}(v_{3i}\ v_{3i+1}), f^*_{orpp}(v_{3i+1}\ v_{3i+2})\}$
= gcd of $\{9i^2+3i+1, 9i^2+9i+3\}$
= gcd of $\{6i+2, 9i^2+3i+1\}$
= gcd of $\{3i+1, 3i(3i+1)+1\}$
= 1, $1 \leq i \leq \frac{n-1}{2}$.

gcin of (v_{3i+3}) = gcd of

$\{f^*_{orpp}(v_{3i+1}\ v_{3i+3}), f^*_{orpp}(v_{3i+3}\ v_{3i+4})\}$
= gcd of $\{9i^2+12i+4, 9i^2+21i+13\}$
= gcd of $\{9i+9, 9i^2+12i+4\}$
= gcd of $\{3i+4, 9i+9\}$
= gcd of $\{3i+1, 3i+4\}$
= gcd of $\{3, 3i+1\}$
= 1, $i = 1,2,\ldots,\frac{n-3}{2}$.

Hence G is one raised product prime graph.

Example 2.8 G = $P_7 \ \Theta \ A(K_1)$.

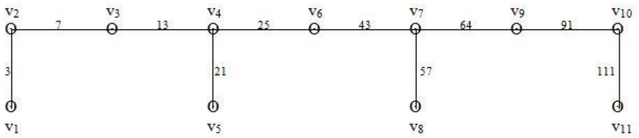

Fig 8

- **Theorem 2.9** Let G be the graph obtained by joining pendant edges alternately to the vertices of path P_n(n > 3) G is denoted by the symbol $P_n \ \Theta \ A(K_1)$. G is one raised product prime graph, if n is even and pendant edges start from the first vertex.

Proof :

Let G be the graph and let $v_1, v_2, \ldots, v_{\frac{3n}{2}}$ are the vertices of G.

Here $|V(G)| = \frac{3n}{2}$ and $|E(G)| = \frac{3n-2}{2}$.

Define a function $f : V \to \{1,2,\ldots,\frac{3n}{2}\}$ by

$f(v_i) = i$, $1 \leq i \leq \frac{3n}{2}$.

Edge labeling f^*_{orpp} is defined as follows

$f^*_{orpp}(v_1\ v_2)$ = 3.

$f^*_{orpp}(v_2\ v_3)$ = 7.

$f^*_{orpp}(v_{3i}\ v_{3i+1})$ = $9i^2+3i+1$, $1 \leq i \leq \frac{n-2}{2}$.

$f^*_{orpp}(v_{3i+1}\ v_{3i+2})$ = $9i^2+9i+3$, $1 \leq i \leq \frac{n-2}{2}$.

$f^*_{orpp}(v_{3i+1}\ v_{3i+3})$ = $9i^2+12i+4$, $1 \leq i \leq \frac{n-2}{2}$.

Clearly f^*_{orpp} is 1-1.

gcin of (v_2) = gcd of $\{f^*_{orpp}(v_1\ v_2), f^*_{orpp}(v_2\ v_3)\}$
= 1.

gcin of (v_3) = gcd of $\{f^*_{orpp}(v_2\ v_3), f^*_{orpp}(v_3\ v_4)\}$
= 1.

gcin of (v_{3i+1}) = gcd of $\{f^*_{orpp}(v_{3i}\ v_{3i+1}), f^*_{orpp}(v_{3i+1}\ v_{3i+2})\}$
= 1, $1 \leq i \leq \frac{n-2}{2}$.

gcin of (v_{3i+3}) = gcd of

$\{f^*_{orpp}(v_{3i+1}\ v_{3i+3}), f^*_{orpp}(v_{3i+3}\ v_{3i+4})\}$
= 1, $1 \leq i \leq \frac{n-4}{2}$.

Hence G is one raised product prime graph.

Example 2.9 G = $P_6 \ \Theta \ A(K_1)$.

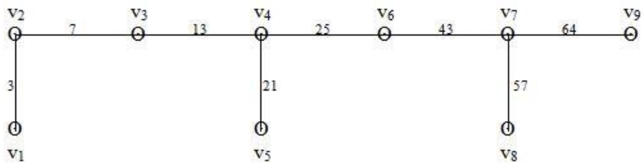

Fig 9

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