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Cube Difference Labeling of Theta Graphs

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Abstract:- In this work, we prove that the theta graph (T_α), Duplication of any vertex of degree 3 in T_α, path union of r copies of T_α, one point union of r(T_α), the fusion of any two vertices of T_α, switching of a central vertex in T_α are Cube difference labeling.

Keywords:- *Cube Difference Labeling (CDL), Fusion, Duplication, Switching, Path Union, One Point Union.*
AMS Subject Classification: 0578.

I. INTRODUCTION

The Cube difference labeling were introduced by Shiama [5]. For our study, we consider a simple, undirected, finite graph and we follow [1,3] for all terminology and notations. A dynamic survey on graph labeling is regularly updated be Gallian [2]. A. Sugumaran and P.Vishnu Prakash [4] proved that theta graph (T_α) admits Prime Cordial labeling.

In section 2, we collect some definitions, which are needed for the present work.

In section 3, we prove the existence of cube difference labeling for some special graphs.

II. DEFINITIONS

➢ *Definition 2.1*

A graph G = (*V*, *E*) is said to be cube difference graph (CDG) [5] if there exist a bijection $g: V(G) \rightarrow \{0,1,2,\dots p-1\}$ such that the induced function g^*: E(G) →N given by $g^*(xy) = |[g(x)]^3 - [g(y)]^3|$ is injective $\forall x, y \in E(G)$.

➢ *Definition 2.2*

A theta graph (T_α) is a block with two non-adjacent vertices of degree 3 and all other vertices of degree 2.

➢ *Definition 2.3*

A vertex v_k' is said to be a duplication of v_k if all the vertices which are adjacent to v_k are now adjacent to v_k' [4].

➢ *Definition 2.4*

A graph G_1 is established by fusing two vertices *x* and *y* (are definite vertices in *G*) by a single vertex *w* in G_1 such that every edge which was incident with either *x* or *y* are now incident with w in G_1 [4].

➢ *Definition 2.5*

A graph *G* is a graph obtained by switching the centreapexv_0 of *G*, removing all the edges incident with *v* and adding edges joining *v* to every other vertex which are not adjacent to *v* in G [4].

III. MAIN RESULTS

A. Theorem 3.1.

The Theta graph (T_α) admits cube difference labeling.

➢ *Proof:*

Consider the graph G = T_α, the theta graph with center v_0 and the edge set be

$E(G) = E_1 \cup E_2 \cup E_3$, where
$E_1 = \{v_i v_{i+1} / 1 \le i \le 5\}$
$E_2 = \{v_0 v_1, v_0 v_4\}$ and
$E_3 = \{v_1 v_6\}$
Now, $|V(T\alpha)| = 7$ and $|E(T\alpha)| = 8$

We explore a vertex valued function as
$f(v_i) = i - 1,\ 1 \le i \le 6$
$f(v_0) = 6$

Then the entire eight edges receiving the labels as,
$f^*(v_0 v_1) = [f(v_0)]^3$
$f^*(v_0 v_4) = |[f(v_0)]^3 - [f(v_4)]^3|$
$f^*(v_i v_{i+1}) = 3i^2 - 3i + 1,\ 1 \le i \le 5$
$f^*(v_1 v_6) = [f(v_6)]^3$

Thus, $f^*(e_i) \neq f^*(e_j)$, $\forall e_i e_j \in E(G)$ and all edge labeling are definite.

Hence the graph T_α admits cube difference labeling. For instance, the graph T_6 is given below.

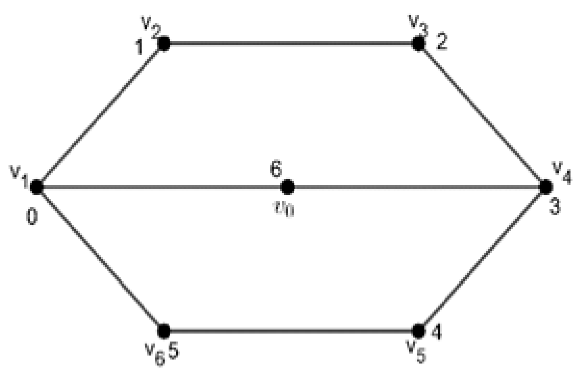

Fig 1:- T_6 Admits CDL

B. Theorem 3.2

The duplication of any vertex v_i of degree 3 in the cycle of T_α is a cube difference graph.

➢ *Proof:*

Let G be the graph obtained by duplication of any vertex v_i in T_α and v_i' be the duplicating vertex of v_i of degree 3. In T_α only two vertices are of degree 3. *i.e.*,v_1 and v_4.

Consider, $V(G) = \{v_j/0 \le j \le 7\}$and $E(G) = \{v_jv_{j+1}/1 \le j \le 5\} \cup \{v_0v_1, v_0v_4, v_1v_6\}$.
Clearly, $|V(G)| = 8$ and $|E(G)| = 11$
Now, define a vertex label $f: V \to \{0,1, \dots 7\}$ as
$f(v_j) = j - 1, 1 \le j \le 6$
$f(v_0) = 6$

$f(v_4') = 7$, where v_4' is the duplicating vertex of v_4. For the above labeling pattern, the edge labels $f^*(v_0v_1)$, $f^*(v_0v_4)$ and $f^*(v_iv_{i+1})$ are same as in the above theorem. Thus, all the edge labeling are distinct. Similarly, $f^*(v_{i-1}v_i)$ and $f^*(v_{i+1}v_i')$ are also distinct. Hence the graph G admits CDL.

Example 3.1

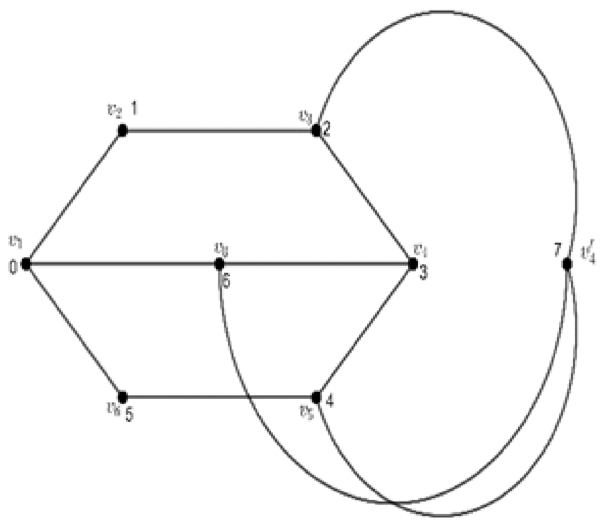

Fig 2:- Duplication of v_4 in T_6 Admits CDL

C. Theorem 3.3

The path union of r copies of theta graph T_α is cube difference graph.

➢ *Proof:*

Let G be the path union of r(T_α) with the vertex set V = $u_i^{(j)}$, $0 \le i \le 5$, $1 \le j \le r$ and the edge set $E(G) = E_1 \cup E_2 \cup E_3$, where
$E_1 = \left\{u_i^{(j)}u_{i+1}^{(j)}/1 \le i \le 5\right\}$
$E_2 = \left\{u_0^{(j)}u_1^{(j)}, u_0^{(j)}u_4^{(j)}/1 \le j \le r\right\}$
$E_3 = \left\{u_3^{(j)}u_2^{(j+1)}/1 \le j \le r-1\right\}$
Note that the cardinality of vertices and edges are*7r* and*9r-1resp.*,

Now, determine the vertex valued function $g: V \to \{0,1, \dots 7r-1\}$ as
$g(u_0^{(j)}) = 7j-1$
$g(u_i^{(j)}) = i+7j-8$

The edge set E is classified into two classes namely E_1 and E_2.

- E_1:

The edge labeled with both of their end vertices have either odd or even, form an increasing sequence of even integer.

- E_2:

The edge labeled with one vertex has odd integer and the other vertex has even integer, form an increasing sequence of odd integer.

It is easily observed that $g^*(e_i) \ne g^*(e_j)$, $\forall e_ie_j \in E(G)$. For the above defined function *f*, the induced function $g^*: E(G) \to N$ satisfies the condition of cube difference labeling. Hence the graph *G*is cube difference graph. The example for the above graph is illustrated below.

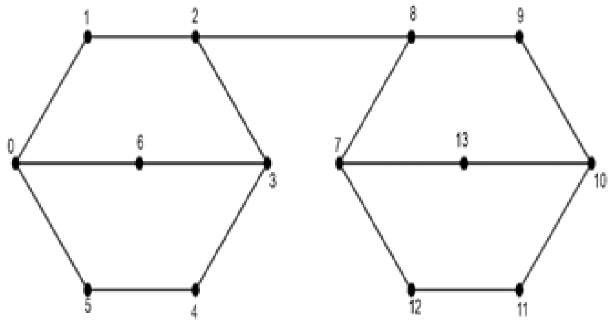

Fig 3:- The Path Union of 2(T_6) is CDL

D. Theorem 3.4.

The one point union of r copies of theta graph admits CDL.

➢ *Proof*

Consider G = (*V*, *E*) be the one point union of *r*(T_α).
Now define $V(G) = \left\{u_i^{(j)}, w/0 \le i \le 5, 1 \le j \le r\right\}$ and
$E(G) = \left\{u_i^{(j)}u_{i+1}^{(j)}\right\} \cup \left\{wu_i^{(j)}\right\} \cup \left\{wu_5^{(j)}\right\} \cup \left\{u_0^{(j)}u_3^{(j)}\right\} \cup \left\{u_0^{(j)}w\right\}$
Clearly, $|V(G)| = 6r+1$ and $|E(G)| = 8r$
Define a bijective function $f: V \to \{0,1, \dots 6r\}$ as follows:
$f(u_0^{(j)}) = 6j-1$
$f(u_i^{(j)}) = i+6j-7$ *for* $1 \le i \le 5, 1 \le j \le r$
$f(w) = 6r$

The edge set E is classified as same as mentioned in the *theorem 3.3*. Thus $f^*(e_i) \ne f^*(e_j)$, $\forall e_ie_j \in E(G)$. Hence the theorem is verified. For instance, the example of 4(T_α) given below.

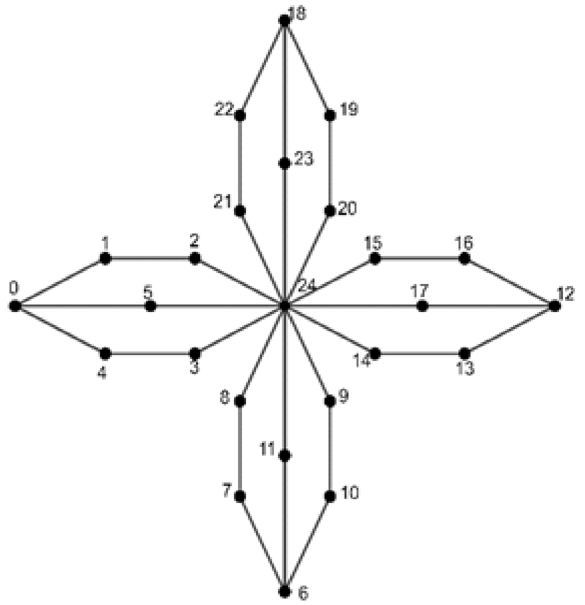

Fig 4:- CDL of 4(T_6)

E. Theorem 3.5

The fusion (identifying) of any two vertices in the cycle of T_α is CDL.

➢ *Proof:*

Let T_α be the graph with centre v_0, the vertex set V = $\{u_0, u_1,\ldots u_6\}$ and the edge set $E = E_1 \cup E_2$, where $E_1 = \{u_i u_{i+1} / 1 \le i \le 6\}$ and $E_2 = \{u_0 u_1, u_0 u_4, u_1 u_6\}$
Note that $|V(T_\alpha)| = 7$ and $|E(T_\alpha)| = 8$

Now, after the fusion of two vertices u_5 and u_6 in the cycle of T_α and we call it as u_5 we obtain the graph G. Clearly, $|V(G)| = 6$ and $|E(G)| = 7$. Define the vertex valued function $f: V \to \{0,1,\ldots 5\}$ as
$f(u_0) = 5$
$f(u_i) = i-1$

Then the induced function f^* yields edge labels as,
$f^*(u_0 u_1) = [f(u_0)]^3$
$f^*(u_0 u_4) = |[f(u_0)]^3 - [f(u_4)]^3|$
$f^*(u_i u_{i+1}) = 3i^2 - 3i + 1,$
$f^*(u_1 u_5) = [f(u_5)]^3$

It is easily observed that all the edge labels are distinct. Hence the graph *G* admits *CDL*. For the above graph, the example mentioned in *fig 5* and *6.*

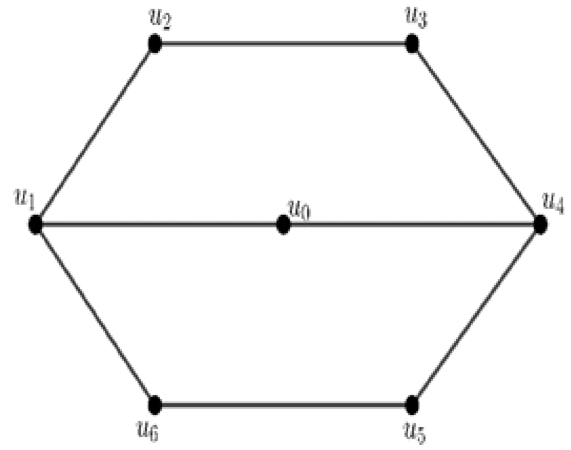

Fig 5:- Theta Graph

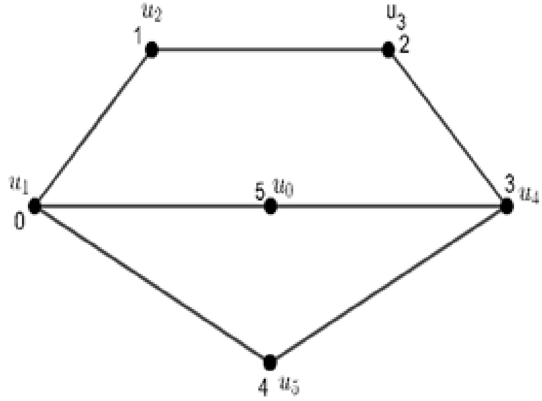

Fig 6:- The Fusion of u_5 and u_6 in Theta Graph is CDG

F. Theorem 3.6

The switching of a central vertex in T_α is cube difference graph.

➢ *Proof*

Let the graph *G* is obtained by switching the central vertex x_0 in T_α with the vertices $x_0, x_1, \ldots, x_6$ and the edges $\{x_j x_{j+1} / 1 \le j \le 6\} \cup \{x_0 x_2, x_0 x_3, x_0 x_5, x_0 x_6\}$.

The cardinality of vertices and edges are 7 and 10 resp.,

Define the bijective function $f: V \to \{0,1,\ldots 6\}$ as
$f(x_0) = 6$
$f(x_j) = j-1$

For the above labeling pattern, the induced function f^* obtain the edge as
$f^*(x_j x_{j+1}) = 3j^2 - 3j + 1, 1 \le j \le 5$
$f^*(x_0 x_2) = 215$
$f^*(x_0 x_3) = 208$
$f^*(x_0 x_5) = 152$
$f^*(x_0 x_6) = 91$

Clearly, the entire 10 edge labels are distinct. Therefore, the graph *G* is cube difference graph.

Example 3.2

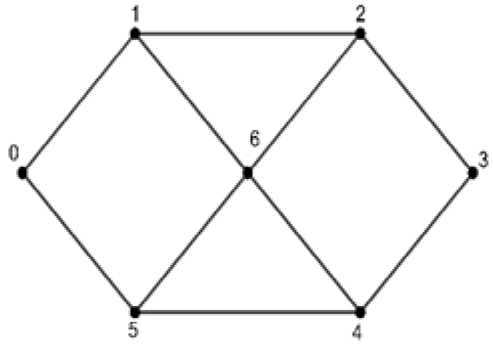

Fig 7:- The Switching of x_0 in Theta Graph is CDG

IV. CONCLUSION

In this paper, we investigated that the theta graph T_α and its associated graphs are cube difference graph.

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