Structure of an Idempotent M-Normal Commutative Semigroups

[1]R.Rajeswari [2]D.Deva Margaret Helen [3]G.Soundharya
[1]Assistant Professor of Mathematics [2][3]Students of I.M.Sc. Mathematics
Thassim Beevi Abdul Kader College For Women
Kilakarai,Ramanathapuram
Tamilnadu,India

Abstract:- This paper concerned with basic concepts and some results on idempotent m-normal commutative semigroup satisfying the identities of the three variables.This is used to frame some structure for m normal commutative semigroups,here we consider the semigroup satisfying some properties of m normal commutative semigroup,left quasi normal and right quasi normal.If it is right and left rectangular then it is semilattice.

Keywords:- *Lattice, RegularSemigroup, Idempotent, Normal, Quasi.*

I. INTRODUCTION

The concept of a semigroup is very simple and plays an important role in the development of mathematics. The theory of semigroup is similar to group theory and ring theory. The earliest major contribution to the theory of semigroups are strongly motivated by comparisons with group and rings. In this paper the results of ring theory were adopted for semigroups.

II. PRELIMINARIES

In this section we present some basic concept of idempotent m normal semigroups and some basic definition of semigroups.

A. *Definition*: A semigroup (S, .) is said to be *singular* if it satisfy the identity ab=a(ab=b) for all a,b in S.

B. *Definition*: A semigroup (S, .) is said to be *rectangular* if it satisfy the identity aba=a for all a,b in S.

C. *Definition*: A semigroup (S, .) is said to be *regular* if it satisfy the identity aba=ab(aba=ba) for all a,b in S.

D. *Definition*: A semigroup (S, .) is said to be *total* if it satisfy the identity S^2=S.

E. *Definition*: A Semigroup S is said to be *normal* if it satisfy the identity abca=acba for all a,b,c ϵ S.

F. *Definition*: A Semigroup S is said to be *left quasi normal* if it satisfy the identity abc=acbc for all a,b,c ϵ S.

G. *Definition*: A Semigroup S is said to be *right quasi normal* if it satisfy the identity abc=abac for all a,b,c ϵ S.

III. MAIN RESULTS

A. Definition:

An idempotent commutative semigroup (S, .) is said to be m-power left normal if it satisfy the identity $ab^mc^m=ac^mb^m$.

B. Definition:

An idempotent commutative semigroup (S, .) is said to be m-power left quasi normal if it satisfy the identity $ab^mc^m=ac^mb^mc^m$.

C. Definition:

An idempotent commutative semigroup (S, .) is said to be m-power regular if it satisfy the identity $ab^mc^ma=ab^mac^ma$.

D. Definition:

An idempotent commutative semigroup (S, .) is said to be m-power left semi regular if it satisfy the identity $ab^mc^ma=ab^mac^mab^mc^ma$.

E. Definition:

An idempotent commutative semigroup (S, .) is said to be m-power left semi normal if it satisfy the identity $ab^mc^ma=ac^mb^mc^ma$.

F. Definition:

An idempotent commutative semigroup (S, .) is said to be total if it satisfy the identity S^2=S

G. Note:

An idempotent m power commutative semigroup is singular then it satisfy the semi lattice condition.

H. Lemma:

An Idempotent m-power commutative semigroup S is left(right) normal if and only if it is left(right) quasi normal.

➢ *Proof:*

Let **(S, .)** be an idempotent m power commutative semigroup and (S, .) be left(right) normal.
We have to prove (S, .) be left(right) quasi normal,

Then ,
$ab^mc^m = ac^mb^m$
$ab^mc^mc^m = ac^mb^mc^m$
$ab^mc^m = ac^mb^mc^m$
$ab^mc^m = ac^mb^mc^m$

Therefore (S, .) is a left quasi normal.
Conversely,

Let (S, .) is a left quasi normal.
We have to prove that (S, .) be a normal.

$ab^mc^m = ac^mb^mc^m$
$= ac^mc^mb^m$
$= ac^mb^m$.
$ab^mc^m = ac^mb^m$.
Hence proves the lemma.

I. Theorem:

An idempotent m power commutative semigroup S is a left quasi normal if and only if it is left semi regular.

- *Proof:*

Let us consider (S, .) is a left quasi normal,
We have to prove it is left semi regular
Then

$ab^mc^m\, a = ab^mc^mc^ma$
$= a.ac^mb^mc^mb^ma \quad (a^2=a)$
$= ab^mac^mb^mc^ma$
$= ab^ma.ac^mb^mc^ma$
$= ab^mac^mab^mc^ma$
$ab^mc^ma = ab^mac^mab^mc\, ^ma$.
Hence (S, .) is left semi regular.

Conversely,
Suppose that (S, .) is left semi regular.
We have to prove that it is left quasi normal

$ab^mc^ma = ab^mac^mab^mc\, ^ma$
$= aab^mc^mab^mc^ma$
$= ab^mac^mab^mc^ma$
$= ab^mc^maab^mc^ma$.
$ab^mc^m = ab^mc^mab^mc^ma$
$= ab^mc^mb^mac^ma$
$= ab^mb^mc^mc^maa$
$= ab^mc^mc^ma$
$= ac^mb^mc^ma$
$= ac^mb^mc^m \quad$ ($a^2=a$ idempotent condition)
$ab^mc^m = ac^mb^mc^m$.

hence (S, .) is a left quasi normal
This proves the theorem.

J. Theorem:

An idempotent m-power commutative semigroup (S, .) is left(right) quasi normal if and only if it is right semi normal.

- *Proof :*

Let (S, .) be an idempotent m power commutative semigroup,assume that (S, .) is left quasi normal.
Then,

$ab^mc^m = ac^mb^mc^m$
$ab^mc^ma = ac^mb^mc^ma$(post multiply by a)
$= ab^mc^mc^ma$
$= ab^mc^ma$
$= ab^mb^mc^ma$
$= ab^mc^mb^ma$.

Hence (S, .) is right semi normal.
Conversely,
Suppose that (S, .) is right semi normal
Then,

$ab^mc^ma = ab^mc^mb^ma$
$a.ab^mc^m = ac^mb^mb^ma$
$ab^mc^m \quad = ac^mb^ma$
$= ac^mc^mb^m$
$= ac^mb^mc^m$.
$ab^mc^m = ac^mb^mc^m$

hence (S, .) is left quasi normal.
This proves the theorem.

K. Theorem:

An idempotent m-power commutative semigroup (S, .) is left(right) quasi normal if and only if it is right semi regular.

- *Proof*:

Let (S, .) be an idempotent commutative semigroup and assume that (S, .) is left quasi normal.
We have to prove (S, .) is right semi-regular.
Then,

$ab^mc^m = ac^mb^mc^m$
$ab^mc^ma = aac^mb^mb^mc^ma$
$= aab^mc^mb^mc^maa$
$= ab^mc^mab^mac^ma$
$ab^mc^ma = ab^mc^mab^mac^ma$.

Hence (S,.) is right semi regular.
Conversely,
Let (S, .) is right semi regular
Then,

$ab^mc^ma = ab^mc^mab^mac^ma$.
$aab^mc^m = ac^mb^mb^mab^mc^maa$
$ab^mc^m = ac^mb^mb^mac^ma$
$= ac^mb^mc^maa$
$= ac^mb^mc^ma$
$ab^mc^m \quad = ac^mb^mc^m$.

Hence it is left quasi normal.
This proves the theorem.

L. Lemma:

An idempotent commutative m power semigroup is right and left singular then it is semi lattice

➢ *Proof :*

Let (S, .) be a semigroup with left and right regular Then,

$ab^m a=ab^m \rightarrow ①$
$ab^m a=b^m a \rightarrow ②$
from ①and②we get
$ab^m=b^m a$
so (S, .) is commutative.
Now to prove (S, .)is a band
$ab^m a=ab^m$
$a(b^m a)=ab^m$
$ab^m(b^m a)=ab^m$
$a(b^m)^2 a=ab^m$
$a^2(b^m)^2=ab^m$
$(ab^m)^2=ab^m$.
So (S, .) is a band.

Hence it is semi lattice since it is a commutative band. This proves the lemma.

M. Corollary 1:

An idempotent m power commutative semigroup satisfies the singular property then it is semi lattice.

N. Corollary 2:

An idempotent m power commutative semigroup is right(left) singular then it is regular.

O. Corollary 3:

An idempotent m power commutative semigroup is rectangular then it is semi regular.

P. Lemma:

An idempotent commutative semigroup(S, .) is m power left singular then it is rectangular

➢ *Proof:*

Let an idempotent commutative semigroup is m power right singular
We have to prove it is rectangular.
$ab^m=a$
post multiply by a on both sides
$ab^m a=a.a$
$=a^2$
$ab^m a=a$(idempotent condition $a^2=a$)
This proves the lemma

IV. CONCLUSION

The structure of m normal commutative semigroup gives the proper structure to the semigroup.This structure is the basement for further works in m normal semigroups.This paper is very useful for the scholars who will do their work in normal semigroup and this will be applied in many chemical industries.

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