Convexity of Y- Edge DominationVariants of Corona Product Graph of a Cycle with a Star

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Abstract:- The theory of Graphs is one of the major areas of combinatorics that has developed into an important branch of Mathematics. The theory of domination in graphs is an emerging area of research in graph theory today. It has been studied extensively and finds applications to various branches of Science & Technology. Frucht and Harary [8] introduced a new product on two graphs G_1 and G_2, called corona product denoted by $G_1 \odot G_2$.

In this paper, some results on convexity of minimal edge,total edge ; minimal signed, total signed ; minimal Roman and total Roman edge dominating functions of corona product graph of a cycle with a star are discussed.

Keywords:- *Corona Product, Edge Dominating Function, Total Edge Dominating Function, Signed Edge Dominating Function, Total Signed Edge Dominating Function, Roman Edge Dominating Function, Total Roman Edge Dominating Function, Convexity of Functions.*

I. INTRODUCTION

The theory of Graphs is one of the important branches of Mathematics. The major development of graph theory has occurred in recent years and inspired to a larger degree and it has become the source of interest to many researchers due to its applications to various branches of Science & Technology.

Domination in graphs has been studied extensively in recent years. It is introduced by Ore [14] and Berge [4] and has become an emerging area of research in graph theory today. Many graph theorists, Cockayne and Hedetniemi [5,6,7], Reji kumar [15], Sampathkumar [16] and others have contributed significantly to the theory of dominating sets, domination numbers and other related topics. Haynes, Hedetniemi and Slater [9,10] presented a survey of articles in the wide field of domination in graphs.

Another type of domination is total domination. Total dominating sets are introduced by Cockayane, Dawes and Hedetniemi [5]. The concept of edge domination was introduced by Mitchell and Hedetniemi [13] and it is explored by many researchers. Arumugam and Velammal [2] have discussed the edge domination in graphs while the fractional edge domination in graphs is discussed in Arumugam and Jerry [1]. The complementary edge domination in graphs is studied by Kulli and Soner [12]. The edge dominating sets are studied by Yannakakis [22]. The edge domination in graphs of cubes and Signed total domination is studied by Zelinka [24, 25].

Product of graphs occurs naturally in discrete mathematics as tools in combinatorial constructions. They give rise to an important classes of graphs and deep structural problems. Frucht and Harary [8] introduced a new product on two graphs G_1 and G_2, called corona product denoted by $G_1 \odot G_2$. This new concept enhances the study of these graphs and it is interesting to study various graph-theoretic parameters of these graphs.

Among the variations of domination, there is an extensive study of Y-domination and its variations. The Y – domination problem was introduced by Bange et al. [3], where Y is a subset of real numbers. A Y – dominating function of a graph $G(V, E)$ is a function $f : V \to Y$ such that $\sum_{u \in N_G[v]} f(u) \geq 1$, for each $v \in V$. Then the Y – domination problem is to find a Y – dominating function of minimum weight for a graph. Analogously Y – edge domination is defined.

Recently, dominating functions in domination theory have received much attention. A purely graph – theoretic motivation is given by the fact that the dominating function problem can be seen, in a clear sense, as a proper generalization of the classical domination problem.Similarly edge dominating functions are also studied extensively.

II. CORONA PRODUCT GRAPH $C_n \odot K_{1,m}$

The **corona product** of a cycle C_n with a star graph $K_{1,m}$ form ≥ 2, is a graph obtained by taking one copy of a n-vertex graph C_n and n copies of $K_{1,m}$ and then joining the ith vertex of C_n to all vertices of ith copy of $K_{1,m}$. This graph is denoted by $C_n \odot K_{1,m}$.

The vertices in C_n are denoted by $v_1, v_2, \dots\dots\dots, v_n$ and the edges in C_n by $e_1, e_2, \dots\dots, e_n$ where e_i is the edge joining the vertices v_i and v_{i+1}, i≠n. For i = n, e_n is the edge joining the vertices $v_n$and v_1.

The vertex in the first partition of $i^{th} copy of K_{1,m}$ is denoted by u_i and the vertices in the second partition of $i^{th} copy of K_{1,m}$ are denoted by $w_{i1}, w_{i2}, \dots\dots\dots, w_{im}$. The edges in the $i^{th} copy of K_{1,m}$ are denoted by l_{ij} where l_{ij} is the edge joining the vertex u_i to the vertex w_{ij}. There are another type of edges, denoted by h_i, h_{ij}. Here h_i is the edge joining the vertex v_i in C_n to the vertex u_i in the $i^{th} copy of K_{1,m}$. The edge h_{ij} is the edge joining the vertex v_i in C_n to the vertex w_{ij} in the $i^{th} copy of K_{1,m}$.

The edge induced sub graph on the set of edges

$E_i = \{h_i, h_{ij}, l_{ij} : j = 1,2,\ldots\ldots, m\}$is denoted by H_i, for i = 1,2,.......,n.

Some basic graph theoretic properties and edge dominating sets of $G = C_n \odot K_{1,m}$. are studied in Sreedevi, J [17]. Also some results on minimal edge and total edge dominating functions (MEDF, MTEDF) of $G = C_n \odot K_{1,m}$. are presented in Sreedevi, J [18,19]. Further the concepts of minimal signed and Roman edge dominating functions (MSEDF, MREDF) and minimal total signed and Roman edge dominating functions (MTSEDF, MTREDF)are studied by Sreedevi, J [20,21].

In this paper we discuss the convexity of variants of Y – edge dominating functions and observed that the convex combination of these functions is also minimal in certain cases and is not minimal in certain other cases.

III. CONVEXITY OF MINIMAL EDGE, SIGNED AND ROMANEDGE DOMINATING FUNCTIONS

A study of convexity and minimality of dominating functions (MDF) are given in Cockayne et al. [5,6] and Yu [23]. Reji kumar [15] developed a necessary and sufficient condition for the convex combination of two MDFs to be again a MDF. Jeelani Begum [11] studied convexity of MDFs of Quadratic Residue Cayley Graphs.

In this section, we discuss the convexity of minimal edge, signed and Roman edge dominating functions of the corona product graph $G = C_n \odot K_{1,m}$. First we define the convex combination of functions and prove some results on the convexity of MEDFs, MSEDFs, MREDFs of G.

Definition: Let G(V,E) be a graph. Let f and g be two functions from E to [0,1] and $\lambda \in (0,1)$. Then the function h: E → [0,1] defined by h(e) = λf(e) + (1-λ) g(e) is called a convex combination of f and g.

First we consider minimal edge dominating sets, minimal edge dominating functions and discuss the convexity of these functions.

Theorem 3.1: Let D_1,D_2 be two MEDSs of $G = C_n \odot K_{1,m}$. Let f_1: E → [0,1] and

f_2: E → [0,1]be defined by

$$f_1(e) = \begin{cases} 1, & \text{if } e \in D_1, \\ 0, & \text{otherwise.} \end{cases} \quad \text{and} \quad f_2(e) = \begin{cases} 1, & \text{if } e \in D_2, \\ 0, & \text{otherwise.} \end{cases}$$

Then the convex combination of f_1 and f_2 becomes a MEDF of $G = C_n \odot K_{1,m}$.

Proof: Let D_1,D_2 be two MEDSs of G. Let f_1 and $f_2$be two functions defined as in the hypothesis. Then these functions are MEDFs of $G = C_n \odot K_{1,m}$.[18]

Let $h(e) = \alpha f_1(e) + \beta f_2(e)$, where $\alpha + \beta = 1, 0 < \alpha < 1$ and $0 < \beta < 1$.

Case 1: Suppose $D_1 \cap D_2 \neq \emptyset$.

Then for $e \in E$, the possible values of h(e) are

$$h(e) = \begin{cases} \alpha, & \text{if } e \in D_1 - D_2, \\ \beta, & \text{if } e \in D_2 - D_1, \\ \alpha + \beta, & \text{if } e \in D_1 \cap D_2, \\ 0, & \text{otherwise.} \end{cases}$$

Now $\sum_{e \in N[l]} h(e)$
$= s\alpha + t\beta$, if s edges of D_1 and t edges of D_2 are in $N[l]$.

Therefore $\sum_{e \in N[l]} h(e) \geq 1$ for each l ∈ E .

This implies that h is an EDF.
Now we check for the minimality of h.

Define $g: E \to [0,1]$ by

$$g(e) = \begin{cases} r, & \text{if } e = e_i \in D_1 \cap D_2, \\ \alpha + \beta, & \text{if } e \in (D_1 \cap D_2) - \{e_i\}, \\ \alpha, & \text{if } e \in D_1 - D_2, \\ \beta, & \text{if } e \in D_2 - D_1, \\ 0, & \text{otherwise,} \end{cases}$$

where $0 < r < 1$.

Since strict inequality holds at the edge $e_i \in E$, it follows that $g < h$.

Then $\sum_{e \in N[l]} g(e)$

$$= \begin{cases} r, & \text{if } l \in i^{th} copy of K_{1,m} \text{ in G}, \\ s\alpha + t\beta + r, & \text{if s edges of } D_1 \text{, t edges of } D_2 \text{ and } e_i \text{ are in } N[l], \\ s\alpha + t\beta, & \text{if s edges of } D_1 \text{ and t edges of } D_2 \text{ are in } N[l]. \end{cases}$$

This implies that $\sum_{e \in N[l]} g(e) = r$
< 1 for the edges in the $i^{th} copy of K_{1,m}$ in G.

So g is not an EDF.

Since g is defined arbitrarily, it follows that there exists no g < h such that g is an EDF.

Thus h is a MEDF.

Case 2: Suppose $D_1 \cap D_2 = \emptyset$.

Then for e$\in E$, the possible values of h(e) are

$$h(e) = \begin{cases} \alpha, & \text{if} e \in D_1, \\ \beta, & \text{if} e \in D_2, \\ 0, & \text{otherwise.} \end{cases}$$

Now $\sum_{e\in N[l]} h(e) = s\alpha + t\beta$, if s edges of D_1 and t edges of D_2 are in $N[l]$.

Therefor $\sum_{e\in N[l]} h(e) \geq 1$ for each l ∈ E.

This implies that h is an EDF.

Now we check for the minimality of h.

Define $g: E \to [0,1]$ by

$$g(e) = \begin{cases} r, & \text{if} e = e_i \in D_1, \\ \alpha, & \text{if} e \in D_1 - \{e_i\}, \\ \beta, & \text{if} e \in D_2, \\ 0, & \text{otherwise,} \end{cases}$$

where $0 < r < \alpha$.

Since strict inequality holds at the edge $e_i \in E$, it follows that $g < h$.

$$\text{Then} \sum_{e\in N[l]} g(e) = \begin{cases} r+\beta, & if l \in i^{th} copy of K_{1,m} \text{ in G}, \\ s\alpha + t\beta + r, & \text{if s edges of } D_1 \text{ ,t edges of } D_2 \text{ and } e_i \text{ are in } N[l], \\ s\alpha + t\beta, & \text{if s edges of } D_1 \text{ and t edges of } D_2 \text{ are in } N[l]. \end{cases}$$

This implies that

$$\sum_{e\in N[l]} g(e) = r + \beta < \alpha + \beta = 1$$

for the edges in the $i^{th} copy of K_{1,m}$ in G.

So g is not an EDF.

Since g is defined arbitrarily, it follows that there exists no g $<$ h such that g is an EDF.

Thus h is a MEDF.

Now we discuss convexity of MSEDFs of $G = C_n \odot K_{1,m}$.

Theorem 3.2: Let f_1 and $f_2$be two minimal signed edge dominating functions of$G = C_n \odot K_{1,m}$defined from E to $\{-1,1\}$ by

f_1 (e) =

$$\begin{cases} -1, & \text{for} \left\lfloor \frac{m+1}{2} \right\rfloor \text{ edges in each copy of } K_{1,m} \text{in } G, \\ 1, & \text{otherwise,} \end{cases}$$

And

$f_2(e))$ =

$$\begin{cases} -1, & \text{for} \left\lfloor \frac{m+1}{2} \right\rfloor \text{ edges in each copy of } K_{1,m} \text{in } G, \\ 1, & \text{otherwise,} \end{cases}$$

Then the convex combination h of f_1 and f_2 becomes a MSEDF, if

$f_1(e) = f_2(e) \forall\ e \in E$. Otherwise h is not a signed edge dominating function of G.

Proof: let f_1 and f_2 be defined as in the hypothesis. Then these functions are MSEDSs of $G = C_n \odot K_{1,m}$. [20]. Let $h(e) = \alpha f_1(e) + \beta f_2(e)$ where $\alpha + \beta = 1$ and $0 < \alpha < 1,\ 0 < \beta < 1$.

For $e \in E$, the possible values of $h(e)$ are

$$h(e) = \begin{cases} \alpha + \beta, & \text{if } f_1(e) = f_2(e) = 1, \\ -(\alpha+\beta), & \text{if } f_1(e) = f_2(e) = -1, \\ -\alpha + \beta, & \text{if } f_1(e) = -1 \text{ and } f_2(e) = 1, \\ \alpha - \beta, & \text{if } f_1(e) = 1 \text{ and } f_2(e) = -1. \end{cases}$$

Now it is clear that in the case $f_1(e) = f_2(e)$, and$h(e)$ takes the value either 1 or -1.Hence h becomes a MSEDF. [20].In the case $f_1(e) \neq f_2(e)$, h is not a signed edge dominating function because the functional values of h are not either 1 or -1.■

Now we discuss convexity of MREDFs of $G = C_n \odot K_{1,m}$.

Theorem 3.3: Let f_1 and f_2 be two minimal Roman Edge Dominating functions of $G = C_n \odot K_{1,m}$from E to $\{0,1,2\}$defined by

$$f_1(e) = \begin{cases} 2, & \text{if} e = h_i \in H_i \text{where} i = 1,2,\dots\dots,n, \\ 0, & \text{otherwise.} \end{cases}$$

$$f_2(e) = \begin{cases} 2, & \text{if} e = h_i \in H_i \text{where} i = 1,2,\dots\dots,n, \\ 0, & \text{otherwise.} \end{cases}$$

Then the convex combination of f_1 and f_2 becomes a MREDF.

Proof: Let f_1,f_2 be two functions defined as in the hypothesis.

Then these functions are MREDFs of $G = C_n \odot K_{1,m}$. [20].

1. **Let h(e) = $\boldsymbol{\alpha f_1(e) + \beta f_2(e)}$, where $\boldsymbol{\alpha + \beta = 1, 0 < \alpha < 1}$ and $\boldsymbol{0 < \beta < 1}$.**

Then by the definition of the functions f_1 andf_2, it follows that

$h(e) = f_1(e) = f_2(e), \forall e \in E$.

Also h becomes a MREDF.[20].

IV. CONVEXITY OF MINIMAL TOTAL EDGE, SIGNED AND ROMAN EDGE DOMINATING FUNCTIONS

A study of convexity and minimality of total dominating functions (MTDFs) are given in Cockayne et al.[5]. Yu.[23] obtained a necessary and sufficient condition for the convex combination of two MTDFs is to be again a MTDF.

In this section we consider minimal total edge dominating sets, minimal total edge dominating functions and discuss the convexity of these functions. Also we discuss the convexity of MTSEDFs and MTREDFs of $G = C_n \odot K_{1,m}$.

Definition: Let $G(V,E)$ be a graph. Let f and g be two functions from E to $[0,1]$ and λ∈(0,1). Then the function $h: E \to [0,1]$ defined by $h(e) = \lambda f(e) + (1-\lambda)\, g(e), \forall e \in E$ is called a **convex combination** of f and g. First minimal total edge dominating sets, minimal total edge dominating functions are considered and the convexity of these functions is discussed.

Theorem 4.1: Let $\mathbf{T_1}$ and $\mathbf{T_2}$ be two MTEDSs of $\boldsymbol{G = C_n \odot K_{1,m}}$. Let $\boldsymbol{f_1: E \to [0,1]}$ and $\boldsymbol{f_2: E \to [0,1]}$ be defined by

$$f_1(e) = \begin{cases} 1, & \text{if } e \in T_1, \\ 0, & \text{otherwise.} \end{cases}$$

and

$$f_2(e) = \begin{cases} 1, & \text{if } e \in T_2, \\ 0, & \text{otherwise.} \end{cases}$$

Then the convex combination of f_1 and f_2 becomes a MTEDF of $G = C_n \odot K_{1,m}$.

Proof: Let T_1 and T_2 be two MTEDSs of G. Let f_1, f_2 be two functions defined as in the hypothesis. Then these functions are MTEDFs of $G = C_n \odot K_{1,m}$. [19]. Let $h(e) = \alpha f_1(e) + \beta f_2(e)$ where $\alpha + \beta = 1$ and $0 < \alpha < 1,\ 0 < \beta < 1$.

Case 1: Suppose $T_1 \cap T_2 \neq \varphi$.
For $e \in E$, the possible values of $h(e)$ are

$$h(e) = \begin{cases} \alpha, & \text{if } e \in T_1 - T_2, \\ \beta, & \text{if } e \in T_2 - T_1, \\ \alpha + \beta, & \text{if } e \in T_1 \cap T_2, \\ 0, & \text{otherwise.} \end{cases}$$

Then $\sum_{l \in N(e)} h(l) = s\alpha + t\beta$, s – edges of T_1 and t – edges of T_2 are in $N(e)$.

Therefore $\sum_{l \in N(e)} h(l) \geq 1, \forall\ e \in E$.

This implies that h is a total edge dominating function. Now we check for the minimalityof h.

Define $g: E \to [0,1]$ by

$$g(e) = \begin{cases} r, & \text{if } e = e' \in T_1 \cap T_2, \\ \alpha + \beta, & \text{if } e \in T_1 \cap T_2 - \{e'\}, \\ \alpha, & \text{if } e \in T_1 - T_2, \\ \beta, & \text{if } e \in T_2 - T_1, \\ 0, & \text{otherwise.} \end{cases}$$

where $0 < r < 1$.

Since strict inequality holds at an edge $e' \in T_1 \cap T_2$, it follows that $g < h$.

Then

$$\sum_{l \in N(e)} g(l) = \begin{cases} r, \text{if } e' \in N(e) \text{ and no edge of } T_1 \text{ and } T_2 \text{ other than } e' \text{ are in } N(e) \\ s\alpha + t\beta + r, \text{ if } s - \text{edges of } T_1, t - \text{edges of } T_2 \text{ and } e' \text{ are in } N(e), \\ s\alpha + t\beta, \text{if } s - \text{edges of } T_1 \text{ and } t - \text{edges of } T_2 \text{ are in } N(e). \end{cases}$$

This implies that $\sum_{l \in N(e)} g(l) = r < 1$ when $e' \in N(e)$ and no edge of T_1 and T_2 other than e' are in $N(e)$.

So g is not a TEDF. Since g is defined arbitrarily, it follows that there exists no $g < h$ such that g is a TEDF. Thus h is a MTEDF.

Case 2: Suppose $T_1 \cap T_2 = \varphi$.
For $e \in E$, the possible values of $h(e)$ are

$$h(e) = \begin{cases} \alpha, & \text{if } e \in T_1, \\ \beta, & \text{if } e \in T_2, \\ 0, & \text{otherwise.} \end{cases}$$

Then $\sum_{l \in N(e)} h(l) = s\alpha + t\beta$, if s – edges of T_1 and t – edges of T_2 are in $N(e)$.

Therefore $\sum_{l \in N(e)} h(l) \geq 1, \forall\ e \in E$.

This implies that h is a TEDF. Now we check for the minimality of h.
Define $g: E \to [0,1]$ by

$$g(e) = \begin{cases} r, & \text{if } e = e' \in T_1, \\ \alpha, & \text{if } e \in T_1 - \{e'\}, \\ \beta, & \text{if } e \in T_2, \\ 0, & \text{otherwise.} \end{cases}$$

where $0 < r < \alpha$.
Since strict inequality holds at an edge $e' \in T_1$ it follows that $g < h$.

Then

$$\sum_{l\in N(e)} g(l) = \begin{cases} r+\beta, \text{ if } e' \in N(e) \text{ and no edge of } T_1 \text{ other than } e' \text{ is in } N(e), \\ s\alpha + t\beta + r, \text{ if } s-\text{edges of } T_1, t-\text{edges of } T_2 \text{ and } e' \text{ are in } N(e), \\ s\alpha + t\beta, \text{if } s-\text{edges of } T_1 \text{ and } t-\text{edges of } T_2 \text{ are in } N(e). \end{cases}$$

This implies that

$$\sum_{l\in N(e)} g(l) = r+\beta < \alpha+\beta = 1$$

where $e' \in N(e)$ and no edge of T_1 other than e' are in $N(e)$.
So g is not a TEDF. Since g is defined arbitrarily, it follows that there exists no $g < h$ such that g is a TEDF.
Thus h is a MTEDF.■

Theorem 4.2: Let f_1 and f_2 be two minimal total signed edge dominating functions of $G = C_n \odot K_{1,m}$ defined from E to $\{-1,1\}$ by

$$f_1(e) = \begin{cases} -1, & \text{for } \frac{m}{2} \text{ edges in each copy of } K_{1,m} \text{ in } G \text{ if } m \text{ is even}, \text{ for } \frac{m-1}{2} \text{ edges in each copy of } K_{1,m} \text{ in } G \text{ if } m \text{ is odd}, \\ 1, & \text{otherwise}, \end{cases}$$

And

$$f_1(e) = \begin{cases} -1, & \text{for } \frac{m}{2} \text{ edges in each copy of } K_{1,m} \text{ in } G \text{ if } m \text{ is even}, \text{ for } \frac{m-1}{2} \text{ edges in each copy of } K_{1,m} \text{ in } G \text{ if } m \text{ is odd}, \\ 1, & \text{otherwise}, \end{cases}$$

Then the convex combination h of f_1 and f_2 becomes a MTSEDF, if $f_1(e) = f_2(e) \forall e \in E$. Otherwise h is not a total signed edge dominating function of G.

Proof: Let f_1 and f_2 be defined as in the hypothesis. Then these functions are MTSEDFs of $G = C_n \odot K_{1,m}$. [21].
Let $h(e) = \alpha f_1(e) + \beta f_2(e)$ where $\alpha + \beta = 1$ and $0 < \alpha < 1$, $0 < \beta < 1$.
For $e \in E$, the possible values of $h(e)$ are

$$h(e) = \begin{cases} \alpha+\beta, \text{if } f_1(e) = f_2(e) = 1, \\ -(\alpha+\beta), \text{if } f_1(e) = f_2(e) = -1, \\ -\alpha+\beta, \text{if } f_1(e) = -1 \text{ and } f_2(e) = 1, \\ \alpha-\beta, \text{if } f_1(e) = 1 \text{ and } f_2(e) = -1. \end{cases}$$

Now it is clear that in the case $f_1(e) = f_2(e)$, and $h(e)$ takes the value either 1 or -1. Hence h becomes a MTSEDF.[21].In the case $f_1(e) \neq f_2(e)$, h is not a total signed edge dominating function because the functional values of h are not either 1 or -1.■

Theorem 4.3: Let f_1 and f_2 be two minimal total Roman Edge Dominating functions of $G = C_n \odot K_{1,m}$ from E to $\{0,1,2\}$ defined by

$$f_1(e) = \begin{cases} 2, & \text{for } e = h_i, i = 1,2,\ldots\ldots\ldots,n, \text{ and for one edge } l_{i1} \text{ in each copy of } K_{1,m} \text{ in G}, \\ 0, & \text{otherwise}, \end{cases}$$

$$f_2(e) = \begin{cases} 2, & \text{for } e = h_i, i = 1,2,\ldots\ldots\ldots,n, \text{ and for one edge } l_{i1} \text{ in each copy of } K_{1,m} \text{ in G}, \\ 0, & \text{otherwise}. \end{cases}$$

Then the convex combination of f_1 and f_2 becomes a MTREDF.

Proof: Let f_1,f_2 be two functions defined as in the hypothesis.
Then $f_1(e) = f_2(e), \forall e \in E$.
Then these functions are MTREDFs of $G = C_n \odot K_{1,m}$. [21].
Let $h(e) = \alpha f_1(e) + \beta f_2(e)$, where $\alpha + \beta = 1, 0 < \alpha < 1$ and $0 < \beta < 1$.
Then by definition of the functions f_1 and f_2, it follows that $h(e) = f_1(e) = f_2(e), \forall e \in E$.
Hence h becomes a MTREDF.[21].

V. CONCLUSIONS

Study of corona product graphs arising from standard graphs is interesting. Edge dominating functions, signed edge dominating functions and Roman edge dominating functions of these graphs are studied by the authors and these works are published. Introducing a new concept i.e., convexity of these functions is quite interesting and gives scope to connect graph theory and LPP. In this paper an attempt is made for the study of convexity of these functions and this throws light on further developments of research in this type of corona product graphs.

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