ISSN No:-2456-2165

A Review - Mathematical Modelling on Water Pollution and Its Effects on Aquatic Species

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Abstract:- This review is intended to summarise the harmful effects of water pollution and the threat to aquatic species to provide a solution using mathematical modelling. The recent developed models help us to predict the changes in water quality and to stop further damage. In this paper, it has been closely observed that the water bodies become polluted due to release of untreated waste, sewage, dissolved oxygen, bacteria and toxic chemical substances from Industries which is worsening the physical, chemical and biological properties of water. This has been clearly explained in the paper with the help of numerous references. The amount of dissolved oxygen in water bodies, their pH values, nutrients, Biochemical Oxygen Demand and the process of self-purification is also discussed. The mathematical concepts used for one-dimensional habitats to solve non-linear differential equations.

Keywords:- *Mathematical Modelling, Water Pollution, Water Quality, One-Dimensional Habitat, Biochemical Oxygen Demand.*

I. INTRODUCTION

Ecosystem comprises of plants, animals, microorganisms and human together with their environment. Environment includes all the factors such as physical, chemical, biological affecting the ecosystem [1] the unique geographical and biological features like soil, climate, flora and fauna create a variety of different types of ecosystem such as forest, fields, Ocean etc. The abnormal and unusual environmental and ecological disturbances can upset the stability and even destroy the ecosystem. Environmental pollution can be defined as any undesirable changes in the physical, chemical or biological characteristics of any component of the environment which can cause harmful effects on various forms of life and properties. One of the major classifications of environmental pollution is water pollution.

Water – the most important constituent for existence of life on earth is being polluted every day. This has become an increasing problem. The water bodies become polluted due to release of untreated waste, sewage, agricultural discharge and toxic metals. The effect of this is that the pollutants impart colour, taste, and odour, to receiving water thus making it unfit for use. The organic matter in the waste decreases the oxygen content of the water. The following are the effects of pollutants in environment: pollutants affect the species in the most noticeable way; direct effect of the toxicant on the organisms is their death, which can be perceived and assessed more readily. The pollutants can even affect the species indirectly by destroying another species of the food chain or food web and thus destabilizing the trophic structure. The most challenging problem to the society is the change in the environment caused by the pollution affecting the long term survival of the aquatic species, human life style and bio-diversity of the habitat [2] a great quantity of the toxicants and contaminants enter into the ecosystems continuously which threaten the survival of the exposed population including human beings. Aquatic environment is getting polluted by many different types of toxic metal which are discharged from the industries and agricultural fields. The chief pollutants which are produced by the industries are heavy metals and radioactive substances. In the agricultural field, fertilizer, pesticides and insecticides are used for more production and control of plant diseases. The fertilizers, pesticides and insecticides containing toxic metals reach the water bodies through surface runoff, causing harms to aquatic life. The effects are explained in brief and solutions are suggested using mathematical modelling in nutshell [3].

II. REVIEW OF LITERATURE

Ganga – one of the holiest rivers of India is polluted. The objective is to improve the water quality of the river to acceptable standards by stopping pollutants entering the river and to make sure that the sewage is treated before it is discharged into the river [4] this paper puts forward the state of the river and the steps that need to be taken to make Ganga pure. The Mathematical modelling on water pollution and self-purification of river Ganges by Rajat Kaushik, 2015, Pelagia Research Library, Ganga- the River, its pollution and what can we do to clean it by Sunita Narain, 2014 Centre for Science and Environment. The Polluted wastewater being discharged in the rivers – A disposal of pollutants in river is studied in the paper Mathematical Modelling of Disposal of Pollutant in Rivers (R. V. Waghmare and S. B. Kiwne, 2017, International Journal of Computational and Applied Mathematics , 835-842). The effects on pollutants in aquatic environmrnt is

ISSN No:-2456-2165

studied in the paper - A Mathematical Approach to Study the Effect of Pollutants/Toxicants in Aquatic Environment (Anita Chaturvedi, Kokila Ramesh, Vatsala 2017) Ecosystem is studied in Environmental science by R Geetha Balakrishna**,** K.G.Lakshminarayana bhatta (2007). Mathematical techniques, classification illustration is studied. Mathematical modelling through ordinary differential equations is studied. J. N. Kapur (2015) Mathematical Modelling, Second Edition, New age international publishers.

III. MATERIALS & METHODS

A. Mathematical modelling on water pollution and self-purification of river Ganges (Rajat Kaushik, 2015, Pelagia Research Library)

In this paper, author throws light on the dying conditions of the holy river Ganga which is highly polluted. It has been closely observed by the author that the cities like Kanpur, Varanasi, and Allahabad are polluted [5] these cities release the untreated waste, sewage, toxic chemical substances from Industries which in turn lead to pollution of the river Ganga. The author also talks about the self-purification of water, as the river moves forward with the distance it gets purified on its own. This has been clearly explained in the paper with the help of numerous examples. The Fig.1. Shows the polluted Kanpur-Varanasi stretch of river Ganga which is unfit for utility.

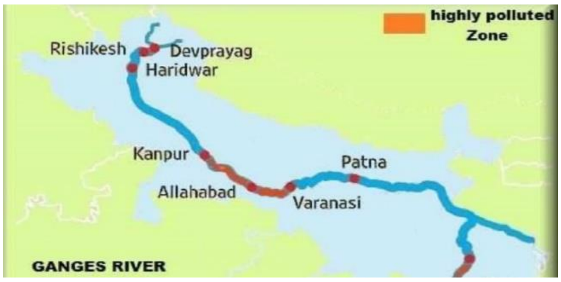

Fig. 1:- Polluted Kanpur and Varanasi stretch [6]

To get a clear picture the author formulates a dynamic problem as an example of mathematical model. The following are the governing equations:

From mass conservation equation,

$$\Delta S_t = \{M + FC - F\frac{(S_0 - S_t)}{2V}\}$$

$$S_t = \frac{Mt + v_1 A_1 C - \frac{S_0}{2V} v_2 A_2}{\frac{1}{t} + \frac{v_2 A_2}{2V}}$$

If velocity flow rate at the initial and the end point of the stretch is same,

$$v_1 = v_2 = v$$

$$S_t = \frac{M + v(A_1 C - \frac{S_0}{2V} A_2)}{\frac{1}{t} + \frac{v_2 A_2}{2V}}$$

S_0 be the initial time quantity of industrial waste, sewage in polluted stretch,
S_t be the quantity of industrial waste, sewage increase after 't' days,
$S_t - S_0 = \Delta S_t$ be the total increment of industrial waste/sewage after't' days,
M be the amount of untreated sewage, industrial waste releasing in the most polluted stretch per day,
F be the daily flow of river,
C be the concentration of pollutants in the water entering the most polluted stretch,
CF be the total sewage, industrial waste, pollutants entering the polluted stretch,
$\frac{S_0}{V}$ be initial concentration of pollutant water in the stretch if total volume of water in stretch is V,
$\frac{S_t}{V}$ be concentration after 't' days,

During period of't' days we can assume that concentration of river in polluted stretch remains as $\frac{(S_0 - S_t)}{2V}$

The author shows mathematically that if sewage treatment is increased, the pollution decreases. Also author suggests to ban the daily release of pollutants.

i.e., $M \rightarrow 0$, from the above equations we get,

$$(S_t - S_0)\{1 + \frac{Ft}{2V}\} = FCt - \frac{FS_0}{2V}t \quad \text{as } M \rightarrow 0$$

$\frac{S_0}{V} = C_0 =$ Initial concentration of sewage/industrial waste of polluted stretch

$$(S_t - S_0) = \frac{F(C - C_0)t}{\{1 + \frac{Ft}{2V}\}}$$

As the author considered Kanpur – Varanasi stretch as the most polluted part of river, therefore $C - C_0 < 0$ *or* $C < C_0$, therefore $S_t - S_0 < 0$ or $S_t < S_0$

This concludes that the river will recover from pollution. If equilibrium state is obtained then $S_t - S_0 = 0$, *i.e.*, $C = C_0$ (After a sufficient time)

From the above analysis, it is concluded that if the industries and populated cities continue to release the waste, harmful pollutants in the river, the pollution will increase and self-purification will not be effective. Mathematically the author shows that if sewage is banned or treated the pollution decreases.

B. Mathematical Modelling of Disposal of Pollutant in Rivers (R. V. Waghmare and S. B. Kiwne, 2017, International Journal of Computational and Applied Mathematics , 835-842)

In this paper, authors talk about the polluted wastewater being discharged in the rivers which contain dissolved oxygen (DO), bacteria, nutrients, PH, toxic which is worsening the physical, chemical and biological

ISSN No:-2456-2165

properties of water [7] This is affecting the water quality, a background of water quality is been provided external inputs such as rainfall, wind, solar radiation which determine the water quality naturally. On the other hand, the water quality is also affected by the addition of waste, harmful substances by man. The concentration of these substances is determined by the dispersion and advection characteristics of the water body.

It has been observed that any natural water body can be seen as a mathematical system, which consists numerous complex interacting systems, here the basic concept of mathematical modelling of one-dimensional habitat is used.

A one-dimensional method is used since the rivers are longer rather than wide or deep and only longitudinal variations of constituent concentrations are resolved in the form of cross-section- averaged values.

The general mass conservation equation is averaged over the cross section of the stream giving, for constituent's subjects to a single first-order decay-process,

$$\frac{\partial C}{\partial t} = -u\frac{\partial C}{\partial x} + \frac{\partial}{\partial x}\left[(D_x + D_L)\frac{\partial C}{\partial x}\right] - KC + \Sigma I$$

x=longitudinal distance along river, L
D_L=longitudinal dispersion coefficient, L^2/ T
To determine the coefficient, the following formula can be used,

$$D_L = 0.011\frac{u^2B^2}{Hu*}$$

DL is the longitudinal dispersion coefficient, L^2/ T
u is the Cross- section-averaged velocity, L T
B is the Stream width, L
H is the stream depth, L
$u*$ =shear velocity,
L T =√gHs,
g =acceleration due to gravity, L^2/ T and s=stream slope, L/L

Further, the authors formulate the analytical solutions of distribution of concentration of pollutants which predicts the changes in water quality in the rivers. Also, calculates the continuous discharge in the river.

The paper describes water pollution related problems using the one-dimensional method which concentrates only on the longitudinal vibrational to resolve in the form of cross-section-averaged values. While, for continuous discharge in the river, it is been concluded that dispersion can be neglected. Also with the prompt release of a component at a point in the stream the maximum concentration decreases with time due to decay and dispersion.

C. Mathematical Modelling of the Survival of a Biological Species in Polluted Water Bodies (J.B. Shukla, A.K. Misra and Peeyush Chandra, 2007)

In this paper, authors study the depletion of dissolved oxygen in a water body caused by the discharge of pollutants with the help of a nonlinear mathematical model. Here, the problem is modelled by using several nonlinear processes which includes organic pollutants, bacteria, protozoa, dissolved oxygen and a biological species totally dependent on it. It is a well-known fact that, when the pollutants are discharged in the water bodies, it endangers the life of the biological species surviving in such a habitat. Further, the effect of the diminished level of dissolved oxygen on the survival of biological species in such an aquatic system is studied.

Authors consider a food chain type system in a water body containing pollutants, bacteria, protozoa, and a biological population whose growth rate is entirely dependent on the concentration of dissolved oxygen [8] the system is governed by the following differential equations:

$$\frac{dT}{dt} = Q - \alpha_0 T - \frac{K_1 TB}{K_{12} + K_{11}T}$$
$$\frac{dB}{dt} = \frac{\lambda_1 K_1 TB}{K_{12} + K_{11}T} - \alpha_1 B - \lambda_{10}B^2 - \frac{K_2 BP}{K_{21} + K_{22}B}$$
$$\frac{dP}{dt} = \frac{\lambda_2 K_2 BP}{K_{21} + K_{22}B} - \alpha_2 P - \lambda_{20}P^2$$
$$\frac{dC}{dt} = q - \alpha_3 C - \frac{\lambda_{12} K_1 BP}{K_{12} + K_{11}T} - \frac{\lambda_{23} K_2 BP}{K_{21} + K_{22}B} - \lambda_{11}\alpha_1 B - \lambda_{22}\alpha_2 P - K_3 CF$$
$$\frac{dF}{dt} = \lambda_3 K_3 CF - \alpha_4 F - \lambda_{30}F^2$$

Where T(0) > 0; B(0) > 0,P(0) > 0, C(0) > 0, F(0) > 0.

Here the α_i depletion rate coefficients K_i are proportionality constants, which are positive. It is seen that for the feasibility of the model, the growth rate of bacteria and protozoa should be positive.

Hence, $\lambda_1 K_1 - K_{11}\alpha_1 > 0$ *and* $\lambda_2 K_2 - K_{22}\alpha_2 > 0$

Let T be the cumulative concentration of organic pollutants
B be the density of bacteria,
P be the density of protozoa,
C be the concentration of dissolved oxygen (DO),
F be the density of a biological population,
Q be the cumulative rate of discharge of organic pollutants into the water body,
T the rate of depletion of cumulative pollutants concentration caused by natural factors is assumed to be proportional to its concentration.

A non-linear model is studies and analysed by the authors for the survival of biological species in presence of the pollutants, bacteria, protozoa, dissolved oxygen. An assumption is made that oxygen is depleted by several non-linear biochemical and biodegradation processes. As per the analysis in this paper, when the rage of discharge of pollutants increases, the equilibrium concentration of dissolved oxygen decreases.

Owing to this the amount of biological population also decreases as it is completed dependent on it. It can be

ISSN No:-2456-2165

concluded that if the amount of discharge of pollutants is very high, the equilibrium concentration of dissolved oxygen becomes negligible, which make the water unsuitable for survival of biological species.

D. A Mathematical Approach to Study the Effect of Pollutants/Toxicants in Aquatic Environment (Anita Chaturvedi, Kokila Ramesh, Vatsala 2017)

Acid accumulates through both dry and wet depositions, these deposited particles get washed from these surfaces by rain and eventually get back to water bodies. Acid lowers the pH level in water bodies and for the survival of aquatic life an optimum pH level is required. When acid lowers the pH level, the toxicity of metals increases. Owing these factors, authors proposed a mathematical model using system of non-linear differential equations with four state variable. The dependent variables are amount of acid and metal in water, density of favourable resources, density of fish population and concentration of nutrients assuming that quantity of metal is less than that of acid present in water [9] The mathematical formulation is as shown below,

$$\frac{dS}{dt} = S_0 - aS - gSP - \alpha(T_1 + qC_m) + kcP + kbF + k_1F^2$$
$$\frac{dP}{dt} = gSP - cP - fFP$$
$$\frac{dF}{dt} = fPF - bF - kF^2$$
$$\frac{d(T_1 + qC_m)}{dt} = Q_0 - \alpha(T_1 + qC_m) - \alpha_1(T_1 + qC_m)S$$

With the initial condition:

$$S(0) = S_{10} > 0, P(0) = P_{10} > 0, F(0) = F_{10} > 0, T(0) = T_{10} > 0$$

It is concluded that the amount of toxic substances present in the nutrient pool increases the equilibrium level of resource and fish populations. When there is instability of non-living equilibrium and fish extinct equilibrium the interior equilibrium exists. With the numerical example, using the set of parameters, authors calculated the interior equilibrium points for all the four state variables. Based on observation, even in the numerical simulation, the variables have reached the equilibrium points. Hence the calculated and the simulated values match with each other. Also the interactions between the variables have been studied in the form of phase plots in this paper.

IV. CONCLUSION

The review of this paper is mainly focused on water pollution and the effects of the pollutants on aquatic species. Mathematical model is studied to discuss the water pollution and the effects of the pollutants on aquatic species, changing rapidly water quality, decreasing levels of oxygen. A one-dimensional method is used for the disposal of pollutants in the river. These mathematical models have been shown to resolve the pollution problem. The model is formulated using the system of non-linear ordinary differential equations. The alternate solution is reducing the release of manmade waste, domestic sewage in water bodies and save the living creatures from harmful pollutants.

- *Advantages:* Mathematical models can simplify complex situations. The mathematical models enable us to make predictions. Mathematical Modelling helps us to get a better understanding of the real world problem.

- *Disadvantages:* A mathematical model is a simplification of real world problem and doesn't include all the parameters of the problem [10] the model may only work in certain situations. There are still an equally large number of situations which have not yet been mathematically modelled because the situations are sufficiently complex.

REFERENCES

[1]. R Geetha Balakrishna**,** K.G.Lakshminarayana bhatta (2007) Environmental science.

[2]. Asha Bharathi A.T Effect of Acid and Metal on Resource Based Population Incorporating Nutrient-Cycling.

[3]. Anil K Dwivedi (2017) Researches in water pollution: a review, *International Research Journal of Natural and Applied Sciences*.

[4]. Sunita Narain, (2014) Ganga- the River, its pollution and what can we do to clean it, Centre for Science and Environment.

[5]. Rajat Kaushik (2015) Mathematical modelling on water pollution and self-purification of river Ganges, *Pelagia Research Library*.

[6]. Fig .1.Is taken from Rajat Kaushik (2015) Mathematical modelling on water pollution and self-purification of river Ganges, *Pelagia Research Library*.

[7]. R. V., W., & S. B., K. (2017). Mathematical Modeling of Disposal of Pollutant in Rivers. *International Journal of Computational and Applied Mathematics* , 835-842.

[8]. J.B.Shukla, A.K.Misra and Peeyush Chandra (2007) Mathematical Modeling of the Survival of a Biological Species in Polluted Water Bodies.

[9]. Anita Chaturvedi, Kokila Ramesh, Vatsala (2017) A Mathematical Approach to Study the Effect of Pollutants/Toxicants in Aquatic Environment.

[10]. J. N. Kapur (2015) Mathematical Modelling, Second Edition, New age international publishers.

[11]. Ahamad S. and Rao M.R.M (1999) *Theory of Ordinary Differential Equations with Applications in Biology and Engineering,* East West Press Privet Ltd. New Delhi.