Alternative Fuzzy Algebra for Fuzzy Linear System Using Cramers Rules on Fuzzy Trapozoidal Number

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Abstract:- In this article will be given alternative positive fuzzy number and negative fuzzy number using the area of positive-x axis and negative-x axis, and then will given algebra modification in multiplying two numbers of fuzzy. The linear system will be solve using Cramer rules.

Keywords:- *Fuzzy trapozoidal numbers, fully fuzzy linear system, Cramer rules*

I. INTRODUCTION

Fuzzy logic is part of mathematics science introduced by L. A Zadeh a professor of electrical engineering UC Barkeley, department of computer science in 1965. L. A Zadeh thinking fuzzy logic can bridge machine language that is precise to human language that emphasizes meaning[5].

Fuzzy linear system has three types, first fuzzy linear system $A\tilde{x} = \tilde{b}$, A is a real matrix and $\tilde{x}, \tilde{b}$ is vektor fuzzy. Second fully fuzzy linear system $\tilde{A}\tilde{x} = \tilde{b}, \tilde{A}$ is a fuzzy matrix and $\tilde{x}, \tilde{b}$ is vektor fuzzy. The last one is dual fully fuzzy linear system $\tilde{A}\tilde{x} + \tilde{c} = \tilde{B}\tilde{x} + \tilde{d}$, $\tilde{A}, \tilde{B}$ is a fuzzy matrix and $\tilde{x}, \tilde{c}, \tilde{d}$ is vektor fuzzy. Fuzzy linear system can be solved by many kind method such as on linear system. In 2010 has been completed a dual fully fuzzy linear using LU factorization rules on triangular fuzzy numbers[7], [11] also has completed fuzzy linear system using LU factirization rules but on trapozoidal fuzzy number. In 2006 was discussed using the Cramer rule in triangular fuzzy number [6,12]. All method can be used in solving fuzzy linear system, but the authors only discusses positive fuzzy, and didn't give a compatible solution. In this article will given modification of fuzzy algebra, begin with define positive fuzzy number and negative fuzzy number so will make have a competible solution. Then it will be applied in a fully fuzzy linear system using Cramer rules. And the end will be given an example as an illustration.

II. PRELIMINARIES

A. Selecting a Template (Heading 2)

The concept of fuzzy numbers has been given by [1,2,7,8] and [9], Cramer method[3].

Definisi 2.1 A fuzzy number is a fuzzy set $\tilde{u}: R \to [0,1]$ which satisfies the following:

- $\tilde{u}$ is upper semicontinuous.
- $\tilde{u}(x) = 0$ outside the interval [a,d]
- There exist real number b, c in $[a, d]$ such that:

- $\tilde{u}(x)$ monotonic increasing in $[a, b]$,
- $\tilde{u}(x)$ monotonic decreasing in $[b, d]$,
- $\tilde{u}(x) = 1$ for $b \le x \le c$.

A more popular equivalent alternative definition of fuzzy number is as follows.

Definition 2.2 A fuzzy number $\tilde{u}$ is a pair $\left(\underline{u}(r), \overline{u}(r)\right)$ of functions $\underline{u}(r), \overline{u}(r), 0 \le r \le 1$ which satisfy the following:

- $\underline{u}(r)$ is a bounded left continuous non decreasing fuction over [0,1],
- $\overline{u}(r)$ is a bounded left continuous non increasing fuction over [0,1],
- $\underline{u}(r) \le \overline{u}(r)$,$0 \le r \le 1$.

A trapozoidal fuzzy number can be written by two ways, first $\tilde{u} = (a, b, c, d)$ with $a < b < c < d$ and the second is $\tilde{u} = (a, b, \alpha, \beta)$ with a, b is the center, α is the left width and β is the right width. A fuzzy number in trapozoidal form $\tilde{u} = (a, b, c, d)$ is the function:

$$\mu_{\tilde{u}}(x) = \mu_{\tilde{u}}(a, b, c, d) = \begin{cases} \dfrac{x-a}{b-a}, & a \le x \le b \\ 1, & b \le x \le c \\ \dfrac{d-x}{d-c}, & c \le x \le d \\ 0, & \text{otherwise} \end{cases}$$

A fuzzy number in trapozoidal form $\tilde{u} = (a, b, \alpha, \beta)$ is the function:

$$\mu_{\tilde{u}}(x) = \mu_{\tilde{u}}(a, b, \alpha, \beta) = \begin{cases} 1 - \dfrac{a-x}{\alpha}, & a \le x \le b \\ 1, & b \le x \le c \\ 1 - \dfrac{x-b}{\beta}, & c \le x \le d \\ 0, & \text{otherwise} \end{cases}$$

On the other ways, a parametric fuzzy number $\tilde{u} = [\underline{u}(r), \overline{u}(r)]$ can be represented as:

$$\underline{u}(r) = a - (1-r)\alpha,$$
$$\overline{u}(r) = b + (1-r)\beta.$$

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Some reviewed for arithmetic between two trapozoidal fuzzy number [1,2,4,6,8,11,12] and [9] .

Definisi 2.3 Arithmetic operations on trapozoidal numbers Let $\tilde{u} = (a, b, \alpha, \beta)$, $\tilde{v} = (c, d, \gamma, \delta)$ and k is a scalar:

- *Addition*:

$$\tilde{u} \oplus \tilde{v} = \left(\underline{u} + \underline{v}, \overline{u} + \overline{v}\right)$$
$$\tilde{u} \oplus \tilde{v} = (a + c, b + d, \alpha + \gamma, \beta + \delta)$$

- *Subtraction*:

$$\tilde{u} \ominus \tilde{v} = \left(\underline{u} + \overline{v}, \overline{u} + \underline{v}\right)$$
$$\tilde{u} \ominus \tilde{v} = (a - d, b - c, \alpha + \delta, \beta + \gamma)$$

- *The Minus of Trapozoidal Fuzzy:*

$$-\tilde{u} = -(a, b, \alpha, \beta)$$
$$-\tilde{u} = (-b, -a, \beta, \alpha)$$

- *Scalar Multiplication:*

$$k \otimes \tilde{u} = k \otimes (a, b, \alpha, \beta) = \begin{cases} (ka, kb, k\alpha, k\beta), & k \geq 0 \\ (kb, ka, k\beta, k\alpha) & k \leq 0 \end{cases}$$

Definisi 2.4 A trapozoidal fuzzy number $\tilde{u} = (a, b, \alpha, \beta)$ is said to be zero trapozoidal fuzzy number if and only if $a = 0, b = 0, \alpha = 0$, and $\beta = 0$.

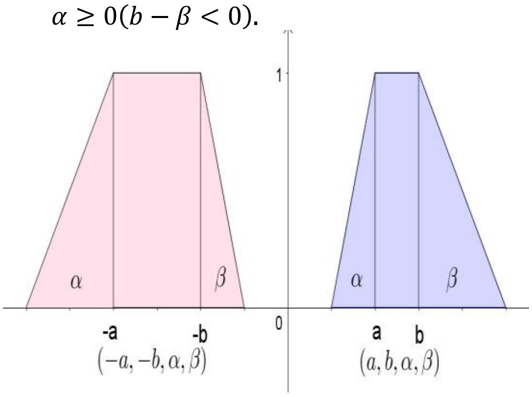
Definisi 2.5 Two fuzzy number $\tilde{u} = (a, b, \alpha, \beta)$ and $\tilde{v} = (c, d, \gamma, \delta)$ are said to be equal if and only if $a = c, b = d, \alpha = \gamma$ and $\beta = \delta$.

III. POSITIVE FUZZY NUMBER AND NEGATIVE FUZZY NUMBER

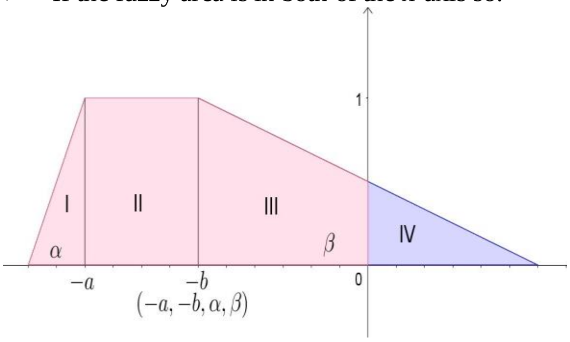
In this section will be given a new definition that fuzzy said to be positive fuzzy number or negative fuzzy number using the area of positive x-axis and negative x-axis, then it will be followed by algebra modifications on two fuzzy number .

Definisi 3.1 A fuzzy number sait to be positive(negative) denoted by $\tilde{u} \geq 0 (\tilde{u} < 0)$ using the area of positive x-axis and negative x-axis:

- If the fuzzy area is in one of the x-axis so fuzzy said to be positive fuzzy number(negative fuzzy number) if $a - \alpha \geq 0 (b - \beta < 0)$.



- If the fuzzy area is in both of the x-axis so:



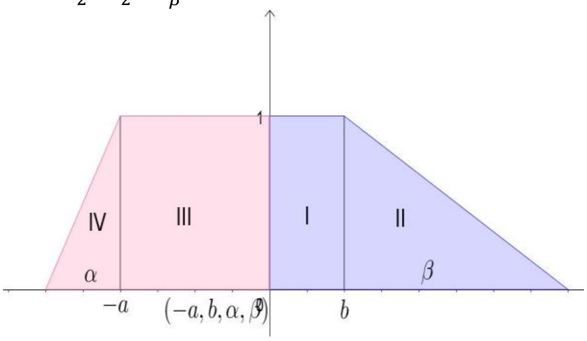
From the picture it appears that there are two regions on the positive x-axis and negative x-axis will be devided into four shapes:

$$L_1 = \frac{1}{2}(\alpha)(1) = \frac{\alpha}{2}$$
$$L_2 = (a - b)(1) = a - b$$
$$L_{34} = \frac{1}{2}(\beta)(1) = \frac{\beta}{2}$$
$$L_4 = \frac{1}{2}(\beta - b)\left(\frac{\beta - b}{\beta}\right) = \frac{(\beta - b)^2}{2\beta}$$
$$L_3 = L_{34} - L_4 = \frac{\beta}{2} - \frac{(\beta - b)^2}{2\beta} = \frac{2\beta b - b^2}{2\beta}$$

Will be shown type of fuzzy with the area of fuzzy:

$$L = L_4 - (L_1 + L_2 + L_3)$$
$$L = \frac{(\beta - b)^2}{2\beta} - \left(\frac{\alpha}{2} + a - b + \frac{2\beta b - b^2}{2\beta}\right)$$
$$L = -a - b + \frac{\beta}{2} - \frac{\alpha}{2} + \frac{b^2}{\beta}$$

A. If $a \leq 0, b \leq 0$ and $b + \beta \geq 0$, so $\tilde{u}$ said to be positive fuzzy number if $-a - b + \frac{\beta}{2} - \frac{\alpha}{2} + \frac{b^2}{\beta} \geq 0$, on the contrary $\tilde{u}$ said to be negative fuzzy number if $= -a - b + \frac{\beta}{2} - \frac{\alpha}{2} + \frac{b^2}{\beta} < 0$.



From the picture it appears that there are two regions on the positive x-axis and negative x-axis will be devided into four shapes:

$$L_1 = (b)(1) = b$$
$$L_2 = \frac{1}{2}(\beta)(1) = \frac{\beta}{2}$$
$$L_3 = (a)(1) = a$$
$$L_4 = \frac{1}{2}(\alpha)(1) = \frac{\alpha}{2}$$

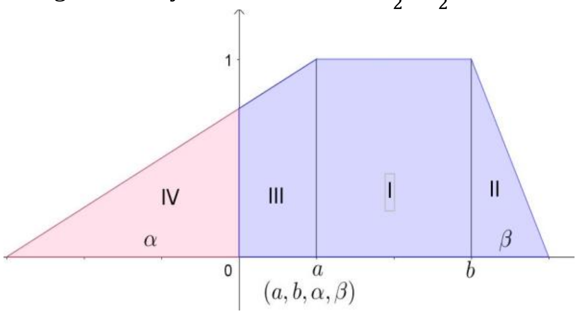
Will be shown type of fuzzy with the area of fuzzy:

$$L = (L_1 + L_2) - (L_3 + L_4)$$

$$L = \left(b + \frac{\beta}{2}\right) - \left(a + \frac{\alpha}{2}\right)$$

$$L = b - a + \frac{\beta}{2} - \frac{\alpha}{2}$$

B. If $a \leq 0$ and $b \geq 0$, so $\tilde{u}$ said to be positive fuzzy number if $b - a - \frac{\alpha}{2} + \frac{\beta}{2} \geq 0$, on the contrary $\tilde{u}$ said to be negative fuzzy number if $b - a - \frac{\alpha}{2} + \frac{\beta}{2} < 0$.



From the picture it appears that there are two regions on the positive x-axis and negative x-axis will be devided into four shapes:

$$L_1 = (b - a1)(1) = b - a$$

$$L_2 = \frac{1}{2}(\beta)(1) = \frac{\beta}{2}$$

$$L_{34} = \frac{1}{2}(\alpha)(1) = \frac{\alpha}{2}$$

$$L_4 = \frac{1}{2}\left(\frac{\alpha - a}{\alpha}\right)(\alpha - a) = \frac{(\alpha - a)^2}{2\alpha}$$

$$L_3 = L_{34} - L_4 = \frac{\alpha}{2} - \frac{(\alpha - a)^2}{2\alpha} = a - \frac{a^2}{2\alpha}$$

Will be shown type of fuzzy with the area of fuzzy:

$$L = (L_1 + L_2 + L_3) - (L_4)$$

$$L = \left(b - a + \frac{\beta}{2} + a - \frac{a^2}{2\alpha}\right) - \left(\frac{(\alpha - a)^2}{2\alpha}\right)$$

$$L = a + b - \frac{\alpha}{2} + \frac{\beta}{2} - \frac{a^2}{\alpha}$$

C. If $a \geq 0$ and $b \geq 0$, so $\tilde{u}$ said to be positive fuzzy number if $a + b - \frac{\alpha}{2} + \frac{\beta}{2} - \frac{a^2}{\alpha} \geq 0$, on the contrary $\tilde{u}$ said to be negative fuzzy number if $a + b - \frac{\alpha}{2} + \frac{\beta}{2} - \frac{a^2}{\alpha} < 0$.

After being defined positive fuzzy and negative fuzzy will be applied on multiplication fuzzy.

Definisi 3.2 Given $\tilde{u} = (a, b, \alpha, \beta) = \left(\underline{u}(r), \overline{u}(r)\right) = (a - (1 - r)\alpha, b + (1 - r)\beta)$ and $\tilde{v} = (c, d, \gamma, \delta) = \left(\underline{v}(r), \overline{v}(r)\right) = (c - (1 - r)\gamma, b + (1 - r)\delta)$ so that:

- If $\tilde{u} > 0$ and $\tilde{v} > 0$ so:
  $$\tilde{w} = \left(ac, bd, (a\gamma + c\alpha), (b\delta + d\beta)\right)$$
- If $\tilde{u} > 0$ and $\tilde{v} < 0$ so:
  $$\tilde{w} = \left(bc, ad, (b\gamma - c\beta), (a\delta - d\alpha)\right)$$
- If $\tilde{u} < 0$ and $\tilde{v} > 0$ so:
  $$\tilde{w} = \left(ad, bc, (d\alpha - a\delta), (c\beta - b\gamma)\right)$$
- If $\tilde{u} < 0$ and $\tilde{v} < 0$ so:
  $$\tilde{w} = \left(bd, ac, -(d\beta + b\delta), -(a\gamma + c\alpha)\right)$$

IV. SOLVING FULLY FUZZY LINEAR SYSTEM

Next will be given the modification algebra on fully fuzzy linear system $\tilde{A}\tilde{x} = \tilde{b}$ for multiplication if $\tilde{u} > 0$ and $\tilde{v} > 0$,

if $\tilde{u} > 0$ and $\tilde{v} < 0$, if $\tilde{u} < 0$ *and* $\tilde{v} > 0$ and the end $\tilde{u} < 0$ and $\tilde{v} < 0$. Start by partition fully fuzzy linear system so that is obtained real matrix and real vektor, to solve it can be aplication to Cramer method. To get the value of $\tilde{x}$ can be considered from positive(negative) matrix $\tilde{A}$ and vektor $\tilde{b}$ so that the multiplication formula to be used is determined.

Given fully fuzzy linear system $\tilde{A}\tilde{x} = \tilde{b}$ written in the form of a matrix:

$$\begin{bmatrix} (a_{11}, b_{11}, \alpha_{11}, \beta_{11}) & (a_{12}, b_{12}, \alpha_{12}, \beta_{12}) & \cdots & (a_{1n}, b_{1n}, \alpha_{1n}, \beta_{1n}) \\ (a_{21}, b_{21}, \alpha_{21}, \beta_{21}) & (a_{22}, b_{22}, \alpha_{22}, \beta_{22}) & \cdots & (a_{2n}, b_{2n}, \alpha_{2n}, \beta_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ (a_{m1}, b_{m1}, \alpha_{m1}, \beta_{m1}) & (a_{m2}, b_{m2}, \alpha_{12}, \beta_{12}) & \cdots & (a_{mn}, b_{mn}, \alpha_{mn}, \beta_{mn}) \end{bmatrix} \otimes \begin{bmatrix} (x_1, y_1, \rho_1, \tau_1) \\ (x_2, y_2, \rho_2, \tau_2) \\ \vdots \\ (x_m, y_m, \rho_m, \tau_m) \end{bmatrix} = \begin{bmatrix} (c_1, d_1, \gamma_1, \delta_1) \\ (c_1, d_1, \gamma_1, \delta_1) \\ \vdots \\ (c_m, d_m, \gamma_m, \delta_m) \end{bmatrix}$$

fuzzy matrix $\tilde{A}$ is partitioned so thats is obtained:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} B = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{bmatrix}$$

$$M = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{m1} & \alpha_{m2} & \cdots & \alpha_{mn} \end{bmatrix} N = \begin{bmatrix} \beta_{11} & \beta_{12} & \cdots & \beta_{1n} \\ \beta_{21} & \beta_{22} & \cdots & \beta_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{m1} & \beta_{m2} & \cdots & \beta_{mn} \end{bmatrix}$$

fuzzy matrix $\tilde{A}$ can be written $\tilde{A} = (A, B, M, N)$.

fuzzy vektor $\tilde{x}$ is partitioned so thats is obtained:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix} \rho = \begin{bmatrix} \rho_1 \\ \rho_2 \\ \vdots \\ \rho_m \end{bmatrix} \tau = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \vdots \\ \tau_m \end{bmatrix}$$

fuzzy vektor $\tilde{x}$ can be written $\tilde{x} = (x, y, \rho, \tau)$.

fuzzy vektor $\tilde{b}$ is partitioned so thats is obtained:

$$c = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_m \end{bmatrix} d = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_m \end{bmatrix} \gamma = \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma_m \end{bmatrix} \delta = \begin{bmatrix} \delta_1 \\ \delta_2 \\ \vdots \\ \delta_m \end{bmatrix}$$

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fuzzy vektor $\tilde{b}$ can be written $\tilde{b} = (c, d, \gamma, \delta)$.
Finally fully fuzzy linear sytem can be written by:

$$\tilde{A}\tilde{x} = \tilde{b}$$
$$(A, B, M, N) \otimes (x, y, \rho, \tau) = (c, d, \gamma, \delta).$$

To solve it can use Cramer method by determining the determinant value in the matrix. Will be shown the multiplication algebra for $\tilde{A} > 0$ and $\tilde{b} < 0$ obtained $\tilde{x} < 0$. Will be used multiplication formula for $\tilde{A} > 0$ and $\tilde{x} < 0$:

$$(A, B, M, N) \times (x, y, \rho, \tau) = (c, d, \gamma, \delta),$$
$$(Bx, Ay, B\rho - Nx, A\tau - My) = (c, d, \gamma, \delta).$$

So that is obtained:

$$Bx = c$$
$$Ay = d$$
$$B\rho - Nx = \gamma$$
$$A\tau - My = \delta$$

To solve $Bx = c$

$$Bx = c$$
$$\begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_m \end{bmatrix}$$

Using Cramer method:

$$x^{(1)} = \begin{bmatrix} c_1 & b_{12} & \cdots & b_{1n} \\ c_2 & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ c_m & b_{m2} & \cdots & b_{mn} \end{bmatrix}$$

$$x^{(2)} = \begin{bmatrix} b_{11} & c_1 & \cdots & b_{1n} \\ b_{21} & c_2 & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & c_m & \cdots & b_{mn} \end{bmatrix}$$

$$\vdots$$

$$x^{(m)} = \begin{bmatrix} b_{11} & b_{12} & \cdots & c_1 \\ b_{21} & b_{22} & \cdots & c_2 \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \cdots & c_m \end{bmatrix}$$

So that is obtained:

$$x_1 = \frac{\det x^{(1)}}{\det B}, x_2 = \frac{\det x^{(2)}}{\det B} \cdots x_m = \frac{\det x^{(m)}}{\det B}$$

To solve $Ay = d$

$$Ay = d$$
$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_m \end{bmatrix}$$

Using Cramer method:

$$y^{(1)} = \begin{bmatrix} d_1 & a_{12} & \cdots & a_{1n} \\ d_2 & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ d_m & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

$$y^{(2)} = \begin{bmatrix} a_{11} & d_1 & \cdots & a_{1n} \\ a_{21} & d_2 & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & d_m & \cdots & a_{mn} \end{bmatrix}$$

$$\vdots$$

$$y^{(m)} = \begin{bmatrix} a_{11} & a_{12} & \cdots & d_1 \\ a_{21} & a_{22} & \cdots & d_2 \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & d_m \end{bmatrix}$$

So that is obtained:

$$y_1 = \frac{\det y^{(1)}}{\det A}, y_2 = \frac{\det y^{(2)}}{\det A} \cdots y_m = \frac{\det y^{(m)}}{\det A}$$

To find the value of ρ, first:

$$B\rho - Nx = \gamma$$
$$B\rho = \gamma + Nx$$

$$B\rho = \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma_m \end{bmatrix} + \begin{bmatrix} \beta_{11} & \beta_{12} & \cdots & \beta_{1n} \\ \beta_{21} & \beta_{22} & \cdots & \beta_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{m1} & \beta_{m2} & \cdots & \beta_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix}$$

$$B\rho = \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma_m \end{bmatrix} + \begin{bmatrix} x_1\beta_{11} + x_2\beta_{12} + \cdots + x_m\beta_{1n} \\ x_1\beta_{21} + x_2\beta_{22} + \cdots + x_m\beta_{2n} \\ \vdots \\ x_1\beta_{m1} + x_2\beta_{m2} + \cdots + x_m\beta_{mn} \end{bmatrix}$$

$$B\rho = \gamma + Nx$$

$$\begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{bmatrix} \begin{bmatrix} \rho_1 \\ \rho_2 \\ \vdots \\ \rho_m \end{bmatrix}$$

$$= \begin{bmatrix} \gamma_1 + x_1\beta_{11} + x_2\beta_{12} + \cdots + x_m\beta_{1n} \\ \gamma_2 + x_1\beta_{21} + x_2\beta_{22} + \cdots + x_m\beta_{2n} \\ \vdots \\ \gamma_m + x_1\beta_{m1} + x_2\beta_{m2} + \cdots + x_m\beta_{mn} \end{bmatrix}$$

Using Cramer method:

$$\rho^{(1)} = \begin{bmatrix} \gamma_1 + x_1\beta_{11} + x_2\beta_{12} + \cdots + x_m\beta_{1n} & b_{12} & \cdots & b_{1n} \\ \gamma_2 + x_1\beta_{21} + x_2\beta_{22} + \cdots + x_m\beta_{2n} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_m + x_1\beta_{m1} + x_2\beta_{m2} + \cdots + x_m\beta_{mn} & b_{m2} & \cdots & b_{mn} \end{bmatrix}$$

$$\rho^{(2)} = \begin{bmatrix} b_{11} & \gamma_1 + x_1\beta_{11} + x_2\beta_{12} + \cdots + x_m\beta_{1n} & \cdots & b_{1n} \\ b_{21} & \gamma_2 + x_1\beta_{21} + x_2\beta_{22} + \cdots + x_m\beta_{2n} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & \gamma_m + x_1\beta_{m1} + x_2\beta_{m2} + \cdots + x_m\beta_{mn} & \cdots & b_{mn} \end{bmatrix}$$

$$\vdots$$

$$\rho^{(m)} = \begin{bmatrix} b_{11} & b_{12} & \cdots & \gamma_1 + x_1\beta_{11} + x_2\beta_{12} + \cdots + x_m\beta_{1n} \\ b_{21} & b_{22} & \cdots & \gamma_2 + x_1\beta_{21} + x_2\beta_{22} + \cdots + x_m\beta_{2n} \\ \vdots & \vdots & & \vdots \\ b_{m1} & b_{m2} & \cdots & \gamma_m + x_1\beta_{m1} + x_2\beta_{m2} + \cdots + x_m\beta_{mn} \end{bmatrix}$$

So that is obtained:

$$\rho_1 = \frac{\det \rho^{(1)}}{\det B}, \rho_2 = \frac{\det \rho^{(2)}}{\det B} \cdots \rho_m = \frac{\det \rho^{(m)}}{\det B}$$

To find the value of τ, first:

$$A\tau - My = \delta$$
$$A\tau = \delta + My$$

$$A\tau = \begin{bmatrix} \delta_1 \\ \delta_2 \\ \vdots \\ \delta_m \end{bmatrix} + \begin{bmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{m1} & \alpha_{m2} & \cdots & \alpha_{mn} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix}$$

$$A\tau = \begin{bmatrix} \delta_1 \\ \delta_2 \\ \vdots \\ \delta_m \end{bmatrix} + \begin{bmatrix} y_1\alpha_{11} + y_2\alpha_{12} + \cdots + y_m\alpha_{1n} \\ y_1\alpha_{21} + y_2\alpha_{22} + \cdots + y_m\alpha_{2n} \\ \vdots \\ y_1\alpha_{m1} + y_2\alpha_{m2} + \cdots + y_m\alpha_{mn} \end{bmatrix}$$

$$A\tau = \begin{bmatrix} \delta_1 + y_1\alpha_{11} + y_2\alpha_{12} + \cdots + y_m\alpha_{1n} \\ \delta_2 + y_1\alpha_{21} + y_2\alpha_{22} + \cdots + y_m\alpha_{2n} \\ \vdots \\ \delta_m + y_1\alpha_{m1} + y_2\alpha_{m2} + \cdots + y_m\alpha_{mn} \end{bmatrix}$$

$$A\tau = \delta + My$$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} \tau_1 \\ \tau_2 \\ \vdots \\ \tau_m \end{bmatrix} = \begin{bmatrix} \delta_1 + y_1\alpha_{11} + y_2\alpha_{12} + \cdots + y_m\alpha_{1n} \\ \delta_2 + y_1\alpha_{21} + y_2\alpha_{22} + \cdots + y_m\alpha_{2n} \\ \vdots \\ \delta_m + y_1\alpha_{m1} + y_2\alpha_{m2} + \cdots + y_m\alpha_{mn} \end{bmatrix}$$

Using Cramer method:

$$\tau^{(1)} = \begin{bmatrix} \delta_1 + y_1\alpha_{11} + y_2\alpha_{12} + \cdots + y_m\alpha_{1n} & a_{12} & \cdots & a_{1n} \\ \delta_2 + y_1\alpha_{21} + y_2\alpha_{22} + \cdots + y_m\alpha_{2n} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \delta_m + y_1\alpha_{m1} + y_2\alpha_{m2} + \cdots + y_m\alpha_{mn} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

$$\tau^{(2)} = \begin{bmatrix} a_{11} & \delta_1 + y_1\alpha_{11} + y_2\alpha_{12} + \cdots + y_m\alpha_{1n} & \cdots & a_{1n} \\ a_{21} & \delta_1 + y_1\alpha_{11} + y_2\alpha_{12} + \cdots + y_m\alpha_{1n} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & \delta_1 + y_1\alpha_{11} + y_2\alpha_{12} + \cdots + y_m\alpha_{1n} & \cdots & a_{mn} \end{bmatrix}$$

$$\vdots$$

$$\tau^{(m)} = \begin{bmatrix} a_{11} & a_{12} & \cdots & \delta_1 + y_1\alpha_{11} + y_2\alpha_{12} + \cdots + y_m\alpha_{1n} \\ a_{21} & a_{22} & \cdots & \delta_2 + y_1\alpha_{21} + y_2\alpha_{22} + \cdots + y_m\alpha_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & \delta_m + y_1\alpha_{m1} + y_2\alpha_{m2} + \cdots + y_m\alpha_{mn} \end{bmatrix}$$

So that is obtained:

$$\tau_1 = \frac{\det \tau^{(1)}}{\det A}, \tau_2 = \frac{\det \tau^{(2)}}{\det A} \cdots \tau_m = \frac{\det \tau^{(m)}}{\det A}$$

Example 1 Given a fully fuzzy linear system **:**

$$\tilde{A}\tilde{x} = \tilde{b}$$
$$(5,6,2,3)\tilde{x}_1 \oplus (4,5,1,2)\tilde{x}_2 \oplus (8,19,7,1)\tilde{x}_3 \oplus = (-36,21,92,16)$$
$$(3,7,2,5)\tilde{x}_1 \oplus (9,11,2,1)\tilde{x}_2 \oplus (5,13,4,2)\tilde{x}_3 \oplus = (-38,26,103,13)$$
$$(7,11,2,5)\tilde{x}_1 \oplus (1,4,1,3)\tilde{x}_2 \oplus (3,7,1,2)\tilde{x}_3 \oplus = (-33,12,63,20)$$

For the first it appears that a matrix fuzzy $\tilde{A}$ is positive fuzzy and vektor $\tilde{b}$ is negative fuzzy, multiplication formula will be used on $\tilde{A} > 0$ and $\tilde{x} < 0$. This fuzzy linear system can be written in matrix:

$$\begin{bmatrix} (5,6,2,3) & (4,5,1,2) & (8,19,7,1) \\ (3,7,2,5) & (9,11,2,1) & (5,13,4,2) \\ (7,11,2,5) & (1,4,1,3) & (3,7,1,2) \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \end{bmatrix} = \begin{bmatrix} (-36,21,92,16) \\ (-38,26,103,13) \\ (-33,12,63,20) \end{bmatrix}$$

fuzzy matrix $\tilde{A}$ and vektor $\tilde{b}$ is partitioned so thats is obtained:

$$A = \begin{bmatrix} 5 & 4 & 8 \\ 3 & 9 & 5 \\ 7 & 1 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 6 & 5 & 19 \\ 7 & 11 & 13 \\ 11 & 4 & 7 \end{bmatrix},$$

$$M = \begin{bmatrix} 2 & 1 & 7 \\ 2 & 2 & 4 \\ 2 & 1 & 1 \end{bmatrix}, \quad N = \begin{bmatrix} 3 & 2 & 1 \\ 5 & 1 & 2 \\ 5 & 3 & 2 \end{bmatrix},$$

$$c = \begin{bmatrix} -36 \\ -38 \\ -33 \end{bmatrix}, \ d = \begin{bmatrix} 21 \\ 26 \\ 12 \end{bmatrix}, \ \gamma = \begin{bmatrix} 92 \\ 103 \\ 63 \end{bmatrix}, \ \delta = \begin{bmatrix} 16 \\ 13 \\ 20 \end{bmatrix}$$

$$Y^{(1)} = \begin{bmatrix} 21 & 4 & 8 \\ 26 & 9 & 5 \\ 12 & 1 & 3 \end{bmatrix}, \quad Y^{(2)} = \begin{bmatrix} 5 & 21 & 8 \\ 3 & 26 & 5 \\ 7 & 12 & 3 \end{bmatrix},$$

$$Y^{(3)} = \begin{bmatrix} 5 & 4 & 21 \\ 3 & 9 & 26 \\ 7 & 1 & 12 \end{bmatrix},$$

$$X^{(1)} = \begin{bmatrix} -36 & 5 & 19 \\ -38 & 11 & 13 \\ -33 & 4 & 7 \end{bmatrix}, \ X^{(2)} = \begin{bmatrix} 6 & -36 & 19 \\ 7 & -38 & 13 \\ 11 & -33 & 7 \end{bmatrix},$$

$$X^{(3)} = \begin{bmatrix} 6 & 5 & -36 \\ 7 & 11 & -38 \\ 11 & 4 & -33 \end{bmatrix},$$

$$\det A = -266 \quad \det B = -1147$$

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$\det X^{(1)} = 2294 \ \det Y^{(1)} = -266$
$\det X^{(2)} = 1147 \ \det Y^{(2)} = -532$
$\det X^{(3)} = 1147 \ \det Y^{(3)} = -266$

Obtained the value of vektor x and vektor y:

$$x_1 = \frac{\det X^{(1)}}{\det B} = \frac{2294}{-1147} = -2$$

$$x_2 = \frac{\det X^{(2)}}{\det B} = \frac{1147}{-1147} = -1$$

$$x_3 = \frac{\det X^{(3)}}{\det B} = \frac{1147}{-1147} = -1$$

Vektor x is

$$x = \begin{bmatrix} -2 \\ -1 \\ -1 \end{bmatrix}$$

$$y_1 = \frac{\det Y^{(1)}}{\det A} = \frac{-266}{-266} = 1$$

$$y_2 = \frac{\det Y^{(2)}}{\det A} = \frac{-532}{-266} = 2$$

$$y_3 = \frac{\det Y^{(3)}}{\det A} = \frac{-266}{-266} = 1$$

vektor y is:

$$y = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

To find the values of ρ, it applies $B\rho = \gamma + Nx$

$$B\rho = \begin{bmatrix} 92 \\ 103 \\ 63 \end{bmatrix} + \begin{bmatrix} 3 & 2 & 1 \\ 5 & 1 & 2 \\ 5 & 3 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 83 \\ 90 \\ 48 \end{bmatrix}$$

$$\rho^{(1)} = \begin{bmatrix} 83 & 5 & 19 \\ 90 & 11 & 13 \\ 48 & 4 & 7 \end{bmatrix}, \quad \rho^{(2)} = \begin{bmatrix} 6 & 83 & 19 \\ 7 & 90 & 13 \\ 11 & 48 & 7 \end{bmatrix},$$

$$\rho^{(3)} = \begin{bmatrix} 6 & 5 & 83 \\ 7 & 11 & 90 \\ 11 & 4 & 48 \end{bmatrix},$$

Obtained the value of vektor ρ and vektor τ:

$$\rho_1 = \frac{\det \rho^{(1)}}{\det B} = \frac{-1147}{-1147} = 1$$

$$\rho_2 = \frac{\det \rho^{(2)}}{\det B} = \frac{-4588}{-1147} = 4$$

$$\rho_3 = \frac{\det \rho^{(3)}}{\det B} = \frac{-3441}{-1147} = 3$$

vektor ρ is:

$$\rho = \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$$

To find the values of ρ, it applies $A\tau = \delta + My$

$$A\tau = \begin{bmatrix} 16 \\ 13 \\ 20 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 7 \\ 2 & 2 & 4 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 27 \\ 23 \\ 25 \end{bmatrix}$$

$$\tau^{(1)} = \begin{bmatrix} 27 & 4 & 8 \\ 23 & 9 & 5 \\ 25 & 1 & 3 \end{bmatrix}, \quad \tau^{(2)} = \begin{bmatrix} 5 & 27 & 8 \\ 3 & 23 & 5 \\ 7 & 25 & 3 \end{bmatrix},$$

$$\tau^{(3)} = \begin{bmatrix} 5 & 4 & 27 \\ 3 & 9 & 23 \\ 7 & 1 & 25 \end{bmatrix},$$

$$\tau_1 = \frac{\det \tau^{(1)}}{\det A} = \frac{-798}{-266} = 3$$

$$\tau_2 = \frac{\det \tau^{(2)}}{\det A} = \frac{-266}{-266} = 1$$

$$\tau_3 = \frac{\det \tau^{(3)}}{\det A} = \frac{-266}{-266} = 1$$

vektor τ is:

$$\tau = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$$

Maka diperoleh vektor $\tilde{x}$ untuk persamaan tersebut sebagai berikut:

$$\tilde{x} = \begin{bmatrix} (-2,1,1,3) \\ (-1,2,4,1) \\ (-1,1,3,1) \end{bmatrix}.$$

V. CONCLUSION

By define it as the type of positive fuzzy number or negative fuzzy number so that it can be determined the type of multiplication of two fuzzy number that will be used and will give the comfertible result.

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