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Mathematical Modeling and Politics: An Overview

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Abstract:- Mathematical study through the modeling is a process of converting real-world problems into mathematical problems and solving them with certain feasible conditions, then interpreting those solutions to obtain the long term results. It has a wide range of applications in the society and of those applications one of them is in politics. Politics is a set of activities associated with the governance of a country. In democratic countries people elect their representative forming a governing body. Hence in this paper a mathematical overview of non-linear mathematical models of political parties using epidemiological approach is discussed. Stability of equilibrium points has also been analyzed.

Keywords:- *Political Parties, Epidemiological Approach, Stability, Equilibria.*

I. INTRODUCTION

Majority rule government is an arrangement of government where the residents practice control by casting a ballot. In majority rule government the fundamental procedure is race. The residents choose agents among themselves. These delegates meet to shape an administering body. This is the procedure by which majority rule government fills workplaces in the lawmaking body and parliament.

In India it is single and uniform citizenship where every person who is a resident and also a major can cast vote[5]. In India election is based single vote method, where in the standing leader wins the election by attaining maximum votes in their favor. In almost all the democratic countries single vote method and generally only two parties fight election. If any other minor parties exist then they join the major party based on the circumstances[2]. Nowadays shifting of parties has become common. If the candidate feels they have not got proper position/weight then they jump to other parties. This is referred to as switching[6]. One of the reasons for switching is corruption which is the biggest social problem today in democratic countries. Modeling is done using the co-infection type epidemiological differential equation.

II. REVIEW OF LITERATURE

A. Hopf bifurcation in multiparty political systems with time delay in switching (Q.J.A. Khan)

In this paper creator has considered the model comprising of four coupled normal differential conditions. A model comprises of three ideological groups and a gathering of voters which does not bolster any of these gatherings. Equilibria and stability analysis are carried out. Time delay is considered to be bifurcation parameter. In this paper author assumes model to be constant. In countries like India, Pakistan, Sri Lanka and many more they have multiparty political system. In these countries switching is the main problem. Author has implemented prey-predator model in this paper.

The model consist of the following ordinary differential equations.

$$\frac{dx_1}{dt} = a_1x_1 - d_1x_1 + \frac{\beta_1 x_1^2 x_3(t-r)}{x_1+x_2} + d_2p_{21}x_2$$

$$\frac{dx_2}{dt} = a_2x_2 - d_2x_2 + \frac{\beta_2 x_2^2 x_3(t-r)}{x_1+x_2} + d_1p_{12}x_1$$

$$\frac{dx_3}{dt} = a_3x_3 - d_3x_3 - \frac{\beta_1 x_1^2 x_3}{x_1+x_2} - \frac{\beta_2 x_2^2 x_3}{x_1+x_2} + d_1p_{13}x_1 + d_2p_{23}x_2$$

$$\frac{dx_4}{dt} = \frac{\beta_1 x_1^2 x_3}{x_1+x_2} + \frac{\beta_2 x_2^2 x_3}{x_1+x_2} - \frac{\beta_1 x_1^2 x_3(t-r)}{x_1+x_2} - \frac{\beta_2 x_2^2 x_3(t-r)}{x_1+x_2}$$

Where,
x_1 – number of people of ruling party
x_2 – number of people of opposition party
x_3 – third party
x_4 – the number of individuals who are at present not an individual from any party at time t
a_i – rates at which members enter into the ruling, opposition and the third party

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d_i – rate at which member individuals from the decision, restriction and the outsider go into different gatherings
$x_3(t - r)$ – the members of the third party who leave the party at time t – r and enter into new party at time t
p_{ij} – the probabilities of successful transition from the i^{th} party to the j^{th} $(i \neq j)$
β_i – conversion rate of members of third party to ruling part x_1 or opposition party x_2
The author has assumed the model to be preservationist on the grounds that the all out number of voters is consistent over a brief timeframe

$$\frac{dx_1}{dt} + \frac{dx_2}{dt} + \frac{dx_3}{dt} + \frac{dx_4}{dt} = 0$$

i.e., $(a_1 - d_1)\, x_1 + (a_2 - d_2)\, x_2 + (a_3 - d_3)x_3 + d_1(p_{12} + p_{13})\, x_1 + d_2 (p_{21} + p_{23})\, x_2 = 0$

In this paper, author has formulated and analyzed a basic mathematical model for multiparty political system. Switching is the biggest problem in democratic countries and which can uproot the governments. Author has examined hopf bifurcation in the neighborhood of nonzero balance by requiring some serious energy delay as bifurcation parameter. What's more, has discovered a locale of precariousness in the area of a nonzero harmony where all the three ideological groups will endure experiencing customary variances.

B. A simple mathematical model for the spread of two political parties (Arvind Kumar Misra)

A non linear mathematical model for the spread of two political parties has been proposed and analyzed in this paper. In the proposed model epidemic approach has been used. Author has assumed the all out populace taking an interest in the framework is steady. In this investigation a predator-prey type model for the association between the gatherings has been considered and the condition for Hopf-bifurcation has additionally been inferred. Keeping in consideration of all the conditions the following epidemiological differential equations are modeled.

$$\frac{dV}{dt} = \mu N - \beta_1 V \frac{B}{N} - \beta_2 V \frac{C}{N} - \mu V$$

$$\frac{dB}{dt} = \beta_1 V \frac{B}{N} - \theta_1 B \frac{C}{N} + \theta_2 C \frac{B}{N} - \mu B$$

$$\frac{dC}{dt} = \beta_2 V \frac{C}{N} + \theta_1 B \frac{C}{N} - \theta_2 C \frac{B}{N} - \mu C$$

Where $V(0) > 0$, $B(0) \geq 0$, $C(0) \geq$)

Adding all the three equations of the model, we obtain that $dN/dt = 0$ showing that the population N is constant over time and $V + B + C = N$. The net shifting of members will be either from party B to part C or vice versa. Put $\theta_1 - \theta_2 = \theta$. With this the above model reduces to

$$\frac{dV}{dt} = \mu N - \beta_1 V \frac{B}{N} - \beta_2 V \frac{C}{N} - \mu V$$

$$\frac{dB}{dt} = \beta_1 V \frac{B}{N} - \theta B \frac{C}{N} - \mu B$$

$$\frac{dC}{dt} = \beta_2 V \frac{C}{N} + \theta B \frac{C}{N} - \mu C$$

Assuming that $\theta > 0$. It implies that net rate of shifting of party members is from political party B to political party C.

In this paper author has proposed and analyzed an epidemiological non linear mathematical model for two parties. For the existence of any party its members and supports play a major role. A party may die out if it does not have members/supporters. Party should set aside a few minutes to time changes in their belief system as per the current individual from the gathering. Numerical reproduction demonstrates a little change in the development of one gathering to another may change the outcomes extremely quick.

C. Mathematical and computer modeling of elections with constant demographic factor (Temur Chilachava and Leila Sulava)

Transformation of bipartisan elections into three is studied and analyzed within inter-election period. Some special cases are studied. In the model considered the coefficients are taken as variable time functions. The non linear mathematical model with variable coefficient is proposed. The model takes into account the change in the total number of voters between two consecutive elections. Exact analytical solutions are obtained. The model is governed by the following differential equations:

$$\frac{dN_1}{dt} = (\alpha_1 - \alpha_2)N_1N_2 + (\alpha_1 - \alpha_3)N_1N_3 - F_1N_1 + \gamma_1N_1$$

$$\frac{dN_2}{dt} = (\alpha_2 - \alpha_1)N_1N_2 + (\alpha_2 - \alpha_3)N_2N_3 - F_2N_2 + \gamma_2N_2$$

$$\frac{dN_3}{dt} = (\alpha_3 - \alpha_1)N_1N_3 + (\alpha_3 - \alpha_2)N_2N_3 + F_1(N_1, t) + F_2(N_2, t) + \gamma_3N_3$$

Where

$N_1(0) = N_{10}$, $N_2(0) = 0$, $N_3(0) = N_{30}$, $N_{30} > N_{10}$

$N_2(t) = 0$, $F_2(N_2,t) = 0$, $t \in [0,T_1)$, $T_1 < T$

$N_2(T_1) = N_{20} > 0$, $N_1, N_2, N_3 \in C[0,T]$

N_1, N_2, N_3 - are the number of supports of two opposition and one ruling party at time t

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$t = 0$ – is the time of previous elections, when one of the parties N_3 won the election and became the ruling party.

$t = T$ – is the time of the following elections

$t = T_1$ $(0 < T_1 < T)$ – time point, when to political arena there is the second opposition party N_2

α_1, α_2, α_3 – are the coefficients of attracting votes by the first and second opposition party and the ruling party at time t.

$F_1(N_1,t)$, $F_2(N_2,t)$ – are the continuous positive functions, that define the scale of used administrative resources.

γ_1, γ_2, γ_3 – are the coefficients that describe demographic changes of the parties.

Author has proposed and analyzed non linear mathematical model of transition of bipartisan elections to three party elections. The obtained exact solution can be used by both the ruling and opposition parties to correct their actions and achieving the desired result. Numerical results obtained can be used by both the ruling and opposition parties to achieve their goals.

III. CONCLUSION

A non linear mathematical model for the connection of ideological groups has been proposed and dissected utilizing epidemiological methodology. Mathematical model may lead to a better understanding of the nature of representative government under different institutional arrangements and electoral rules. Switching is the big problem in all the democratic countries. Party members switch from one party to another due to no proper weight/position in their party. The survival of any parties depends on retaining the existing members than new recruitment of members from voter class. It is very important to study mathematical models to investigate the effect of switching on the future strength of various political parties. Models have been proposed for two and three political parties with time delay as bifurcation parameter. Conditions for the co-existence of the political parties have been obtained. Numerical simulation is also performed to support the analytical results.

FUTURE SCOPE

Further the research could be proposed to know when a non voter turns out to be a voter class, what would be the impact on the political parties. The another model is when a third party forms a collation government how would political scenario change keeping the model constant.

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