# Modeling and Performance Analysis of Armature-Controlled DC Motor

Nyein Nyein Soe Department of Electrical Power Engineering Technological University (Mawlamyine) Mawlamyine, Myanmar

Abstract:- In this paper, performance analysis of dc motor is described. The selected dc motor is armaturecontrolled motor with fixed field. Moment of inertia with damper (friction) is connected directly to the dc motor as mechanical load. From the dynamic equations of this system to state space form, the roots of the system are evaluated and determine the motor is stable or unstable in open-loop operation. Mathematical modeling and Transfer Function modeling of the dc motor, with and without load torque are built by using MATLAB Simulink Software. Armature current, speed and electromagnetic torque of dc motor are simulated as the output results. This paper is mainly focus to apply MATLAB Software to the operation and modeling of fixed field dc motor in open-loop operation.

*Keywords:- DC Motor; Open-Loop Operation; Mathematical Modeling; Transfer Function Modeling; MATLAB Simulink Software.* 

# I. INTRODUCTION

DC drives are widely used in applications requiring adjustable speed, good speed regulation and frequent starting, braking and reversing. Some important applications are rolling mills, paper mills textile mills, printing presses and so on. Fractional horsepower dc motors are widely used as servo motors for positioning and tracking. DC motor drives are mostly used in variable speed application because of low cost, reliability and simple control.

# II. SELECTION OF DC MOTOR

In this paper, separately excited or armature controlled with fixed field dc motor is selected for modeling. Field winding is physically and electrically separate from the armature winding. The field flux is controlled only by the control of the field current. The field is assumed as constant and the torque is proportional only to the armature current.

# III. EQUIVALENT CIRCUIT AND ELECTROMAGNETIC TORQUE

The electromagnetic torque of dc motor is proportional to the magnitude of the flux due to the field current and armature current. For the armature control mode, the field current is held constant and an adjustable voltage is applied to the armature. Aye Aye Maw Department of Electrical Power Engineering Technological University (Mawlamyine) Mawlamyine, Myanmar

A constant field current is obtained by separately exciting the field from a fixed dc source. The flux produced by the field current is therefore constant. The electromagnetic torque produced by dc motor is

$$T(t) = K_t i_m \tag{1}$$

where T = electromagnetic torque (Nm)  $K_t$  = torque constant  $i_m$  = armature current (A)

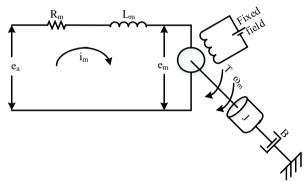


Fig 1:- Circuit diagram of a fixed field dc motor

When the motor armature is rotating, back emf voltage is induced that is proportional to the product of flux and speed. Since the flux is constant, the induced voltage is directly proportional to the speed  $\omega_m$ .

$$\mathbf{e}_{\mathrm{m}} = \mathbf{K}_{\mathrm{b}}\boldsymbol{\omega}_{\mathrm{m}} \tag{2}$$

where  $e_m = induced \text{ or back emf (V)}$   $K_b = emf \text{ constant}$  $\omega_m = speed (rad/sec)$ 

The torque constant  $K_t$  and emf constant  $K_b$  are the same if they are expressed in volt-sec/rad. The voltage equation of armature circuit is

$$L_m Di_m + R_m i_m + e_m = e_a \tag{3}$$

where  $L_m$  = armature inductance (H)  $R_m$  = armature resistance ( $\Omega$ )  $e_a$  = applied voltage (V) D = operator (d/dt)

# IV. MODELING OF DC MOTOR WITHOUT LOAD TORQUE (TL)

As shown in Fig.1, if the load consists only of a moment of inertia (J) and damper (friction) (B), the torque equation is

$$JD\omega_m + B\omega_m = T \tag{4}$$

A. Parameters of DC Motor

For mathematical modeling of fixed field dc motor, selected motor parameters as shown in Table 1.

Specification	Value	Unit
R <sub>m</sub>	0.5	Ω
L <sub>m</sub>	0.003	Н
K <sub>b</sub>	0.8	volt-sec/rad
J	0.0167	kgm <sup>2</sup>
В	0.01	Nm/rad/sec
ea	220	V

Table 1:- Selected DC Motor Parameters

B. Mathematical Modeling of DC Motor without  $T_L$  by MATLAB Simulink

For steady-state,  $d\omega_m/dt = 0$  and  $i_m = 0.0125 \,\, \omega_m$ 

By substitution in (3),  $\omega_m = 272.868 \text{ rad/sec}$ T = 2.728 Nm

Form (3) and (4), mathematical modeling of dc motor is obtained by using MATLB Simulink Software and shown in Fig. 2.

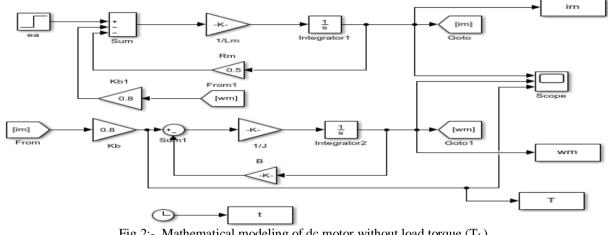


Fig 2:- Mathematical modeling of dc motor without load torque (T<sub>L</sub>)

The simulation outputs of armature current, speed and torque are shown in Fig. 3.

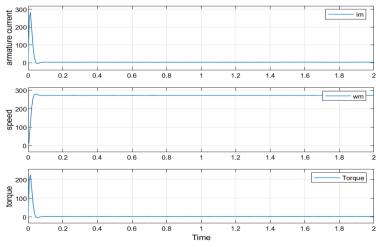


Fig 3:- Output results of simulation without load torque (T<sub>L</sub>)

Form Fig. 3, it can be seen that the speed of dc motor is 272.868 rad/sec in steady-state condition. Steady-state armature current and torque can also be seen clearly from the simulation result.

Torque-current curve or performance curve of dc motor is shown in Fig. 4.

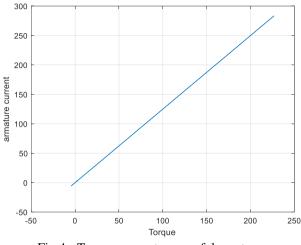


Fig 4:- Torque-current curve of dc motor

C. Transfer Function Modeling of DC Motor without  $T_L$  by MATLAB Simulink

Equation (3) and (4) are converted to Laplace Transform and become

$$I_{m}(s) = \frac{e_{a}(s) - K_{b}\omega_{m}(s)}{R_{m} + sL_{m}}$$
(5)

$$\omega_{\rm m}(s) = \frac{K_{\rm b} I_{\rm m}(s)}{J_{\rm S} + B} \tag{6}$$

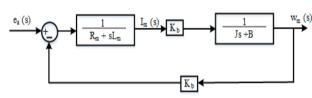


Fig 5:- Block diagram for DC motor without T<sub>L</sub>

Without load torque, transfer function of dc motor system is

$$\frac{\omega_{\rm m}(s)}{e_{\rm a}(s)} = \frac{K_{\rm b}}{s^2(JL_{\rm m}) + s(R_{\rm m}J + L_{\rm m}B) + (K_{\rm b}^2 + R_{\rm m}B)}$$
(7)

By substituting the dc motor parameters, the open-loop transfer function of Fig. 6 is obtained.

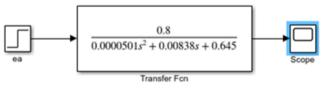
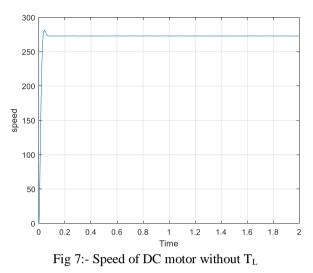


Fig 6:- Open-loop transfer function of DC motor without  $$T_{\rm L}$$ 

The speed of motor output is got by simulation and shown in Fig. 7.



# D. State-space Modeling of DC Motor without $T_L$

The dynamic equations are transformed to state-space form and

$$\begin{bmatrix} D\omega_m \\ Di_m \end{bmatrix} = \begin{bmatrix} -\frac{B}{J} & \frac{K_b}{J} \\ -\frac{K_b}{L_m} & -\frac{R_m}{L_m} \end{bmatrix} \begin{bmatrix} \omega_m \\ i_m \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L_m} \end{bmatrix} \begin{bmatrix} e_a \end{bmatrix}$$
(8)

From substituting the motor parameters,

$$A = \begin{bmatrix} -0.599 & 47.904 \\ -266.67 & -166.67 \end{bmatrix}$$
$$B = \begin{bmatrix} 0 \\ 333.33 \end{bmatrix}$$

The roots of the system are evaluated and  $\lambda_1$ ,  $\lambda_2 = -83.63 \pm j$  76.68. These eigenvalues will always have a negative real part, indicating that the motor is stable in open-loop operation.

# E. Performance of DC Motor without $T_L$

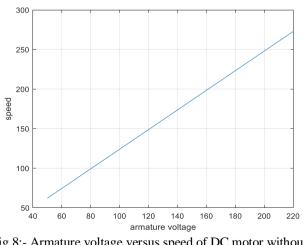


Fig 8:- Armature voltage versus speed of DC motor without  $$T_{\rm L}$$ 

When supplied armature voltage of dc motor is varied from 50V to 220V, the speed will be also varied from 62.016 rad/sec to 272.868 rad/sec. The performance characteristics curve of armature voltage versus speed is shown in Fig. 8.

# V. MODELING OF DC MOTOR WITH LOAD TORQUE $(T_L)$

When the load torque  $\left(T_L\right)$  is constituted, torque equation becomes

$$JD\omega_m + B\omega_m + T_L = T \tag{9}$$

A. Mathematical Modeling of DC Motor with  $T_L$  by MATLAB Simulink

For the load torque (T<sub>L</sub>) of 100Nm, the performance of dc motor changes.

For steady-state,  $d\omega_m/dt = 0$   $i_m = 125 + 0.0125 \omega_m$ By substitution in (3),  $\omega_m = 195.35 \text{ rad/sec.}$  $T = 100 + 0.01 \omega_m$ 

 $1 = 100 + 0.01\omega_{\rm m}$ = 101.954 Nm

Form (3) and (9), mathematical modeling of dc motor is obtained by using MATLB Simulink Software and shown in Fig. 9. Load torque ( $T_L$ ) is unit step input of 100 Nm and connected at 0.5 second after starting.

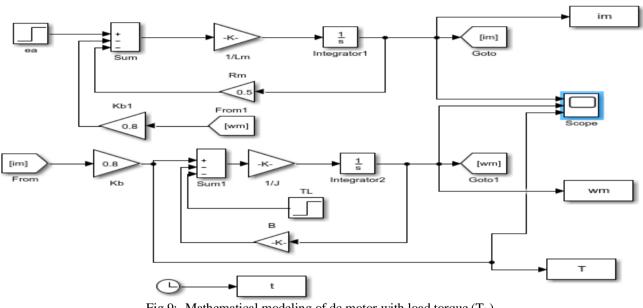
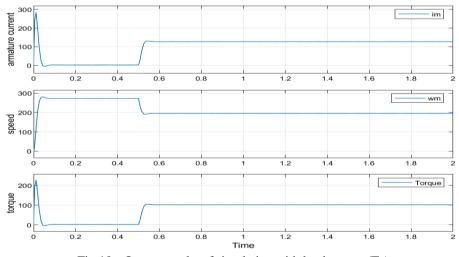
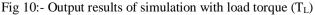


Fig 9:- Mathematical modeling of dc motor with load torque (T<sub>L</sub>)

The simulation outputs of armature current, speed and torque are shown in Fig. 10. Form Fig. 10, it can be seen that the speed of dc motor is 272.868 rad/sec in 0 to 0.5 second period. At 0.5 second, load torque of 100Nm is connected and the speed starts to decrease and reaches 195.35 rad/sec at steady-state condition.





B. Transfer Function Modeling of DC Motor with T<sub>L</sub> by MATLAB Simulink

By converting to Laplace Transform,

$$\omega_{\rm m}(s) = \frac{K_{\rm b} I_{\rm m}(s) - T_{\rm L}(s)}{J_{\rm s} + B}$$
(10)

From (5) and (10), block diagram of dc motor with  $T_{\rm L}$  can be obtained.

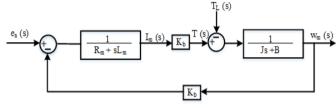


Fig 11:- Block diagram for DC motor with TL

With load torque, transfer function of dc motor system is

$$\omega_{\rm m}(s) = G_1(s) + G_2(s)$$
 (11)

where  $G_1(s)$  is

 $\frac{\omega_m(s)}{e_a(s)} = \frac{K_b}{s^2(JL_m) + s(R_mJ + L_mB) + (K_b^2 + R_mB)}$ 

 $G_2(s)$  is

$$\frac{\omega_{m}(s)}{T_{L}(s)} = \frac{-R_{m}-sL_{m}}{s^{2}(JL_{m})+s(R_{m}J+L_{m}B)+(K_{b}^{2}+R_{m}B)}$$

By substituting the dc motor parameters, the openloop transfer function of Fig. 12 is obtained.

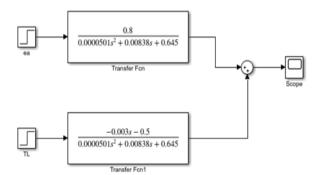
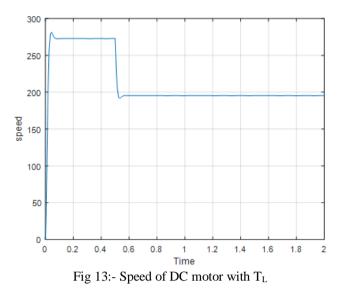


Fig 12:- Open-loop transfer function of DC motor with T<sub>L</sub>

The speed of motor output is got by simulation and shown in Fig. 13.



### C. State-space Modeling of DC Motor with $T_L$

The dynamic equations are transformed to state-space form and

$$\begin{bmatrix} D\omega_{m} \\ Di_{m} \end{bmatrix} = \begin{bmatrix} -\frac{B}{J} & \frac{K_{b}}{J} \\ -\frac{K_{b}}{L_{m}} & -\frac{R_{m}}{L_{m}} \end{bmatrix} \begin{bmatrix} \omega_{m} \\ i_{m} \end{bmatrix} + \begin{bmatrix} \frac{1}{J} \\ \frac{1}{L_{m}} \end{bmatrix} \begin{bmatrix} T_{L} \\ e_{a} \end{bmatrix}$$
(12)

From substituting the motor parameters, the roots of system can be obtained.

$$A = \begin{bmatrix} -0.599 & 47.904 \\ -266.67 & -166.67 \end{bmatrix},$$
$$B = \begin{bmatrix} 59.88 \\ 333.33 \end{bmatrix}$$

These roots will always have a negative real part, indicating that the motor is stable in open-loop operation.

### D. Performance of DC Motor with $T_L$

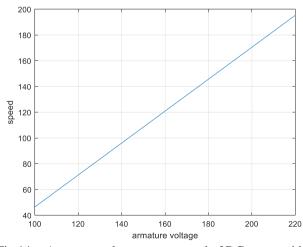


Fig 14:- Armature voltage versus speed of DC motor with  $T_L$ 

When supplied armature voltage of dc motor is varied from 100V to 220V, the speed will be also varied from 46.511 rad/sec to 195.35 rad/sec. The performance characteristics curve of armature voltage versus speed is shown in Fig. 14.

### VI. CONCLUSIONS

This paper is intended to the beginner who wants to study the basic usage of MATLAB Simulink Software. Furthermore, the basic theory and operation of separately excited or armature controlled dc motor is also expressed so that everybody can understand without any trouble. Performances of DC motor with and without load torque are described in this paper. Simulation results are obtained by using mathematical modeling and transfer function modeling. DC motors are widely used in industrial and commercial applications and it is needed to know the characteristics and performance of them. Simulation of speed control systems of dc motor with MATLAB Simulink Software is recommended for further study.

# ACKNOWLEDGMENT

The authors would like to thank all people who give their tremendous moral supports during our study.

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