The Simple Linier Modeling of Habitable Zone for the Main Sequence Stars

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Abstract:- The discovery of extrasolar planets, especially the habitable planets to be one of the most exciting and continuous studies has developed rapidly hitherto. The habitable planet must be in a habitable zone. The habitable zone that has been modeled previously has a non-linear form by considering some parameters of albedo, temperature, surface pressure, semi-major axis, starfish mass, spectral star type, and so on. In this model, we simplify the previous model into a linear model and determine the mathematical equations for inner and outer boundaries of habitable zone, we obtain 0.54 SA for inner boundary and 1.66 SA for outer borders only considering the semi-major axis, the mass of the parent star, and spectral type of the parent star. This linear model has a high accuracy in predicting the habitable planets so that the results do not vary much with the previous non-linear models.

Keywords:- Habitable zone, Habitable Planet, Linear Model, Non-linear Model, Extrasolar Planet.

I. INTRODUCTION

The search for extrasolar planets, especially planets located within the habitable zone and its life has become the most interesting and progressive discussion (Bean et al, 2010). Over time, research on extrasolar planets, specifically using NASA's Kepler mission, has revealed that generally dwarf planet are habitable (Williams et al, 1997; Raymond et al, 2006; Petigura et al, 2013; Burke et al, 2015; Dressing and Charbonneau, 2015;), although there are large planets categorized in it (Bloh et al, 2009; Heller and Armstrong, 2014).

Habitable zone is defined as an area that allows the planets to retain water in a liquid phase so as to support the existence and survival of living things in their environment. (Huang, 1959; Hart, 1978; Kasting et al, 1993; Selsis et al, 2007; Kopparapu et al, 2013; Kane et al, 2013; Ramirez and Kaltenegger, 2016; Gonzalez, 2014), but not all habitable planets can be inhabited by living things because to be able to fully support life so that it can be called habitable, the planet must be filled up the other requirements. (Seager, 2013). To find out how many planets are trapped in the habitable zone, some researchers have modeled the inner and outer limits of the habitable zone. Since the mass and luminosity of the star increase over time, no habitable zone model is static in space and time (Rushby et al, 2013; Ramirez and Kaltenegger, 2016).

Many of the estimated value is debatable, and finally come to various estimates on the value of the limit in inner and the outer of the habitable zone. According to the calculations of the Stellar Habitable Zone Calculator from the University of Washington, the estimate of habitable zones in the solar system is 0.5 - 3 AU. Dole (1964) argues that the inner limit of the habitable zone is 0.725 AU assuming the thin atmosphere and albedo is fixed. In this case, Venus placed in the aphelion. Hart (1979) claimed that the inner limit on 0.96 AU around the sun like stars. The previous model found in the limits of the habitable zone is located at a distance of 0.88 AU for Earth-like planet with a pressure of 10 bar N₂ (Rasool and De Bergh. 1970). Another case with Gonzalez (2014) that specify the limits of inner and outer are respectively 0.95 AU and 1.37 AU, this model is based on the greenhouse effect. maximizing the cooling effect of clouds in Earth-like planets, the inner limit is known located at 0.87 SA around the sun according to the 1D cloud model. Using the energy equilibrium model and various surface pressures between 0.3 and 3 bars, Vladilo et al (2013) shows that the inner limit lies at 0.85 AU. Abe (2011) argues that the limits in the habitable zone of the Sun like stars lie at 0.77 SA for Earth-like planets with water reservoir boundaries based on 3D circulation models. Kopparapu (2013) fixes the estimate of habitable zone boundaries by adding the algorithm of the greenhouse effect and water loss. With this estimate, the limit in the habitable zone located in 0.99 AU. Taking into account the albedo and greenhouse effect, LoPresto and Ochoa (2017) get the limits in the habitable zone around the Sun like stars at 0.45 AU.

Here we focus the habitable zone for stars around the main sequence stars. Using data from the extrasolar planet catalogs and re-analyzing some parameters as criteria for habitable planets, we have found simple models and equations that can be used as models to acquire the planets in the habitable zone.

We have categorized the habitable planets into two types, the planets in the habitable zone and Earth-like planets. Which will be discussed in this article are specific to the habitable zone of a simple will determine the position of the habitable planet, which will be the initial capital for the determination of Earth-like planets. We describe our modeling method in chapter 2 and determine the formula for outer boundary, inner boundaries, and simple model of habitable zone in chapter 3, discussions in chapter 4, and conclusions to be discussed in chapter 5.

II. METHOD

As explained previously, habitable planet is a planet that is possible to accommodate the estimated life, but still there are gas planets or dwarf planets therein. The planet is already supposed to be located in the habitable zone in their solar system. In the process of finding habitable planets will be making a simple model from the previous model (Selsis et al, 2007; Kopparapu et al, 2013). Therefore, we can be made stages to find the habitable planets as follows.

* The Model of Habitable Zone

A. Making the Star Mass vs Semi Major Axis Diagram

In the diagrams that we make or that already exist, the reference star is the sun and the planets that are referenced are the Nine planets in our solar system. Figure 1 shows a diagram created by Kasting et al (1993) dan kopparapu, et al (2014). The diagram that make by Waltham (2017) which is a basic diagram is generally used in modeling the habitable zone, also used as a reference model in our model. However, not all use the same parameters. The model made by Braun et al (2011) states that in particular the habitable zone can be determined by the size of the star and its surface temperature.



Fig. 1:- The diagram illustrates the habitable zone in the Zero Age Main Sequence (ZAMS). The stars in this series can be determined habitable zone by using parameters such as star mass and distance (AU).

In the process of searching the planet habitable, the mass of the sun and the sun's class must be determined first to know the central point of the habitable zone, so that the planets in the solar system can be aligned according to the distance of each planet and the location of the star. After all arranged, then the mass of the diagram can be formed semimajor axis vs. planet to be used in subsequent processes.

B. Finding the Habitable Zone

In the process of finding a habitable zone, is requires a star mass diagram versus the semi-major axis of a previously described. But in this second phase, what should be examined is the distance or semi-major axis at the center of the habitable zone in our solar system. As it is commonly known, the planets in the habitable zone of our solar system are Venus, Earth, and Mars. The planets make our habitable zone at the interval distance of 0.7 AU - 1.5 AU, even according to Szom et al (2013) its inner limit could be closer to 0.38 AU. This data will be the key to finding the habitable zone of extrasolar planets. After finding the key of the habitable zone, it can be made a line that separates between the planets in the habitable zone with the planets beyond the habitable zone. The line is a transverse trapezoid. Before determining the equation of the line, the transverse trapezoid must be determined its coordinates based on the mass of the star and the distance of the planet. The coordinates will be used to determine the line equation of the inner and outer values of the habitable zone of the extrasolar planets. The line equation can be determined by ordinary mathematical processes.

C. Determining the Inner and Outer Values of the Habitable Zone

Nakajima et al (1992) determines the inner edge of a habitable zone based on the planet's greenhouse effect that will help evaporate water into the atmosphere. This reason also will help us to define the deep limits of the habitable zone for the main sequence stars. Some models have been made by some researchers and show little resemblance between one model and another (Dole, 1964; Hart, 1979; Rasool dan De Bergh, 1970; Gonzalez, 2014; Vladilo et al, 2013; Abe, 2011; Kopparapu, 2013; Lo Presto dan Ochoa, 2017). To simplify the modeling, we make the models through the line equations that have been made in the second step, so it is necessary in determining this value. Two parallel lines in the transverse trapezoid already illustrated will help to show the inner and outer positions of the extrasolar planets to which already determine the value. Before the inner and outer positions are known, the existing extrasolar planet data must be grouped by the mass of its parent star (mass more than 1 or less than 1) for completion of the next step.

D. The Addition of the Area of the Habitable Zone

This stage aims to see how far can still be accommodated by habitable planet. By using a mathematical process that is still quite simple, we can be explained the process of adding the area in the habitable zone of extrasolar planet. The required data is the semimajor axis of the central planet. We are using data from semi major axis of Venus and Mars. This data will be searched for the difference to find out the width of the habitable zone of the solar system. The value of increment will be increased by 10% for inner and 10% for outer, and then will result in inner and outer expansion values.

E. Determining the Planet in the Habitable Zone

To find planets located in the habitable zone, semimajor axis of the extrasolar planets that have been recorded in the extrasolar planet catalog should be known. This is the last step for searching planets located in the habitable zone. The semi-major axis data will be used to determine whether the planets are trapped within the inner and outer limits in the habitable zone of the solar system. Solar system that are known to be trapped in the habitable zone must then be re-recorded in the new catalog. The data from this new catalog will be used to create a development model for the previous habitable zone model, before it will eventually be recreated into a histogram to see the progress of a habitable planet.

F. Re-modeling the Habitable Zone

Re-modeling the habitable zone requires the mass data of the parent star and the semi-major axis data of the extrasolar planets. This re-modeling aims to see how the habitable zone lines formed with data - data that has been in though with the four previous step. This stage is done with the help of graphics from Microsoft Excel and change the graphical model to logarithm to re-generate two parallel lines that are trapezoidal. Its only that the shape and the resulting slope are different from previous models.

The parameters used and the impact on the modeling of the habitable zone we made a qualitative description of some parameters that required to modeling this simple habitable zone. Considering the mass of the parent star, the semi-major axis, the stellar type. The following will be explained one by one about these parameters.

> Mass of the Parent Star

The mass of the parent star is one of the important parameters that affect the location of the habitable zone. In modeling the sun's habitable zone, we use the mass of stars by assuming that the larger the mass of a star, the habitable zone distance will be further or it can be said that the mass of a star is proportional to the emitted energy. If the mass of the parent star increases, the habitable zone is away from it.

The relationship between the mass of the star and the semi-major axis in the main sequence is given by the equation Waltham (2017)

$$\overline{a} = 1.3 \, \left(\frac{M}{M_{\odot}}\right)^{2.03} \tag{1}$$

Semi Major Axis

A determining factor for the planet in a solar system habitable or not is seen from the side of the semi major axis. If it as seen from the distance of Venus, Earth, and Mars respectively are 0.7, 1, and 1.5 SA, it can be calculated that the three planets never receive too high flux from their parent star, so it can be said that the effective distance of the planet to retain water on the surface of a habitable planet located around the main sequence star is to make Venus and Mars as the base (Elkins – Tanton, 2011; Gomes et al, 2005).

III. RESULT

According to the definition made by Huang (1959) the habitable zone is the zone where the planets can retain water in liquid phase. Instead, the habitable zone is implicitly defined as the star-planet split region where conditions permit processes that lead to some conditions. The process can be conditioned to be warm enough to sustain liquid water. Therefore, the location of the outer limits of the habitable zone will be identical to the two definitions. Values for the inner limit can result in different assumptions, since the maximum temperature for sustaining life is likely to be less than 60 ° C. According to Kasting et al (1993), the temperature for maintaining a greenhouse is not more than 60° C. Therefore, the inner limit of the habitable zone also does not differ significantly between the two definitions. We use these definitions and a simple mathematical model and assume that the inner and outer boundaries of the habitable zone which is two parallel lines it can then be made new equation to determine the limits of the habitable zone.

$$M_{\odot} = \frac{M'_{\odot}}{d_{Ven} - d_i} (i - d_i)$$
⁽²⁾

where i is the limit in the habitable zone of sun (AU), d_{ven} Venus is a semi-major axis (AU), and d_i is the boundary line in which intersect with the line separation between stars - planets (AU).

As for the outer boundary of the Sun's habitable zone, used the semi-major axis of Mars as a boundary. By using the same method to determine the limit, then obtained a simple equation for the outer limits of the Sun's habitable zone is as follows.

$$M_{\odot} = \frac{M_{O}}{d_{Mars} - d_o} \left(o - d_o \right)$$
(3)

With do is the outer perimeter that intersects the semimajor axis (AU) and d_{Mars} is the value of the semi-major axis of Mars (AU). Both di and do are fixed, depending on the intersecting location of the line with the semi-major axis.

From the diagram known that the di is 0.0002 AU and do 0.0005 AU. Knowing the value of the major axes of Venus and Mars is 0.7 AU and 1.5 AU, then a new equation can be made to find out the inner limit of the extrasolar planetary habitable zone as follows.

$$i = i = (d_{Ven} - d_i) M_{\odot} + d_i$$
 (4)

or can be rewritten

$$i = i = K_a M_{\odot} + d_i \tag{5}$$

 K_a is a constant value to the limit in the amount of habitable zone is 0.6998 AU/kg and to the outer limit of the habitable zone exoplanet can be created also a new simple equation as follows.

$$o = (d_{Mars} - d_o) M_{\odot} + d_o \tag{6}$$

or can be rewritten

$$o = o = K_b M_{\odot} + d_o \tag{7}$$

With Kb is the value of the boundary constant in the habitable zone whose the value is 1.4995 AU/kg. This simple equation can be used to find the value of inner and outer boundaries of other solar systems with varying conditions. By using a simple equation, it can be modeled to be habitable zone exoplanet with a logarithmic scale as in Figure 3b.

However, by reconsidering the surface temperatures of Venus and the Aphelium Distance of Venus used as a constraint, we add the distance for inner and outer boundary values of 0.16 AU respectively. From the diagram known that di value is 0.0002 AU and do is 0.0005 AU. If the value of semi major axis of Venus and Mars is 0.7 AU and 1.5 AU, then when our addition of inner and outer values of the habitable zone of 0.16 AU in obtaining new values for semi-major axis used in this model is 0.54 AU and 1.66 AU. Then a new equation can be made to find out the inner limits of the renewable zone of extrasolar planets that have been updated as follows.

$$i = K_c \ M_{\odot} + d_i \tag{8}$$

 K_c is the value for the outer boundary of the renewable habitable zone, of which the value is 0.5398 *AU/kg*, and for the outer limit of the renewable habitable zone can be written as

$$o = K_d \ M_{\odot} + \ d_o \tag{9}$$

K_d is the constant for the outer boundary of the habitable zone whose value is 1.6595 AU/kg. To date, the value for the outer limits of the habitable zone can not yet be determined with certainty. According to Franck et al (2000) the value of the outer limits of the habitable zone of the Sun is 1.2 AU, and in our opinion this value is still too close as the outer limit. Meanwhile, according to Kopparapu et al (2013) the limit is 1.7 AU, even according to Mischna et al (2000) the absolute external boundary for the sun's habitable zone is 2.4 AU. This outer boundary value is too far away so we do not use both values as a reference. Assuming that the outer boundary value for the Sun's habitable zone is 1.66 AU, the limit will be more effective than the inner limit value set by the stellar habitable zone calculator of 3 AU. This is because the distance of 3 AU is considered far enough and can not maintain water in liquid phase.



Fig. 2:- Modeling of the habitable zone using the simple equation. Assuming that the inner and outer limits are 0.7 SA - 1.5 AU

This expanded simple equation can be used to redefine the inner and outer boundary values of the extrasolar planet, and produce a new model of habitable zone.

IV. DISCUSSION

A. Line Model of Habitable Zone

In this model, we used a simple model that follows the model of the line equation. This model is also applied by Chameron (1963) and the casting et al (1993) which explains the cause of the accretion disk of habitable zone around the Jupiter system. Our model follows the pattern of the habitable zone model around the Jupiter system that is used as a helper line because it is still able to model the habitable zone around the main sequence stars and with small-mass planet systems.

B. The Excess of the Habitable Zone Linier Model

This linear model has also provided some excess that will help future researchers to determine the habitable zone around the main sequence stars. The following is explained about the excess of our model.

➤ Easier and Simpler

This model is able to simplify the previous model (nonlinear model) of the habitable zone created by some previous researchers (Kasting, 1993). Not only from the side of the decent zone model, but the resulting equation is also more difficult than the linear model.

➢ Faster in Determining the Habitable Zone

Along with technological developments, new stars and their planets will soon be able to determine their decent zone. Only with the assistance of a simple program using Microsoft Excel, the researchers will be helped with a simple equation that we have made only by putting the parameters in the form of the mass of the parent star which alone can determine where the habitable zone around the star.

> Having a High Accuracy

Compared to the model made by Selsis (2007) which received 327 habitable planets, the simple model accuracy for this simple linear habitable zone is 92%, with a total of 303 planets. This proves that the accuracy made based on a linear lunar zone model can still be used for both Jupiter systems and small mass planet systems.

C. Estimates of Habitable Planet Detection Results by Type of Spectral Class

Estimates of habitable zones around other stars, along with the discovery of extrasolar planets and new knowledge



Semi major axis (AU)

of extreme habitats on Earth, it is suggested that there may be other places in the habitable universe than might be expected to date (Hornect and Rettberg, 2007). On November 4, 2013, based on Kepler's mission, astronomers concluded that there are billions of Earth-sized planets orbiting the Sun like stars in the habitable zone and the possibly of nearby planet is 12 light-years away (Pettigura et al, 2013b). Based on data from the encyclopedia catalog of extrasolar planets, new data of habitable planets have been obtained which can be seen in Appendix. The planets



Fig. 3:- The habitable zone takes into account the surface temperatures and Venus aphelium. From the diagram di is known 0.0002 AU and do is 0.0005 AU. If the value of the major axis of Venus and Mars is 0.7 AU and 1.5 AU (figure 3a), then when the addition of inner and outer values of the habitable zone of 0.16 AU is obtained a new value for the semi-major axis used in this model is 0.54 AU and 1.66 AU (figure 3b).

Spectrum Class	Semi Major Axis (AU)	Inner (SA)	Outer (SA)	Numbers
В	0.6	0.32408	0.9962	1
А	1.63	0.880074	2.705485	1
F	0.81 - 1.54	0.43 - 0.83	1.34 - 2.556	26
G	0.75 - 3.09	0.405 - 1.668	1.245 - 5.128	102
K	0.65 - 3.04	0.35 - 1.64	1.079 - 5.045	103
М	0.08 - 1.6	0.04 - 0.86	0.13 - 2.65	18
Unknown	0.46 - 3.9	0.248 - 2.105	0.76 - 6.47	45

Table 1:- Data on the detection of planets is habitable according to the semi-major axis. It is known from the table that most habitable planets have stars with spectral classes of G and K.

Mostly orbit the sun like stars with the spectral class F-M, although there are several planets orbiting their parent stars with spectral classes A and B.



Fig. 4:- Diagram of discovery of habitable planets per star class. Most habitable planets orbits their parent star with a spectral class K type that has a temperature of about 4000 K.

In Figure 4, its shown that most habitable planets orbit more of their parent star with a spectral class K that has a temperature of about 4000 K because the planets have temperatures that allow it to retain water in liquid phase on its surface. The planets also remain steadily longer in the main phase than the Sun (Shiga, 2014), allowing a longer time to life to form on the planet that surrounds the main sequence stars in the spectral class K (Vieru, 2014). However, planets orbiting stars in older-age spectrum K are thought to be unable to sustain life on its surface. This is because the planet can be trapped in synchronous rotation due to tidal attenuation, caused by the high incidence of corona mass ejection, which together produce little protection from high-energy particle radiation (Khodacenko et al, 2007). To detect a habitable planet, the radial velocity method has dominated the detection for a habitable planet orbiting the parent star with

spectral class F, G, K, and M. in the class spectrum, the planet will be able to maintain its surface temperature remains stable.

D. Estimated Results Habitable Planet Detection by Semi-Major Axis

Based on its semi-major axis, the most heavily detected planet in a habitable zone is a planet that has a semi-major axis of 0.75 SA - 3.04 AU with total planets detected is 205 planets. Based on table 1, the habitable planets that have the semi-major axis surround the parent star with the spectral class G and K, and have inner and outer limits of 0.405 SA - 5.545 AU. The limits within the habitable zone still include the inner and outer limits of the habitable zone used are 0.54 SA - 1.66 AU, so it can be said that the limits are quite effectively used as inner and the outer limits of habitable zones.

V. CONCLUSION

We have made a simple linear model of the habitable zone using a simple program Microsoft Excel. This model uses the inner and outer limit in succession is 0.54 - 1.66 SA. Some of the advantages of this method are easier and simpler in determining the equations for the inner and outer limits of the habitable zone, and faster in determining which planets are trapped inside the habitable zone in a solar system, and has a high accuracy of 92% of the model we use as a comparison (Selsis, 2007). With the model used, it can be determined that more habitable planets orbit the parent star with spectral types G and K as much as 102 and 103 planets and the most prominent methods for detection are radial and transit velocities.

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